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Grundlagen der Quadralektik

Vorwort

Rudolf Kaehrs wegweisender Aufsatz “Quadralectic Diamonds, Fourfoldness of Beginnings: Semiotic Studies with Toth´s 'Theory of the Night' ” darf als der Beginn einer neuen Wissenschaft betrachtet werden, die ihre vorläufig ausführlichste Darstellung in meiner „Theory of the Night“ (2016, 668 S.) gefunden hat. Informell könnte man Quadralektik als eine Art von kybernetischer Systemtheorie definieren, die so verschiedene Gebiete wie polykontexturale Logik und Ontologie, Semiotik, Ontik, qualitative und ortsfunktionale Arithemetik, Zahlentheorie, Algebra, Ordnungstheorie und Topologie durch die fundamentale Feststellung vereinigt, daß die klassische aristotelische Dichotomie L = (0, 1), die von nur zwei unvermittelten und sich reflektierenden Werten ausgeht, durch ein Quadrupel L* = (0(1), 1(0), (0)1, (1)0), eine vierstellige Relation also, ersetzt wird, die weder absolute Objekte noch absolute Subjekte, sondern objektive und subjektive Objekte und subjektive und objektive Subjekte kennt.

Zentral ist innerhalb der Quadralektik, daß sie im Gegensatz zur polykontexturalen Logik und der auf ihr basierenden, von Engelbert Kronthaler begründeten Mathematik der Qualitäten (1973/1986) die drei Grundgesetze des Denkens und in ihnen vor allem das Gesetz des Tertium non datur nicht aufhebt und damit also trotz des Ersatzes der fundamentalen Dichtomie durch eine Tetratomie aristotelisch bleibt. Indessen wird statt eines materialen dritten Wertes ein Einbettungsoperator verwendet, der als Vermittler zwischen den beiden Werten von L fungiert und dadurch L* erzeugt, d.h. in L* gibt es eine hierarchische Vermittlung, während es in L eine heterarchische Nicht-Vermittlung gibt. Man könnte somit die Quadralektik in ihrer allgemeinsten Form auch als eine formale Theorie hierarchischer Vermittlung definieren.

Die im folgenden vereinigten Aufsätze zeigen ferner den formalen Weg von der Semiotik von Peirce und Bense zu der von mir inaugurierten Ontik, also einer allgemeinen Objekttheorie, auf, welche der Semiotik als einer allgemeinen Zeichentheorie an die Seite gestellt wurde, so daß Ontik und Semiotik durch ein sehr komplexes System von Isomorphien miteinander verbunden sind.

Tucson, AZ/Basel, 26. Juli 2018 Prof. Dr. Alfred Toth

E.T.A. Hoffmanns chiastischer Karneval

> Wer ist der Ich, der aus dem Ich gebären
> Das Nicht-Ich kann, die eigne Brust zerspalten
> Und schmerzlos hoch Entzücken mag bewähren?
>
> E.T.A. Hoffmann, *Prinzessin Brambilla*, S. 80

1. E.T.A. Hoffmann als Philosoph

Nach von Matt steht Ernst Theodor Amadeus Hoffmann (1776-1822) "nicht im Ruf, ein grosser Denker zu sein" (1971, S. 1)[1]. Als Schriftsteller, Musiker und Maler war er dennoch Zeitgenosse von Novalis (1772-1801), von Chamisso (1781-1838), Ludwig Tieck (1773-1853), Kant (1724-1804), Hegel (1770-1831), Fichte (1762-1814) und Schelling (1775-1854), d.h. seine Lebenszeit fällt literarisch in die Romantik, philosophisch in die Zeit des kritischen Rationalismus und vor allem des transzedentalen Idealismus. Im folgenden beabsichtige ich nicht, eine neue Interpretation von einigen Werken Hoffmanns vorzulegen, sondern ich versuche, einige für Hoffmann typische Motive auf ihre philosophische Herkunft und heutige philosophische Einordnung hin zu

[1] Hinzu kommen die für die folgenden Argumentationen nicht unwichtigen Fehleinschätzungen von Hoffmanns Person: "Alkoholismus hat sich bei H[offmann] nicht auf rein zufällige Art entwickelt. Er war mit einem neuropathischen Erbgut schwer belastet und war selbst allezeit, trotz seiner bemerkenswerten intellektuellen Fähigkeiten, ein Anormaler, ein Psychopath. Der Alkohol wirkte auf seinen Geisteszustand in doppelter Weise: er verstärkte seinen schon vorher bestehenden Zustand der inneren Unausgeglichenheit, und er fügte noch die ihm eigentümlichen Stigmen hinzu, unter denen Wahnträume bei Tag und Nacht den ersten Platz einnahmen. Mehr noch als sein Geist wurde die physische Gesundheit H.'s angegriffen, und er erlag in fünf Monaten einer fortschreitenden Alkoholpolyneuritis. Die meisten Werke, die H. hinterlassen hat, wurden in den letzten fünfzehn Jahren seines Lebens geschrieben, d.h. in der Zeit, in der er regelmässig trank. Das erklärt, dass ihnen der Stempel des Alkohols aufgeprägt ist, und dass man überall die Spuren des Wahnsinns findet, deren Opfer er war (Lange-Eichbaum 1967, S. 391f.). Der gegenwärtige Autor kann sich hier eines Kommentars nicht enthalten: Wer – wie in diesem Aufsatz nachgewiesen werden wird – wie Hoffmann in zwei Kontexturen lebt, der muss schon deshalb auch in der "Halbwelt" leben, weil die platonische Dyas ja sowohl das Verhältnis 2:1 als auch dasjenige 1:2 einschloss (und damit die 2 bereits als polykontexturale, weil hermeneutisch relevante, Zahl auswies).

prüfen. Dabei wird sich ergeben, dass Hoffmann sehr wohl ein Philosoph war – allerdings keiner, der monokontextural-aristotelisch argumentierte, sondern einer der frühesten Pioniere einer polykontextural-nichtaristotelischen Philosophiekonzeption. Wie aus der Arbeit von Hohmann über Kierkegaard (Hohmann 1999) und meiner eigenen zu Panizza (Toth 2006) hervorgeht, sind die drei wichtigsten Kriterien für Texte, welche polykontexturales Gedankengut vermitteln:

1. Die Aufhebung der Grenze zwischen Subjekt und Objekt (Kap. 2.)
2. Das Auftreten von Reflexionsresten (Kap. 3)
3. Die Aufhebung der Individualität von Personen (Kap. 4)

Diese drei Kriterien bedingen sich gegenseitig insofern, als Kriterium 1 erfüllt sein muss, bevor Kriterium 2 erfüllt sein kann, und ohne die Kriterien 1 und 2 kann auch das Kriterium 3 nicht erfüllt sein.[2]

2. Die Aufhebung der Grenze zwischen Subjekt und Objekt

Zwischen Subjekt und Objekt, Zeichen und Bezeichnetem, Ich und Du, Leben und Tod, usw. verläuft in der klassisch-zweiwertigen Logik eine Kontexturgrenze, die als unüberschreitbar bzw., einmal überschritten, als irreversibel betrachtet wird. Solche Kontexturüberschreitungen gehören geradezu zu der "in jenem Serapionischen Prinzip endültig fixierte[n] Erkenntnis von der wechselseitigen Spiegelung der inneren und der äusseren Welt" (Stegmann 1976, S. 67); entsprechend gehört der Topos des bei Hoffmann immer wieder erweckten Traumbildes ausdrücklich "beiden Welten" an (Stegmann 1976, S. 67). Über Hoffmanns Weltbild heisst es später im Hegelschen Sinne: "Es ist ein dialektisches Zugleich" (Stegmann 1976, S. 68). Sehr modern im Sinne der von Gotthard Günther (1900-1984) geschaffenen Polykontexturalitätstheorie mutet auch die folgende Feststellung an: "Die

[2] Ich verwende neben den üblichen folgende Abkürzungen: ET = Die Elixiere des Teufels; GT = Der goldne Topf; PB = Prinzessin Brambilla; ZZ = Klein Zaches, genannt Zinnober. Seltener zitierte Werke Hoffmanns werden ausgeschrieben.

Wirklichkeit als ganze ist vieldeutig und offen. Sie ist der unendliche Kreislauf vom Ich zur Welt und von der Welt zum Ich” Stegmann 1976, S. 69).

Das Heraustreten aus dem Spiegel ist eine der Möglichkeiten, die Überschreitung der Kontexturgrenze zwischen Diesseits und Jenseits bildhaft zu machen: “Die drei goldgrünen Schlänglein tanzten und hüpften. Und wenn die schlanken, in tausend Funken blitzenden Leiber sich berührten, da erklangen herrliche Akkorde wie Kristallglocken, und die mittelste streckte wie voll Sehnsucht und Verlangen das Köpfchen zum Spiegel heraus” (GT, S. 217). “ ‘Mirakel, Mirakel!’ schrie das Volk immerfort, ‘seht ihr wohl den alten Mann im violetten Mantel? – Der ist aus dem Bilde des Hochaltars herabgestiegen” (ET, S. 581). “An den Anselmus musste sie [Veronika Paulmann] denken, und als sie immer fester und fester den Gedanken auf ihn richtete, da lächelte er ihr freundlich aus dem Spiegel entgegen wie ein lebhaftes Miniaturporträt. Aber bald war es ihr, als sähe sie nicht mehr das Bild – nein, sondern den Studenten Anselmus selbst leibhaftig” (GT, S. 237f.).

Auch die Loslösung des Spiegelbildes von seinem Träger folgt aus der Aufhebung der Subjekt-Objekt-Dichotomie: “ ‘Lass mir dein Spiegelbild, du innig Geliebter, es soll mein und bei mir bleiben immerdar’. – ‘Giulietta’, rief Erasmus ganz verwundert, ‘was meinst du denn? – Mein Spiegelbild?’ [...]. Da rief Erasmus, wahnsinnig vor tötendem Liebesschmerz: ‘Muss ich denn fort von dir? – Muss ich fort, so soll mein Spiegelbild dein bleiben auf ewig und immerdar. Keine Macht – der Teufel soll es dir nicht entreissen, bis du mich selbst hast mit Seele und Leib’. Giuliettas Küsse brannten wie Feuer auf seinem Munde, als er dies gesprochen, dann liess sie ihn los und streckte sehnsuchtsvoll die Arme aus nach dem Spiegel. Erasmus sah, wie sein Bild unabhängig von seinen Bewegungen hervortrat, wie es in Giuliettas Arme glitt, wie es mit ihr im seltsamen Duft verschwand” (Die Abenteuer der Silvesternacht, S. 284). Aus einem Dialog zwischen dem Teufel und Peter Schlemihl in dem gleichnamigen Werk Adelbert von Chamissos erfahren wir: “Er zog sogleich meinen Schatten aus seiner Tasche, und ihn mit einem geschickten Wurf auf die Heide entfaltend, breitete er ihn auf der Sonnenseite zu seinen Füssen aus, so, dass er zwischen den beiden ihm aufwartenden

Schatten, dem meinen und dem seinen, daher ging; denn meiner musste ihm gleichfalls gehorchen und nach allen seinen Bewegungen sich richten und bequemen" (von Chamisso, Bd. II, S. 322).

Die Urvorstellung der Aufhebung der Grenze zwischen Zeichen und Objekt scheint das Pygmalion-Motiv zu sein: Der kyprische König Pygmalion schafft sich selbst eine Statue einer Frau, welches Aphrodite lebendig werden lässt. Sie heiraten und haben eine Tochter; vgl. Ovid, Metamorphosen X 250-252: "Virginis est verae facies, quam vivere credas, / Et, si non obstet reverentia, velle moveri; / Ars adeo latet arte sua". Hier übersetzt die Budé-Ausgabe falsch: "tant l'art se dissimule à force d'art", gemeint ist natürlich nichts anderes als die Aufhebung der Kontexturgrenze. 281ff.: "visa tepere est. / Admovet os iterum, manibus quoque pectora temptat; / Temptatum mollescit ebur positoque rigore / Subsidit digitis ceditque [...]. / Rursus amans rursusque manu sua vota retractat; / Corpus erat; saliunt temptatae pollicae venae [...] / Sensit et erubuit timidumque ad lumina lumen / Attollens pariter cum caelo vidit amantem". Bömer (1980, S. 93) vermerkt in seinem Kommentar, die Pygmalion-Geschichte sei "eine der wenigen Metamorphosen, in denen nicht, wie üblich, der Wandel einer menschlichen Gestalt in ein lebloses Wesen, sondern das genaue Gegenteil Gegenstand der Erzählung ist". Auch Hoffmann hat diesen Topos in die ET eingebaut: "[Francesko] heulte vor wahnsinniger Begier, er gedachte des heidnischen Bildhauers Pygmalion, dessen Geschichte er gemalt, und flehte so wie er zur Frau Venus, dass sie seinem Bilde Leben einhauchen möge. Bald war es ihm auch, als finge das Bild an sich zu regen, doch als er es in seine Arme fassen wollte, sah er wohl, dass es tote Leinwand geblieben. Dann zerraufte er sein Haar und gebärdete sich wie einer, der von dem Satan besessen. Schon zwei Tage und zwei Nächte hatte es Francesko so getrieben; am dritten Tag, als er wie eine erstarrte Bildsäule vor dem Bilde stand, ging die Tür seines Gemachs auf, und es rauschte hinter ihm wie mit weiblichen Gewändern. Er drehte sich um und erblickte ein Weib, das er für das Original seines Bildes erkannte" (ET, S. 537).

Geradezu das Leitmotiv schlechthin ist aber die Durchstossung der Kontexturgrenze zwischen Ich und Du in Hoffmanns Erzählung "Klein Zaches, genannt Zinnober". Ich habe insgesamt dreizehn Fälle gezählt, wobei im folgenden nur auf drei besonders charakteristische hinzuweisen ist: "Balthasar griff herab nach dem Kleinen, ihm aufzuhelfen, und berührte dabei unversehens sein Haar. Da stiess der Kleine einen gellenden Schrei aus, dass es im ganzen Saal widerhallte und die Gäste erschrocken auffuhren von ihren Sitzen. Man umringte den Balthasar und fragte durcheinander, warum er denn um des Himmels willen so entsetzlich geschrien" (ZZ, S. 310). Obwohl also Klein Zaches schreit, wird der Schrei dem Balthasar angelastet. Doch es kommt noch schöner: "Balthasar glaubte, dass der rechte Augenblick gekommen, mit seinem Gedicht von der Liebe der Nachtigall zur Purpurrose hervorzurücken [...]. Sein eignes Werk, das in der Tat aus wahrhaftem Dichtergemüt mit voller Kraft, mit regem Leben hervorgeströmt, begeisterte ihn mehr und mehr. Sein Vortrag, immer leidenschaftlicher steigernd, verriet die innere Glut des liebenden Herzens. Er bebte vor Entzücken, als leise Seufzer – manches leise Ach – der Frauen, mancher Ausruf der Männer: 'Herrlich – vortrefflich, göttlich!' ihn überzeugten, dass sein Gedicht alle hinriss. Endlich hatte er geendet. Da riefen alle: 'Welch ein Gedicht! – Welche Gedanken – welche Phantasie, was für schöne Verse – welcher Wohlklang – Dank – Dank Ihnen, bester Herr Zinnober, für den göttlichen Genuss" (ZZ, S. 311ff.).

Doch Hoffmann begnügt sich nicht mit dem simplen Austausch eines Subjektes durch ein Objekt bzw. umgekehrt, wie es etwa Oscar Wilde in seinem "Bildnis des Dorian Gray" oder Edgar Allan Poe im "Oval Portrait" getan hatten: Im folgenden Fall ist Mosch Terpin sogar Subjekt und Objekt zugleich: "Als sie eintraten, stand der Professor Mosch Terpin allein in der Mitte, die Instrumente noch in der Hand, womit er irgendein physikalisches Experiment gemacht, starres Staunen im Gesicht. Die ganze Gesellschaft hatte sich um den kleinen Zinnober gesammelt, der, den Stock untergestemmt, auf den Fussspitzen dastand und mit stolzem Blick den Beifall einnahm, der ihm von allen Seiten zuströmte. Man wandte sich wieder zum Professor, der ein anderes sehr artiges Kunststückchen machte. Kaum war er fertig, als wiederum alle, den Kleinen umringend, riefen: 'Herrlich – vortrefflich, lieber Herr Zinnober!'. – Endlich sprang auch Mosch Terpin zu dem Kleinen hin und rief zehnmal stärker als die übrigen: 'Herrlich – vortrefflich, lieber Herr Zinnober!'" (ZZ, S. 313f.). (Wie alle angeführten und auch die hier unterdrückten Beispiele zeigen, befindet sich von allen Partizipanten des ZZ offenbar einzig Balthasar in der monokontexturalen Welt. Er dient quasi als “Verbindungsmann” zum ebenfalls in der Monokontexturalität lebenden Lesenden.)

Im Zusammenhang mit der Durchbrechung der Subjekt-Objekt-Dichotomie entdeckt man immer wieder, dass Kontexturgrenzen mitten durch unsere vermeintlich monokontexturale Wirklichkeit verlaufen. Das bekannteste Beispiel der Weltliteratur steht in Lewis Carroll’s “Through the Looking-Glass” und wurde von Günther wie folgt kommentiert: “No matter how loud the discourse between Alice and the Tweedle brothers may get, it will not wake the Red King, because the existence or mode of Reality of Alice and the Twins is discontextural with the physical body of the King who is – or seems at least – to be lying in front of them in the grass” (1976-80, II, S. 253). Bei Hoffmann lesen wir etwa: “Ungeachtet des weiten Weges bis in die einsame Strasse, in der sich das uralte Haus des Archivarius Lindhorst befand, war der Student Anselmus vor zwölf Uhr an der Haustür. Da stand er und schaute den grossen schönen, bronzenen Türklopfer an; aber als er nun auf den letzten, die Luft mit mächtigem Klange durchbrechenden Schlag der Turmuhr an der Kreuzkirche den Türklopfer ergreifen wollte, da verzog sich das metallene Gesicht im ekelhaften Spiel blauglühender Lichtblicke zum grinsenden Lächeln. Ach! Es war ja das Apfelweib vom Schwarzen Tor! [...] Die Klingelschnur senkte sich hinab und wurde zur weissen, durchsichtigen Riesenschlange; sie umwand und drückte ihn, fester und fester ihr Gewinde schnürend, zusammen, dass die mürben zermalmten Glieder knackend zerbröckelten und sein Blut aus den Adern spritzte, eindringend in den durchsichtigen Leib der Schlange und ihn rot färbend (GT, S. 208). Im Gegensatz zu Alice kommt Anselmus aber der polykontexturalen Wahrheit auf den Grund: “ ‘Er kann aber auch selbst in Person davongeflogen sein, der Herr Archivarius Lindhorst’, sprach der Student Anselmus zu sich selbst, ‘denn ich sehe und fühle nun wohl, dass alle die fremden Gestalten aus einer fernen wundervollen Welt, die ich sonst nur in ganz besonders merkwürdigen Träumen schaute, jetzt in ein waches, reges Leben geschritten sind und ihr Spiel mit mir treiben” (GT, S. 218f.).

Polykontexturale Welten können sich verändern; sie sind ja nicht wie die eine (vermeintlich) monokontexturale Welt unveränderlich. Diese Einsicht kommt bei Hoffmann sehr gut zum Ausdruck, als Anselmus den Garten des Archivarius Lindhorst betritt: "Anselmus schritt gestrost hinter dem Archivarius her; sie kamen aus dem Korridor in einen Saal oder vielmehr in ein herrliches Gewächshaus, denn von beiden Seiten bis an die Decke hinauf standen allerlei seltene wunderbare Blumen, ja grosse Bäume mit sonderbar gestalteten Blättern und Blüten. Ein magisches blendendes Licht verbreitete sich überall, ohne dass man bemerken konnte, wo es herkam, da durchaus kein Fenster zu sehen war. Sowie der Student Anselmus in die Büsche und Bäume hineinblickte, schienen lange Gänge sich in weite Ferne auszudehnen. – Im tiefen Dunkel dicker Zypressenstauden schimmerten Marmorbecken, aus denen sich wunderliche Figuren erhoben, Kristallstrahlen hervorspritzend, die plätschernd niederfielen in leuchtende Lichtkelche [...] (GT, S. 227f.). Dann aber später: "Als er nun mittags durch den Garten des Archivarius Lindhorst ging, konnte er sich nicht genug wundern, wie ihm das alles sonst so seltsam und wundervoll habe vorkommen können. Er sah nichts als gewöhnliche Scherbenpflanzen, allerlei Geranien, Myrtenstöcke u. dgl. Statt der glänzenden bunten Vögel, die ihn sonst geneckt, flatterten nur einige Sperlinge hin und her, die ein unverständliches unangenehmes Geschrei erhoben, als sie des Anselmus gewahr wurden. Das blaue Zimmer kam ihm auch ganz anders vor, und er begriff nicht, wie ihm das grelle Blau und die unnatürlichen, goldnen Stämme der Palmbäume mit den unförmigen, blinkenden Blättern nur einen Augenblick hatten gefallen können" (GT, S. 251).

Polykontexturale Welten sind ferner eindeutig-mehrmöglich bzw. multi-ordinal im Sinne Korzybskis (vgl. Kronthaler 1986, S. 60). Als Fabian und Balthasar den Garten des Doktors Prosper Alanus betreten, lesen wir: "Fabian bemerkte zwei Frösche von ungewöhnlicher Grösse, die schon von dem Gartentor an zu beiden Seiten der Wandelnden mitgehüpft waren. 'Schöner Park', rief Fabian, 'in dem es solch Ungeziefer gibt!' und bückte sich nieder, um einen kleinen Stein aufzuheben, mit dem er nach den lustigen Fröschen zu werfen gedachte. Beide sprangen ins Gebüsch und guckten ihn mit glänzenden, menschlichen Augen an. 'Wartet, wartet!' rief Fabian, zielte nach dem einen und warf. In dem Augenblick quäkte aber ein kleines hässliches Weib, das am Wege sass: 'Grobian! Schmeiss Er nicht auf ehrliche Leute, die hier im Garten mit saurer Arbeit ihr bisschen Brot verdienen müssen'" (ZZ, S. 325). Ob Frosch oder Mensch, ob Einhorn oder Pferd – in polykontexturalen Welten sind die Zuordnungen zwischen Objekten und Funktionen, zwischen Personen und Erscheinungen, zwar nicht eindeutig, aber auch nicht willkürlich, sondern eben eindeutig-mehrmöglich.

Es ist eben die Aufklärung, der Rationalismus, der – in getreuer Weiterführung des aristotelischen Konzepts der reinen Quantität gegenüber der qualitativ-quantitativen bzw. quantitativ-qualitativen Konzeption Platons, unter dem verblendenden Namen der Illumination das Organische ins Anorganische, das Prozessuale ins Statische, das Eindeutig-Mehrmögliche ins Eineindeutige, kurz: das Leben in den Tod gefüht hat: "In der unglücklichen Zeit, wenn die Sprache der Natur dem entarteten Geschlecht der Menschen nicht mehr verständlich sein, wenn die Elementargeister, in ihre Regionen gebannt, nur aus weiter Ferne in dumpfen Anklängen zu den Menschen sprechen werden, wenn, dem harmonischen Kreise entrückt, nur ein unendliches Sehnen ihm die dunkle Kunde von dem wundervollen Reiche geben wird, das er sonst bewohnen durfte, als noch Glaube und Liebe in seinem Gemüte wohnten [...] (GT, S. 243). Novalis ging sogar noch weiter und fragte: "Könnte die Natur nicht über den Anblick Gottes Stein geworden seyn? Oder vor Schrecken über die Ankunft des Menschen?" (ed. Samuel 1978, S. 224). Anders als Novalis, für den galt: "Das höchste Leben ist Mathematik". "Echte Mathematik ist das eigentliche Element des Magiers", usw. (vgl. Hamburger 1966, S. 16), machte aber Hoffmann den Schritt vom transzendentalen Idealismus zu einem "magischen Realismus" nicht mit, denn nach Hoffmann lässt sich diese Welt "durch Zählen, Messen und Wiegen allein nicht in ihrer Ganzheit erklären. Genau das ist aber der offenbar bis heute unausrottbare Aberglaube der Aufklärung. Romantik heisst für Hoffmann, der Welt den Zauber zu belassen [...]. Hoffmann erkennt, dass die Früchte der Aufklärung abgeerntet sind und erklärt die Aufklärung daher zum Mittelalter seiner Gegenwart und die Vernunft zum schwarzen Tod der Phantasie" (Driesen 1997, S. 87f.).

Lewis Carroll brachte es fertig, mit dem “Lied vom Weissen Ritter” ein Gedicht zu schreiben, das aus Wörtern bzw. Abschnitten besteht, die im Satz- bzw. Textzusammenhang betrachtet multi-ordinale Zeichen sind. Bei ihm wird offenbar eine polykontexturale Semiotik vorausgesetzt, in der Zeichen (“Name” bzw. “heissen”) und Objekt (“Lied” bzw. “sein”) nicht länger durch Kontexturgrenzen voneinander geschieden sind, so dass sich insgesamt vier Möglichkeiten der Bezeichnung ergeben: “ ‘Der Name des Liedes heisst ‘Heringsköpfe’. – ‘Ach! Das ist wirklich sein Name?’ fragte Alice, damit es nicht so aussähe, als wäre ihr das gleichgültig. – ‘Nein, du hast mich falsch verstanden’, sagte der Ritter etwas unmutig. ‘So *heisst* sein Name nur. Der Name selbst ist ‘Der uralte Mann’.’ – ‘Dann hätte ich also sagen sollen: ‘So heisst das Lied also?’ verbesserte sich Alice. – ‘Aber nein doch, das ist wieder etwas anderes. Das *Lied* heisst ‘Trachten und Streben’; aber freilich *heisst* es nur so.’ – ‘Ja, aber welches Lied *ist* es denn?’ fragte Alice, die sich nun gar nicht mehr auskannte. - ‘Das wollte ich dir eben sagen’, erwiderte der Ritter. ‘Es ist das Lied ‘Hoch droben auf der Pforten’”. Des Weissen Ritters Erläuterungen lassen sich also wie folgt gliedern:

	heissen	sein
Name	Heringsköpfe	Der uralte Mann
Lied	Trachten und Streben	Hoch droben auf der Pforten

Hier wird also sowohl von der Unterscheidung zwischen Name vs. Lied als auch von derjenigen zwischen heissen und sein die monokontexturale Zeichen-Objekt- und das heisst die Subjekt-Objekt-Relation proömiell durchbrochen. Wir werden im 4. Kapitel anlässlich der Besprechung des Chiasmus im Zusammenhange mit der Auflösung der Identität bzw. Individualität von Personen darauf zurückkommen: “Er sprache: ‘Ich pflücke Heringsköpf’ / Auf Äckern, Flur und Raine / Und mache daraus Hosenknöpf’ / Beim trauten Lampenscheine; / Und dafür gibt man mir nicht Gold / Und auch nicht Silber teuer, / Zwei Heller, wenn ihr geben wollt, / Dann sind drei Dutzend Euer. / Auch grab ich manchmal nach Kakao / Und fisch im See die Zeder / Und sammel auf der grünen Au / Für Kutschen Speichenräder. / Auf diese Weis', so zwinkert er, / 'Bin ich zu Geld gekommen / Und leer dies Glas auf Euch, mein Herr, / Wohl mög es Euch bekommen!'" (Carroll 1974, S. 118ff.).

Konersmann hat in seiner schönen Arbeit über René Magritte sogar gesagt: "Zwischen den Bildern und den Dingen klafft eine Lücke, die zu schliessen auch die Kunst nicht vermag. Sie bietet jedoch Raum für Gestaltungsmöglichkeiten, in denen die Differenz zwischen der Welt und ihrem Abbild, oder sagen wir genauer: zwischen der Welt des Bildes und der Welt der Dinge sich variantenreich erörtern lässt. Hier nistet das Mysterium, von dem Magritte immer wieder spricht" (1991b, S. 17). Dieses "Mysterium" erlebt etwa auch ein Kunsterzieher, der zwanzig Schüler dieselbe Rose abzeichnen lässt – er wird am Ende zwanzig verschiedene Rosen-Zeichnungen haben, denen doch etwas Invariantes gemein ist. Theoretisch ausgedrückt: Den n verschiedenen Zeichen des einen Rosen-Objektes korrespondieren die n-1 ontologischen und logischen Standpunkte einer n-wertigen polykontexturalen Logik mit 1 Objekt und n-1 Subjekten.

3. Das Auftreten von Reflexionsresten

Reflexionsreste, die nach Günther als "Obdachlosenasyle" für die aus dem zweiwertigen Denken ausgegliederten Denkreste fungieren, treten in einer zweiwertigen Logik deshalb auf, weil die Negation das blosse Spiegelbild der Position ist und diese daher bloss kopiert. Sobald wir aber eine Logik haben, in der Platz ist für mehr als ein Subjekt, entsteht eine Unbalanciertheit zwischen Subjekt und Objekt, die in Form von sich unklassisch gebärdenden Objekten zum Ausdruck kommt, wie etwa Drachen, Hexen und Meerjungfrauen in den Volksüberlieferungen. So sagt der Berater des Fürsten Paphnutius: "Nicht alle Feen, gnädiger Herr, wollen wir fortschicken nach Dschinnistan, sondern einige im Lande behalten" (ZZ, S. 293). Genauso wie der Volksglaube in Märchen, Sage und Legende neben unserem rationalen Weltbild nebenher läuft, genauso wie neben der Astronomie noch immer die Astrologie und neben der Chemie noch immer die Alchemie weiterleben, erkennt auch Balthasar: "Wahr, dass Fürst Paphnutius die Aufklärung einführte zu Muss und Frommen seines Volkes, seiner Nachkommenschaft, aber manches Wunderbare, Unbegreifliche ist doch noch zurückgeblieben” (ZZ, S. 319). “Die Wunder sind geblieben, denn wenn wir selbst das Wunderbarste, von dem wir täglich umgeben, deshalb nicht mehr so nennen wollen, weil wir einer Reihe von Erscheinungen die Regel der zyklischen Wiederkehr abgelauert haben, so fährt doch durch jenen Kreis ein Phänomen, das all unsere Klugheit zuschanden macht und an das wir, weil wir es nicht zu erfassen vermögen, in stumpfsinniger Verstocktheit nicht glauben. Hartnäckig leugnen wir dem innern Auge deshalb die Erscheinung ab, weil sie zu durchsichtig war, um sich auf der rauhen Fläche des äusseren Auges abzuspiegeln. – Jenen seltsamen Maler rechne ich zu den ausserordentlichen Erscheinungen, die jeder erlauerten Regel spotten; ich bin zweifelhaft, ob seine körperliche Erscheinung das ist, was wir wahr nennen” (ET, S. 530).

Dass alles, was jemand in der Gegenwart von Klein Zaches tut, diesem; was Klein Zaches aber macht, einem andern angelastet wird, für die Aufhebung oder Permeabilisierung der Kontexturgrenze zwischen Ich und Du also, dafür ist ja gerade ein solches prä-rationalistisches Relikt verantwortlich: Die Fee Rosabelverde, welche offiziell das “säkularisierte” Stiftsfräulein von Rosengrünschön ist (ZZ, S. 291). Also muss nach Hoffmanns Auffassung die vorkartesische Zeit die polykontexturale Zeit gewesen sein (in Wirklichkeit beginnt die monokontexturale Zeit bereits mit der Metaphysik des Aristoteles), denn bei Descartes lesen wir klipp und klar: “Nun bemerke ich hier erstlich, dass ein grosser Unterschied zwischen Geist und Körper insofern vorhanden ist, als der Körper seiner Natur nach stets teilbar, der Geist hingegen durchaus unteilbar ist” (1994, S. 74). Unteilbar ist der Geist nach der irrigen Auffassung des Cartesius einzig deshalb, weil es in einer zweiwertigen Logik zwar unendlich viele Objekte gibt, aber Platz nur für ein einziges Subjekt hat, für das meistens “Ich” eingesetzt wird. Descartes berühmtes (wenigstens in dieser Gestalt kolportiertes) “Cogito, ergo sum” wird so auch verständlich, insofern derjenige, welcher denkt, trivialerweise deshalb mit dem Ich identisch sein muss, weil die zweiwertige Logik gar keinen dritten Wert für ein Du, Er, Wir, usw. hat, dessen Existenz durch das Denken bewiesen werden könnte.

Als Antizipation von Reflexionsresten finden wir ein besonders eindrückliches Beispiel in Oskar Panizzas "Liebeskonzil": Der Teufel, von Gott, Maria und ihrem Sohn mit der Aufgabe betraut, die Menschheit für ihre sexuellen Ausschweifungen mit einem besonderen Gift zu bestrafen, zieht sich in seine Wohnung zurück, versucht nachzudenken, kommt aber zu keinem Resultat und schläft darüber ein. Während er noch schläft, wechselt das Bühnenbild im Hintergrund: "Man erblick ein ungeheures Totenfeld, auf dem eine schier unfassbare Zahl, wie es scheint lauter Weiber, in Leibesgestalt, mit fahlen Gewändern, die einen hockend, die anderen hingestreckt, teils die Arme aufgestützt, teils das Gesicht in den Armfalten vergraben, wie schlafend dortliegen". Plötzlich erwacht der Teufel: "Ah! – Ihr seid mir vorausgeeilt, Gedanken!" Er betrachtet lange mit Entzücken die Szene: "Ihr habt euch verwirklicht, meine guten Gedanken!" (Panizza 1991, S. 75f.).

In einer polykontexturalen Logik, welche n Werte besitzt, gibt es aber, wie bereits gesagt, Platz für n-1 Subjekte. Schon im vergleichsweise trivialen Fall einer dreiwertigen Logik lässt sich unterscheiden zwischen einem subjektiven Subjekt, einem objektiven Subjekt und einem Objekt: "Das Subjekt begegnet sich im Modus der Differenz, und nun stellt sich die Frage nach der Verbindung, die die geforderte Einheit des Subjekts mit dieser Differenz von Subjekt und Objekt versöhnt, die es doch zugleich auch übergreift. Die Darstellung dieses komplizierten Zusammenhangs stellt hohe Anforderungen an die lebendige Sprache. Sie muss das prekäre Selbstverständnis in seiner besonderen Struktur fasslich werden lassen. Darzustellen ist eine Relation, in der das Subjekt sich als sein Gegenstand reflektiert, der sich umgekehrt in ihm reflektiert, so dass er, der es selber ist, ihm, und in eins damit es sich, in dieser seiner puren Gegenständlichkeit sofort entgeht, denn das Subjekt ist immer auch schon mehr als das, als was es sich erblickt, nämlich es selbst. Verlangt wird also ein Modus uneigentlichen Sprechens" (Konersmann 1991a, S. 25). Damit hat Konersmann – offenbar unbeeinflusst durch die Polykontexturalitätstheorie – die Proömialrelation vorweggenommen, denn ein Subjekt, das sich selbst als sein Gegenstand reflektiert, ist gänzlich nicht-aristotelisch und führt in der klassisch-monokontexturalen Logik zu Paradoxien qua Selbstreferenz.

Doch ganz zentral wird die Unterscheidung zwischen subjektivem und objektivem Subjekt bei der Doppelgänger-Problematik. So sagt Medardus: "Mein eignes Ich, zum grausamen Spiel eines launenhaften Zufalls geworden und in fremdartige Gestalten zerfliessend, schwamm ohne Halt wie in einem Meer all der Ereignisse, die wie tobende Wellen auf mich hineinbrausten [...]. Aber das Verhältnis mit der Baronesse, welches Viktorin unterhält, kommt auf mein Haupt, denn ich bin selbst Viktorin. Ich bin das, was ich scheine, und scheine das nicht, was ich bin, mir selbst ein unerklärlich Rätsel, bin ich entzweit mit meinem Ich!" (ET, S. 283). "Es ist das eigne wunderbare Heraustreten aus sich selbst, das die Anschauung des eignen Ichs vom andern Standpunkte gestattet, welches dann als ein sich dem höheren Willen schmiegendes Mittel erscheint, *dem* Zweck zu dienen, den er sich als den höchsten, im Leben zu erringenden gesetzt" (ET, S. 387). Für Panizza liegt der Reiz des menschlichen Lebens gerade darin, "dass unser Willens-Impuls das Resultat der gegensätzlichsten Motive und Neigungen ist, heute so, morgen so, und das Zusehen des 'Ich' bei diesem Kampfe ist ja eben das, was wir Leben nennen" (1981, S. 63).

Doch auch hier geht Hoffmann noch einen entscheidenden Schritt weiter, wenn er das objektive Subjekt – wieder unter Durchbrechung der Kontexturgrenze – zum Objekt werden lässt: " 'Du bist nicht ich, du bist der Teufel!', schrie ich auf und griff wie mit Krallen dem bedrohlichen Gespenst ins Gesicht, aber es war, als bohrten meine Finger sich in die Augen wie in tiefe Höhlen, und die Gestalt lachte von neuem auf in schneidendem Ton. In dem Augenblick erwachte ich, wie von einem plötzlichen Ruck emporgeschüttelt. Aber das Gelächter dauerte fort im Zimmer. Ich fuhr in die Höhe, der Morgen brach in lichten Strahlen durch das Fenster, und ich sah vor dem Tisch, den Rücken mir zugewandt, eine Gestalt im Kapuzinerhabit stehen. – Ich erstarrte vor Schreck, der grauenhafte Traum trat ins Leben" (ET, S. 423).

Und wie soll man das folgende, im rätoromanischen Dialekt des Unterengadins geschriebene Gedicht "La Mort" ("Der Tod") von Andri Peer (1921-1985) verstehen (deutsche Übersetzung vom gegenwärtigen Autor):

Cur ch'eu'm sdasdet,
staiv'la tschantada
al pè da meis let,
la grifla dad öss
süilla litera.

Eu n'ha fat finta da durmir.
Cur ch'eu divrit igl ögls,
d'eir'la davent.

Als ich erwachte,
stand er da,
am Fuss meines Bettes,
die Klaue aus Knochen
auf dem Bettgestell.

Ich tat so, als schliefe ich.
Als ich die Augen öffnete,
war er weg.

Während der grause Kapuziner-Doppelgänger des Medardus-Viktorin aus dem Traum, wo er noch blosses objektives Subjekt (qua Doppelgängertum) ist, über die Kontexturgrenze ins reale Reale als Objekt hinübertirtt, gehe ich davon aus, dass das “Ich” im Gedicht von Peer die Augen erst dann öffnet, wenn es die Kontexturgrenze aus dem Diesseits in Richtung Jenseits bereits überschritten hat, also erst in der der Ontik korrespondierenden Meontik.

Dass also auch Reflexionsreste proömiell-chiastische Relationen darstellen, hat bereits Lewis Carroll erkannt, obwohl er sich in dem folgenden einschlägigen Zitat gleichzeitig darüber lustig macht: “ ‘Ich bin ganz deiner Meinung’, sagte die Herzogin, ‘und die Moral davon ist: Scheine, was du bist, und sei, was du scheinst’ – oder einfacher ausgedrückt: ‘Sei niemals ununterschieden von dem, als was du jenen in dem, was du wärst oder hättest sein können, dadurch erscheinen könntest, dass du unterschieden von dem wärst, was jenen so erscheinen könnte, als seiest du anders!’” (Carroll 1981, S. 93).

4. Die Aufhebung der Individualität

Während in einer zweiwertig-aristotelischen Logik die Individualität eines Menschen durch den Tod als Negation seiner Existenz aufgehoben wird, ist es zumindest unklar, ob dies auch in einer mehrwertig-nichtaristotelischen Logik gilt; so besitzt ja bereits eine dreiwertige Logik drei Negationen. Daher ist “erst noch zu untersuchen, ob der Fortfall der ersten Identität im Tode wirklich die ichhafte Identität des Individuums endgültig auflöst" (Günther 1976-80, III, S. 2, 11f.). Die Aufhebung der Individualität kann so in einer mehrwertigen Logik zur Existenz von Parallel-Personen, Doppelgängern, Figuranten, seltsamen Spiegelbildern, Personen ohne Schatten, usw. führen: "Hoffmann vermag das Leitmotiv des Doppelgängers ins Unendliche zu varrieren, von Signor Formica, der dank einem ganzen Apparat von Verkleidungen und theatralischen Machenschaften mit Salvator Rosa zusammen nur ein Einziger ist, bis zu Meister Floh, in dem die doppelte Natur eines einzigen Wesens sich in der Gestalt von zwei verschiedenen Personen manifestiert. Sofern es sich nicht um die Spaltung in drei Personen handelt, von denen jede doch ein Ganzes bleibt, wie in dem Fall von Aline, Dörtje Elverdink und der Prinzessin Gamaheh. Hier hat Hoffmann meisterhaft auszudrücken und zu suggerieren verstanden, dass es sich nicht um zeitlich sich folgende Verwandlungen, sondern um simultane Manifestationen handelt; und darauf beruht gerade das Rätsel, das der bis ins tiefste Innere verstörte Leser wahrnimmt. Wo sind sind die anderen Doppelgänger, was tun sie, wenn sie nicht gerade vor dem Leser agieren?" (Wittkop-Ménardeau 1997, S. 40).

Im Falle der Aufhebung der Individualität bzw. der Identität von Personen kommen wir nun nicht mehr darum herum, die proömiell-chiastische Struktur des Hoffmannschen Werkes aufzuzeigen, auf die bereits in den vorangehenden Kapiteln jeweils kurz hingewiesen worden war. Am nächsten – doch offenbar ohne die Polykontexturalitätstheorie zu kennen – kommt der Wahrheit Detlef Kremer: "Viktorin und Medardus sind zwei unterschiedliche Romanfiguren, die dennoch über ihre zahlreichen Beziehungen zu den gegensätzlichen Teilen einer einzigen Person zusammenlaufen. Ihre Kreuzsymmetrie regelt eine doppelte Perspektivführung, die sich gegenseitig bedingt und ausschliesst. Immer wenn Medardus den Doppelgänger Viktorin als Phantom seines Wahns verstehen will, dann wird er mit einer konkreten eigenständigen Figur konfrontiert, wenn er ihm hingegen Realität zubilligt, dann behauptet das Phantom Viktorin seine Identität mit Medardus und rückt letzteren in die Position des Phantasmas. Beide haben sie Recht und beide täuschen sich, wenn sie die Balance von Identität und Differenz einebnen wollen" (1993, S. 234).

“Da rührte es sich unter meinem Fuss, ich schritt weiter und sah, wie an der Stelle, wo ich gestanden, sich ein Stein des Pflasters losbröckelte. Ich erfasste ihn und hob ihn mit leichter Mühe vollends heraus. Ein düstrer Schein brach durch die Öffnung, ein nackter Arm mit einem blinkenden Messer in der Hand streckte sich mir entgegen. Von tiefem Entsetzen durchschauert, bebte ich zurück. Da stammelte es von unten heraus: ‘Brü-der-lein! Brü-der-lein, Me-dar-dus ist da-da, herauf ... nimmt, nimm! ... brich ... brich in den Wa-Wald ... in den Wald!’ – Schnell dachte ich Flucht und Rettung; alles Grauen überwunden, ergriff ich das Messer, das die Hand mir willig liess und fing an, den Mörtel zwischen den Steinen des Fussbodens emsig wegzubrechen. Der, der unten war, drückte wacker herauf. Vier, fünf Steine lagen zur Seite weggeschleudert, da erhob sich plötzlich ein nackter Mensch bis an die Hüften aus der Tiefe empor und starrte mich gepenstisch an mit des Wahnsinns grinsendem entsetzlichem Gelächter – ich erkannte mich selbst – mir vergingen die Sinne” (ET, S. 480).

Auch der – ebenfalls von der Polykontexturalitätstheorie unabhängige – Kommentar des Philosophen Safranski kommt der Wahrheit der strukturellen Logik, die Hoffmanns Texten zu Grunde liegt, ein gutes Stück näher: “Unmerklich nistet es [das ‘falsche’ Selbst, A.T.] sich zunächst in die Aktivitäten des ‘wahren’ Selbst ein und lässt sie zweideutig werden. Dann endlich setzt es sich in einer Art ‘Implosion’ gänzlich an die Stelle des zur Gegenwehr nicht mehr fähigen ‘wahren’ Selbst. Hoffmann gibt diesem Umschlag durch die Machtergreifung des Doppelgängers eine sinnfällige Darstellung. Auch die Infiltration erhält ein grelles Signal: das Teufelselixir, das Medardus langsam vergiftet. Nach der Machtergreifung des ‘falschen’ Selbst kehren sich die Rollen um: Jetzt ist es das ‘wahre’ Selbst, das sich als schlechtes Gewissen und Selbstbeobachtungsmanie in die Aktivitäten des ‘falschen’ Seins einschleicht. Der Prozess der Spaltung wird rückwärts durchlaufen: Das ‘wahre’ Selbst erobert sich wieder seine Vorrangstellung, während dem ‘falschen’ Selbst nur noch die Kraft der Anfechtung bleibt” (1984, S. 342). “Das ist die Umkehrung: Das ‘wahre’ Selbst ist zur Maske geworden, das bisher Ausgegrenzte, der Geist Viktorins, das durch Ausgrenzung zum feindlichen Prinzip gewordene Triebleben, rückt in den Mittelpunkt. Doch das ‘wahre’ Selbst ist jetzt nicht nur

Maske, es hat sich – vorerst noch ohnmächtig – auf eine Beobachtungsposition zurückgezogen. Der ‘alte’ Medardus sieht dem ‘neuen” zu und kann sich für dessen greuliche Taten nicht verantwortlich fühlen. Wenn Medardus für Augenblicke in sein altes Selbst zurückkehrt, dann ist ihm, als seien die Verbrechen von jemand anderem, eben dem Doppelgänger, verübt worden. So aber ist er am tiefsten in seinen Wahn verstrickt: Er hält sein anderes Selbst für jemand anderes als er selbst. Projiziert Medardus seine Verbrechen auf den Doppelgänger, dann verliert er das Bewusstsein der Gespaltenheit: Er versinkt im Abgrund eines fragmentierten Ichs, dem sich die anderen Ich-Fragmente als andere Personen darstellen. So paradox es klingen mag: Nur wenn sich Medardus in seiner Gespaltenheit erfährt, ist er sich nahe. Diese Nähe, diese Augenblicke der Selbstbegegnung sind schrecklich; und das Schicksal der Seele steht auf des Messers Schneide: Die Person kann völlig zerbrechen, aber sie kann auch zusammenfinden im erfahrenen und gelebten Widerspruch” (1984, S. 344). Wer je Kierkegaard – einen anderen transklassischen Denker (vgl. Hohmann 1999) – gelesen hat, erinnert sich der folgenden berühmten Definition aus der “Angst zum Tode”: “Und das Verhältnis zu sich selbst kann ein Mensch nicht loswerden, so wenig wie sein eigenes Selbst, was im übrigen ein und dasselbe ist, da ja das Selbst das Verhältnis zu sich selbst ist” (vgl. dazu Toth 1995).

In all dem ist nichts mehr zu spüren von der Ontologie, Metaphysik und Erkenntnistheorie der klassisch-zweiwertigen, monokontexturalen Logik aristotelisch-chrysippischer Prägung. Sehr richtig hat Gabrielle Witkopp-Ménardeau auch den Zusammenhang zwischen Spiegeln und Doppelgängern erkannt: “So ist auch das Leitmotiv des Spiegels, des Spiegelbildes oder seines Fehlens nur eine subtile Variation des Doppelgängermotivs” (1997, S. 40): “Es ist nun höchst fesselnd zu sehen, wie [Jacob] Böhme versucht, den Sündenfall des ersten Menschen als Spiegelschau zu deuten. Vor der Versuchung ist Adam androgyn, Mann und Weib in eins verschmolzen. In seiner Seele lebt die Jungfrau Sophia als klarer Spiegel der Gottheit. Seine Sünde besteht nach Böhme darin, dass er begehrt, statt Gott zu spielen, sich selbst im Spiegel zu betrachten. Die erste subjektivistische Ich-Spaltung ist damit vollzogen: der erste Mensch unterliegt der ‘Selbheit’ und begehrt gleich Luzifer göttliches

Vorrecht, d.h. sein eigenes Ich im Spiegel zu sehen. Denn der Fall beider entsteht dadurch, ‘dass sie das Licht des Verstandes in die Selbheit scheinen hatten, in welchem sie sich bespiegeln und beschauen konnten’ [Der Weg zu Christo. Jakob Böhme’s sämtliche Werke, hrsg. von K.W. Schiebler, Neudruck Leipzig 1922, Bd. I, S. 78]” (Langen 1940, S. 276).

Der Karneval ist es nun, welcher “die multiple Person [erlaubt]. Die Verwandlungslust, im bürgerlichen Alltag unter dem Druck eines strengen, auf Widerspruchsfreiheit angelegten Identitätsideals zumeist niedergehalten, jetzt darf sie gelebt werden” (Safranski 1984, S. 445). “Auf dem Höhepunkt des karnevalistischen Treibens begegnen sich also Giglio und Giacinta, ohne sich zu erkennen, doch sie tanzen miteinander, und dieser Tanz ist eine ekstatische Entfesselung aller Verwandlungskunst, ein wahrer Dionysios-Tanz über den Trümmern einer sonst ängstlich festgehaltenen Identität” (Safranksi 1984, S. 448). Allerdings – so ergänzt Kremer – muss vom Leser der ET die “Fähigkeit zum differenzierten Umgang mit einer mindestens dreifachen Spiegelung der Fiktion erwartet werden” (1993, S. 250). Bei der PB werden wir es, wie zu zeigen sein wird, “bloss” mit einer zweifachen Spiegelung zu tun haben, allerdings einer, die stärker chiastisch (weil absolut symmetrisch) strukturiert ist als diejenige, die den ET zugrunde liegt. Diese “Zumutung” an den Lesenden, auf die Kremer (ohne freilich dieses Wort zu gebrauchen) abhebt, basiert natürlich auf der polykontexturalen Struktur der ET, vielleicht das in dieser Hinsicht komplexeste aller Werke Hoffmanns. Vom monokontexturalen Standpunkt aus wird es daher empfunden als Schöpfung “ohne Gewissheit oder Visionen der Essenz, ohne Ordnung, aber auch ohne Kapitualtion vor der Unordnung” (Claudio Magris, cit. ap. Kremer 1993, S. 255, Anm. 146).

Ähnlich schrieb Heine in seinen “Briefen aus Berlin”: “Über Hoffmanns ‘Meister Floh’ versprach ich Ihnen in meinem Vorigen mehreres zu schreiben [...]. Das Buch hat keine Handlung, keinen grossen Mittelpunkt, keinen innern Kitt. Wenn der Buchbinder die Blätter desselben willkürlich durcheinander geschossen hätte, würde man es sicher nicht bemerkt haben [...]. Die Strenge und Bitterkeit, womit ich über diesen Roman spreche, rührt eben daher, weil ich Hoffmanns frühere Werke so sehr schätze und liebe. Sie gehören zu den merkwürdigsten, die unsere Zeit hervorgebracht. Alle tragen sie das Gepräge des Ausserordentlichen, jeden müssen die Phantasiestücke ergötzen. In den Elixieren des Teufels liegt das Furchtbarste und Entsetzlichste, das der Geist sich erdenken kann [...]. In Göttingen soll ein Student durch diesen Roman toll geworden sein. In den Nachtstücken ist das Grässlichste und Grauenvollste überboten. Der Teufel kann so teuflisches Zeug nicht schreiben [...]. Aber Prinzessin Brambilla ist eine gar köstliche Schöne, und wem diese durch ihre Wunderlichkeit nicht den Kopf schwindlicht macht, der hat gar keinen Kopf. Hoffmann ist ganz originell" (ed. Windfuhr, Bd. 6, 1973, S. 51f.).

Einer der Herausgeber Hoffmanns schrieb über die PB: "Es ist ein Karneval gigantischen Ausmasses" (Leber, in: Hoffmann 1985, Bd. II, S. 8). Kremer (1993, S. 318) übertitelt: "Ein hermeneutischer Tanz": "Auf Schritt und Tritt kreuzen sich in Hoffmanns Erzählung Beschreibungen und paradoxe Konstellationen, werden Erwartungen getäuscht und Wahrnehmungen gestört. Vom Leser erwartet sie nichts weniger, als sich ihrer Widerspruchslogik zu fügen und als Strukturprinzip des Textes anzunehmen, dass zu einem Satz leicht auch der Gegensatz, zu einem Bild eben auch ein Gegenbild gehört" (Kremer 1993, S. 318). Wenn Kremer hier treffend von einer "Widerspruchslogik" spricht, stellt sich die Frage, wem diese Hoffmannsche Logik denn widerspreche. Die Antwort dürfte klar sein: Die Hoffmannsche Logik widerspricht der klassisch-monokontexturalen Logik, und gerade die PB weist eine im folgenden zu demonstrierende chiastische Struktur auf, wie sie nur transklassisch-polykontexturalen Logiken eigen sein können.

Bevor wir zur chiastischen Struktur kommen, ist es noch wichtig, die folgende Feststellung Kremers zu berücksichtigen: "Der simulierte Tanz des Prinzen mit der Prinzessin vollzieht sich zugleich als hermeneutische Selbstreflexion" (1993, S. 321). Kremer weist ferner darauf hin, dass Luhmann in seinen "Beobachtungen der Moderne" "im Zusammenhang von Paradoxie, die aus Selbstreferenz resultiert, erstaunlicherweise auf Hoffmanns 'Prinzessin Brambilla' verweist" (1993, S. 322, Anm. 173). Luhmanns Original-Wortlaut: "Die Beobachtung derjenigen Oppositionen, die das re-entry erster oder zweiter Ordnung vollziehen, läuft auf die Beobachtung der Erzeugung und

Entfaltung einer Paradoxie hinaus. Das Aussen ist nur innen zugänglich. Die Beobachtung beobachtet die Operation der Beobachtung; sie beobachtet sich selbst als Objekt und als Unterscheidung, oder, nach den Vorstellungen der Romantik, als Doppelgänger oder asymmetrisiert als Maske, im Spiegel, von innen und von aussen, aber immer mit eigenen Operationen, also höchst individuell. Ihre mathematische Darstellung würde einen 'imaginären Raum' erfordern, der nur für diesen Zweck erfunden ist. Jedenfalls würde es nicht genügen, in eine 'Typenhierarchie' auszuweichen, die nichts weiter leistet als eine Verschleierung der Paradoxie durch eine dafür erfundene Unterscheidung von 'Ebenen'"(Luhmann 1992, S. 75) – das Versagen der Typentheorie angesichts von Selbstreferenz und daraus resultierenden Paradoxien ist einer der Hauptgründe, weshalb die polykontexturale Logik eingeführt worden war.

Da anzunehmen ist, dass am Ende des Prozesses einer unendlichen Selbstreflexion, dann also, wenn alle Hamilton-Kreise der subjektiven Negativität durchlaufen sind, diejenige strukturlogische Form erreicht ist, wo die Individualität des selbst zu Reflektierenden ausgelöscht ist, hat Kremer wohl auch darin recht, dass er die Brambilla als eine Prinzessin beschreibt, "die ihre Kontur und Identifikation in einem unendlichen mythischen Tanz abwerfen möchte" (1993, S. 324). Es ist auch wahr, dass sich die PB "jeder hermeneutischen Zudringlichkeit entzieht" (1993, S. 324), denn der hermeneutisch-formale Prozess der polykontexturalen Logik nimmt mit jedem neu zu durchlaufenden Hamiltonkreis ab. Hoffmann selbst hat diesen Sachverhalt wie folgt ausgedrückt: "Ich denke mir mein Ich durch ein Vervielfältigungsglas – und alle Gestalten, die sich um mich herum bewegen, sind Ichs" (E.T.A. Hoffmann, Tagebücher. Nach der Ausgabe Hans v. Müllers mit Erläuterungen hrsg. von Friedrich Schnapp. München 1971, S. 107 [Tagebucheintrag vom 6.11.1809])[3].

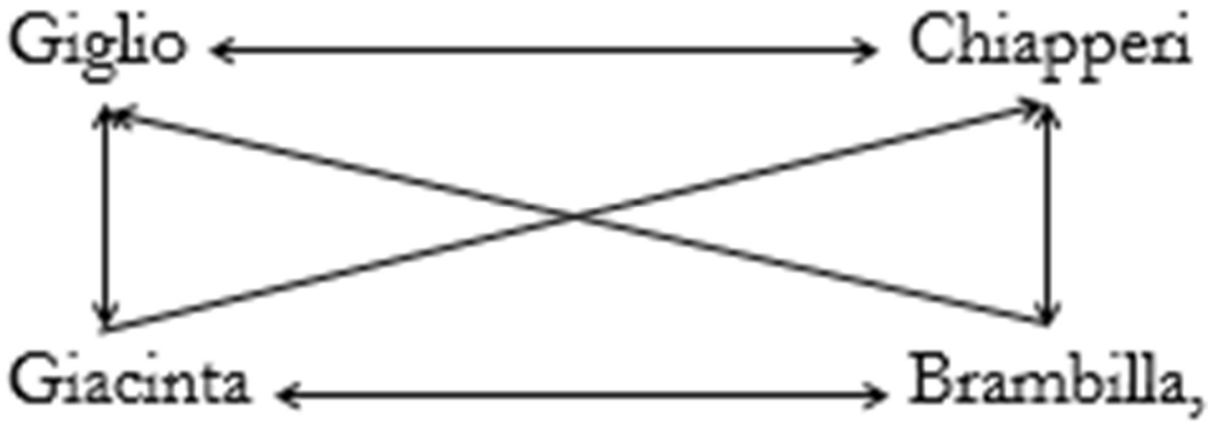
Die Putzmacherin Giacinta ist verlobt mit dem armen Schauspieler Giglio Fava (PB, S. 11). Es ist die Zeit kurz vor dem römischen Karneval, und es geht das Gerücht, dass "die weltberühmte Prinzessin Brambilla aus dem fernen

[3] Die Quellenangabe dieses Zitates verdanke ich Herrn Prof. Dr. Bernhard Schemmel (Bamberg).

Äthiopien” bereits in die Stadtmauern eingezogen sei, und zwar deshalb, “weil sie glaubt, unter den Masken des Corso ihren Herzensfreund und Bräutigam, den assyrischen Prinzen Cornelio Chiapperi, aufzufinden” (PB, S. 20). Giglio trachtet nun “mehrere Tage hintereinander vergebens darnach [...], auch nur das mindeste von der Prinzessin Brambilla zu erspüren [...]. Nur sein Traum war sein Leben, alles übrige ein unbedeutendes, leeres Nichts” (PB, S. 27). Doch Giacinta erscheint ihm auf dem Balkon des Meisters Belcapi als Brambilla, und Brambilla, mit der er am Karneval maskiert tanzt, erkennt er nicht als Brambilla. Giglio ist also hinter Brambilla her, während Giacinta davon träumt, dass Chiapperi sie heimführe. Hinzukommt, dass sich Giglio selbst für Chiapperi hält (PB, S. 55) und von Belcapi auch für Chiapperi gehalten wird (PB, S. 72). Schliesslich wird Giglio von dem Zauberer Celionati, der ihn ebenfalls für Chiapperi hält, wie folgt aufgeklärt: “ ‘Wisst, mein Fürst, dass diejenige Person, die man Euch unterschob statt der Prinzessin niemand anders ist als eine artige Putzmacherin, Giacinta Soardi geheissen!’ – ‘Ist es möglich?’ rief Giglio. – ‘Aber mich dünkt, dies Mädchen hat zum Liebhaber einen miserablen bettelarmen Komödianten, Giglio Fava?’ – ‘Allerdings’, erwiderte Celionati; ‘doch könnt ihr euch wohl denken, dass eben diesem miserablen bettelarmen Komödianten, diesem Theaterprinzen die Prinzessin Brambilla nachläuft auf Stegen und Wegen und eben nur darum Euch die Putzmacherin entgegenstellt, damit Ihr vieleicht gar in tollem wahnsinnigem Missverständnis Euch verlieben in diese und sie abwendig machen sollt dem Theaterhelden?’” (PB, S. 27).

Noch mehr Verwirrung entsteht, als dann der offenbar “richtige” Chiapperi auftaucht: “ ‘Ich weiss nicht’, erwiderte der junge artige Mensch, indem er beide, den Abbate und den Impresario, ganz verwundert anblickte, ‘ich weiss nicht, meine Herren, was ihr eigentlich von mir wollt. – Ihr redet mich mit einem fremden Namen an, ihr sprecht von mir ganz unbekannten Dingen – ihr tut, als wäre ich euch bekannt, unerachtet ich mich kaum erinnere, euch jemals in meinem Leben gesehen zu haben” (PB, S. 96). “Wäret Ihr doch früher gekommen, bester Signor Celionati, um mich von zwei Überlästigen zu befreien, die mich durchaus für den Schauspieler Giglio Fava halten, den ich – ach, Ihr wisst es ja – gestern in meinem unglücklichen Paroxismus auf dem Korso niederstiess, und die mir allerlei abscheuliche Dinge zumuteten. – Sagt, bin ich denn wirklich jenem Fava so ähnlich, dass man mich für ihn ansehen kann?' – 'Zweifelt', erwiderte der Ciarlatano höflich, ja beinahe ehrerbietig grüssend, 'zweifelt nicht, gnädigster Herr, dass Ihr, was Eure angenehmen Gesichtszüge betrifft, in der Tat jenem Schauspieler ähnlich genug sehet, und es war daher sehr geraten, Euern Doppelgänger aus dem Weg zu räumen" (PB, S. 98). "Der junge Mann leidet nämlich an dem chronischen Dualismus" (PB, S. 100). Es stellt sich auch noch heraus, dass der Capitan Pantalon, der den Giglio Fava in jenem Duell auf dem Corso niedergestreckt hatte, niemand anders war als der Prinz Chiapperi (PB, S. 104).

Hoffmann löst die Verwirrung, die er durch sein ganzes Buch zwischen Giacinta und Brambilla, zwischen Fava und Chiapperi, eingeschlossen den Capitan Pantalon, angerichtet hatte, auf unnachahmlich subtile Weise: "Mitternacht war vorüber, das Volk strömte aus den Theatern. Da schlug die alte Beatrice das Fenster zu [...]. Die Türe ging auf, und herein trat Giglio Fava mit seiner Giacinta". Diese spricht dann: " 'Aber denkst du denn nicht daran, welch ein Tag heute ist? Ahnst du nicht, in welchen verhängnisvollen Stunden die besondere Begeisterung uns erfasste? Erinnerst du dich nicht, dass es heute gerade ein Jahr her ist, da wir in den herrlichen hellen Urdarsee schauten und uns erkannten?' – 'Giacinta', rief Giglio in freudigem Erstaunen, 'Giacinta', was sprichst du? – Es liegt wie ein schöner Traum hinter mir, das Urdarland - der Urdarsee! – Aber nein! – es war kein Traum – wir haben uns erkannt! – O meine teuerste Prinzessin!' – 'O', erwiderte Giacinta, 'mein teuerster Prinz'" (PB, S. 110f.). Ohne weiteren Kommentar erhalten wir damit das folgende chiastische Schema:



dem die polykontexturale Proömial-Relation zugrunde liegt, welche jede Relation – also auch diejenigen der monokontexturalen Logik – als solche konstituiert. Sie "definiert den Unterscheid zwischen Relation und Einheit oder – was das gleiche ist – zwischen der Unterscheidung und dem, was unterschieden ist, was wiederum das gleiche ist wie der Unterschied zwischen Subjekt und Objekt" (Günther 1999, S. 22f.). Kaehr formalisierte die Proömialrelation wie folgt (1978, S. 6):

$$\begin{tikzcd}
\mathrm{PR}(R_{i+1}, R_i, x_i, x_{i-1}) = & & R_i \arrow[r] \arrow[d, leftrightarrow] & x_{i-1} & m-1 \\
& R_{i+1} \arrow[r] \arrow[d, leftrightarrow] & x_i & & m \\
R_{i+2} \arrow[r] & x_{i+1} & & & m+1
\end{tikzcd}$$

Die Proömialrelatiom durchkreuzt somit die Unterscheidung von Subjekt und Objekt, indem sie die jeweiligen dichotomischen Glieder austauschbar macht. Da in dem obenstehenden Diagramm sowohl Giglio und Chiapperi einerseits, als auch Giacinta und Brambilla anderseits in einer Austauschrelation stehen und da jeweils eine männliche Person mit einer weiblichen in einer

Ordnungsrelation steht, können wir die vier Personen des chiastischen Schemas für die relationalen Glieder (R_{i+1}, R_i, x_i, x_{i-1}) einsetzen. Ein wesentlich komplizierteres Schema aus mindestens dreimal drei relationalen Gliedern liegt den ET zu Grunde. Alle drei Kriterien, welche für polykontexturale Konzeptionen charakteristisch sind – Aufhebung der Grenze zwischen Subjekt und Objekt, das Auftreten von Reflexionsresten sowie die Aufhebung der Individualität – münden also in den Chiasmus; anderseits bildet dieser aber die Basis für die drei Kriterien: Relator und Relatum, Operator und Operand sind also dialektisch vermittelt und somit selbst wiederum proömiell-chistisch strukturiert.

Sicherlich wäre es lohnenswert, Hoffmanns Werk einmal nicht vom literarischen bzw. literarhistorisch-interpretierenden, sondern von den seinem Werk zugrunde liegenden philosophischen (logischen, ontologischen und metaphysischen) Grundlagen her zu analysieren. Mit dem Vorurteil aufgeräumt zu haben, dass es mit der Philoosphie des E.T.A. Hoffmannn nicht weit her sei und ihn als transklassischen Denker ausgewiesen zu haben, war das Ziel der vorliegenden Abhandlung.

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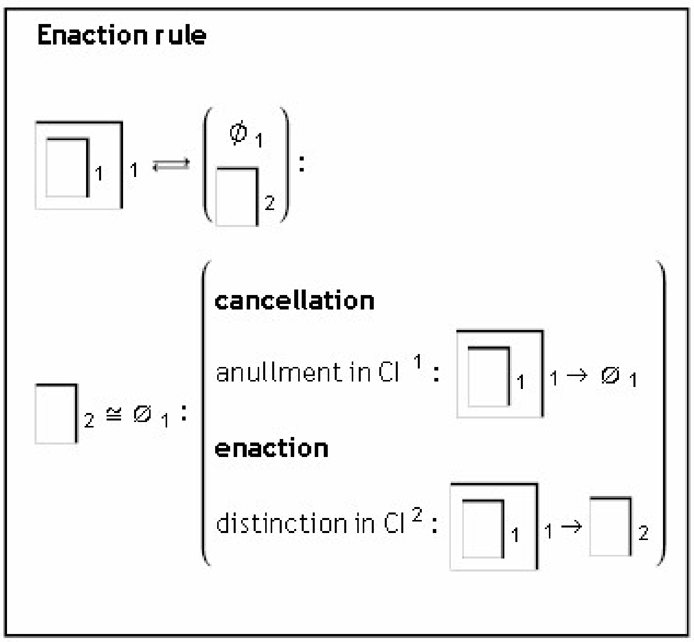
Annullierung und Distinktion

1. Nach den klassischen „Laws of Form“ von Spencer Brown (1969) bedeutet der Unterschied eines Unterschieds dessen Aufhebung („cancellation“); da der Calculus zweiwertig ist, kann es gar nichts anderes geben. In dem von Rudolf Kaehr soeben erarbeiteten „quadralektischen“ Modell eröffnen sich jedoch zwei vom klassischen Standpunkt aus grundverschiedene Möglichkeiten:

> A distinction of a distinction is conceived as both at once: as *annulation* and as *reflection* (enaction). Therefore, annulation is eliminating and destroying distinctions while reflection as enaction is not only creating new distinctions but also a new domain, i.e. world of distinctions, in which the new distinction and its further applications is realized.



Neben der Aufhebung gibt es hier also die „Enaktion“ genannte Bereitstellung einer neuen Unterscheidung; das ist natürlich nur darum möglich, weil der quadralektische Calculus nicht zweiwertig ist. Theoretisch ist es also möglich, die aus den Anfängen der qualitativen Mathematik bekannten von Gerhard G. Thomas geschaffenen „Permutogramme“ in die Form quadralektischer Diamanten zu transformieren; neben den ebenfalls seit langem bekannten mehrwertigen Negationszyklen, in der Form von Hamiltonkreisen darstellbar, wird man hochkomplexe enaktive Zyklen konstruieren können. Für die Semiotik gilt einmal mehr, dass die jüngsten Errungenschaften von Kaehrs

Forschungen zu kaum schon absehbaren völlig neuen Errungenschaften führen werden.

2. Unter einem Zeichen sei der Unterschied der Form ⌉ verstanden. Im Rahmen des quadralektischen Calculus sind danach folgende Kombinationen möglich:

⌉ ⌉	⌈ ⌉	⌋ ⌉	⌊⌉
⌉ ⌈	⌈ ⌈	⌋ ⌈	⌊ ⌈
⌉ ⌋	⌈ ⌋	⌋ ⌋	⌊ ⌋
⌉ ⌊	⌈ ⌊	⌋ ⌊	⌊ ⌊

Bedeute nun (vgl. Kaehr 2011a, S. 12):

⌉ := inside of inside   ⌈ := outside of outside

⌋ := inside of outside   ⌊ := outside of inside,

dann können wir die obige Tabelle mit Kaehr (2011, S. 22) wie folgt interpretieren:

II-II	OO-II	IO-II	OI-II
II-OO	OO-OO	IO-OO	OI-OO
II-IO	OO- IO	IO-IO	OI- IO
II-OI	O-O-OI	IO-OI	OI-OI

Diese Tabelle entspricht nun, wie Kaehr (2011b) sehr richtig bemerkt hat, der epistemologischen Basis-Matrix meiner "Theorie der Nacht" und damit einer vierwertigen handlungstheoretischen semiotischen Matrix, wie Kaehr (2011b) ebenfalls festgestellt hat. D.h. wir müssen ausgehen von einer Zeichendefinition

ZR = (Q, M, O, I) = (a.b c.d e.f g.h) mit a, ..., h ∈ {0, 1, 2, 3},

von denen jedes Primzeichen die vier quadralektischen Positionen einnehmen kann.

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Die vollständige Komposition von Subzeichen aus Primzeichen

1. Wie Rudolf Kaehr bereits in (2008), zuletzt aber ausführlich in einer soeben erschienenen Studie zum Vierfachen Anfang in quadralektischen Diamanten (2011), die man nicht anders als genial zu bezeichnen vermag, klargemacht hat, sind morphismische Abbildungen in der Semiotik nicht nur in der Zweitheit (z.B. $\alpha := 1 \rightarrow 2$) und in der Drittheit (z.B. $\alpha \rightarrow \beta \circ \beta \rightarrow \gamma = \alpha \rightarrow \gamma$) vertreten, sondern ebenfalls in der Erstheit. Diese vom klassischen Standpunkt aus völlig unvorstellbare Eigenschaft betrifft bedingt jedoch nicht nur eine Redefintion der Erstheit, sondern auch der Zweitheit und Drittheit, da im Zeichenmodell ja die doppelte Inklusion $ZR = (1 \rightarrow ((1 \rightarrow 2), (1 \rightarrow 2 \rightarrow 3))$ gilt. Und da somit die Subzeichen betroffen sind, folgen Neudefinitionen aller höheren semiotischen relationalen Gebilde, in Sonderheit der Zeichenklassen und Realitätsthematiken.

2. Wegen

$1 = 1 \circ 1$

gilt

$1. = 1. \circ 1.$ oder $1. \circ .1$

$.1 = .1 \circ .1$ oder $.1 \circ 1.$

Damit gilt also für konverse Subzeichen-Paare, wie z.B. $(1.2)° = (2.1)$:

$1.2 = .2 \circ 1.$ oder $.2\ .1$

$2.1 = .1 \circ 2.$ oder $.1 \circ .2$

und damit

$1.2 = [(.2 \circ .2 \ / \ .2 \circ 2.) \circ [(1. \circ 1. \ / \ 1. \circ .1) \ / \ (.1 \circ .1 \ / \ .1 \circ 1.)]]$

$2.1 = [(.1 \circ .1 \ / \ .1 \circ 1.) \circ [(2. \circ 2. \ / \ 2. \circ .2) \ / \ (.2 \circ .2 \ / \ .2 \circ 2.)]].$

Entsprechend erhalten wir z.B. für (3.1)

$3.1 = [(.1 \circ .1 \ / \ .1 \circ 1.) \circ [(3. \circ 3. \ / \ 3. \circ .3) \ / \ (.3 \circ .3 \ / \ .3 \circ 3.)]]$

und somit für die Zeichenklasse (3.1 2.1 1.2):

$$3.1\ 2.1\ 1.2 = \quad [[(.1 \circ .1\ /\ .1 \circ 1.) \circ [(3. \circ 3.\ /\ 3. \circ .3)\ /\ (.3 \circ .3\ /\ .3 \circ 3.)]],$$

$$[(.1 \circ .1\ /\ .1 \circ 1.) \circ [(2. \circ 2.\ /\ 2. \circ .2)\ /\ (.2 \circ .2\ /\ .2 \circ 2.)]],$$

$$[(.2 \circ .2\ /\ .2 \circ 2.) \circ [(1. \circ 1.\ /\ 1. \circ .1)\ /\ (.1 \circ .1\ /\ .1 \circ 1.)]]].$$

Was nun die duale Realitätsthematik angeht, so bekommen wir

$$\times(3.1\ 2.1\ 1.2) = (2.1\ 1.2\ 1.3) =$$

$$\times[[(.1 \circ .1\ /\ .1 \circ 1.) \circ [(3. \circ 3.\ /\ 3. \circ .3)\ /\ (.3 \circ .3\ /\ .3 \circ 3.)]],$$

$$[(.1 \circ .1\ /\ .1 \circ 1.) \circ [(2. \circ 2.\ /\ 2. \circ .2)\ /\ (.2 \circ .2\ /\ .2 \circ 2.)]],$$

$$[(.2 \circ .2\ /\ .2 \circ 2.) \circ [(1. \circ 1.\ /\ 1. \circ .1)\ /\ (.1 \circ .1\ /\ .1 \circ 1.)]]] =$$

$$[[(.1 \circ .1\ /\ .1 \circ 1.) \circ [(2. \circ 2.\ /\ 2. \circ .2)\ /\ (.2 \circ .2\ /\ .2 \circ 2.)]],$$

$$[(.2 \circ .2\ /\ .2 \circ 2.) \circ [(1. \circ 1.\ /\ 1. \circ .1)\ /\ (.1 \circ .1\ /\ .1 \circ 1.)]],$$

$$[(.3 \circ .3\ /\ .3 \circ 3.) \circ [(1. \circ 1.\ /\ 1. \circ .1)\ /\ (.1 \circ .1\ /\ .1 \circ 1.)]]],$$

also strukturell keine direkt ablesbare Inversion der Ordnung der Subzeichen und der Primzeichen.

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Semiotische Funktionentheorie kontexturierter Conway-Zahlen

1. Wir gehen aus von der in Toth (2011b) eingeführten dyadisch-trivalenten Semiotik. Sie basiert auf dem Zeichenmodell

ZR* = ((a.b), (c.d)) mit a, ..., ∈ {1, 2, 3}.

Triaden werden aus Dyaden nach dem Schema

$$\begin{array}{lll} (a.b) \circ (b.c) \rightarrow & (a.b) & \\ & (b.c) \rightarrow & (a.b.c) \end{array}$$

Tetraden werden aus Dyaden (oder direkt aus Triaden) nach de Schema

$$\begin{array}{ll} (a.b) \circ (b.c) \circ (c.d) \rightarrow (a.b.c) & \\ & \circ\ (c.d) \rightarrow (a.b.c.d), \text{usw.} \end{array}$$

konstruiert. Nun hat jede n-stellige Relation $\binom{n}{k}$ k-stellige Partialrelationen, die sich nach dem Schema (n · (n-1) · (n-2) · ... · (n- (k-1)) / k! errechnen lassen, wozu noch (k! – 1) Konversen kommen (vgl. z.B. Menne 1991, S. 152).
konkateniert

2. Die Partialrelationen semiotischer Relationen bestimmen nun auch Art und Anzahl semiotischer Funktionen, eine Konzeption, die durch Bense (1981, S. 150 ff.) in die Semiotik eingeführt worden war. Im folgenden gehe ich von einer tetravalenten anstatt von einer trivalenten Semiotik aus, um Werte und Anzahl der Relationen in der dyadischen Zeichenstruktur ((a.c), (c.d)) zu balancieren. Ferner werden die Subzeichen in den Partialrelationen kontexturiert, um den Anschluss an den neusten Stand der Semiotik zu gewährleisten (vgl. Kaehr 2009). Schliesslich verwende ich, um den Anschluss an meine "Theory of the Night" (Toth 2011a) zu erarbeiten, Conway-Zahlen („surreale" Zahlen), eine Art Dedekindscher Schnitte mit der Einschänkung, dass weder an der linken noch an der rechten Zahlengrenze die leere Menge aufscheinen darf (bei den surrealen Zahlen ist dies zugelassen; was sowohl links als auch rechts von der leeren Menge ist, ist per definitionem die Zahl 0).

Was wir nun noch benötigen, sind maximale und minimale Variablen-und Kontexturen-Schemata, um ja alle möglichen Fälle der Partialrelationen zu erfassen:

Maximales Variablenschema: $w = f(x_{i,j,k}, y_{i,j,k}, z_{i,j,k})$
Minimales Variablenschema: $w = f(x_{i,j,k}, y_{i,j,k})$

Maximales Kontexturschema: $w = f(x_{i,j,k}, y_{i,j,k}, z_{i,j,k})$
Minimales Kontexturschema: $w = f(x_{i,j}, y_{i,j})$

mit $i, j, k \in \{1, 2, 3, 4\}$

2.1. Funktionen mit $w = (0.1_{1,3})$

a. $({\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}) = f({\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, {\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

b. $({\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}) = f({\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, {\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, {\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

c. $({\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}) = f({\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, {\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

d. $({\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}) = f({\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, {\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}.\{\{0\}$

| {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{3\} \mid \{5\}\}}$)

e. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{3\} \mid \{5\}\}}$)

f. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{3\} \mid \{5\}\}}$)

g. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{3\} \mid \{5\}\}}$)

h. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{3\} \mid \{5\}\}}$)

i. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{3\} \mid \{5\}\}}$)

j. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{3}$

$_{| \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

k. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

l. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

2.2. Funktionen mit w = $(0.2_{1,2})$

a. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

b. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

c. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

d. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

e. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

f. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

g. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

h. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

i. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid}$

{{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

j. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

k. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

l. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

m. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

n. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}})

o. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2}

| {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

p. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

q. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

r. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

s. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

t. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

u. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid}$

$_{\{5\}\}\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

v. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

w. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

x. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

y. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

z. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

aa. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid}$

{{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}})

bb. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}})

cc. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}})

dd. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}})

ee. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}})

ff. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

gg. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}{{2} | {{{3} | {5}}}},{{3}

| {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}})

hh. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}})

ii. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}})

jj. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

kk. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}})

ll. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}})

mm. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2}

$_{\mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

nn. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

oo. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

2.3. Funktionen mit w = $(0.3_{2,3})$

a. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

b. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

c. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

d. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} |

{{{2} | {{{3} | {5}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

e. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

f. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

g. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

h. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

i. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

j. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} |

{{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

k. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}})

l. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

m. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

n. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

o. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}})

p. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

q. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

r. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

s. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

t. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

u. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

v. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

w. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} |

{{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}})

x. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

y. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}})

z. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

aa. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

bb. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

cc. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

dd. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{0} | {{{1} |

{{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

ee. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

ff. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

gg. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

hh. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

ii. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

jj. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

kk. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

ll. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

mm. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}})

nn. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

oo. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

pp. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

qq. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{1} | {{{2} |

{{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

rr. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

ss. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

tt. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

uu. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

vv. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

ww. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{1} | {{{2} |

{{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

xx. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

yy. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

zz. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

aaa. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

bbb. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

ccc. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} |

{{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

ddd. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

eee. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

fff. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

ggg. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

hhh. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

iii. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

jjj. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} |

{5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

kkk. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

lll. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

mmm. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

nnn. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

ooo. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

ppp. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

qqq. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

rrr. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

sss. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

ttt. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

uuu. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

vvv. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

www. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} |

{5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

xxx. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

yyy. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

zzz. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

aaaa. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

bbbb. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

cccc. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

dddd. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{2} | {{{3} |

{5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

eeee. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

ffff. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

gggg. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

hhhh. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

iiii. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

jjjj. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} |

{5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

kkkk. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

llll. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

mmmm. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

nnnn. ({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

2.4. Funktionen mit w = $(1.0_{1,3})$

a. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

b. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} |

{5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

c. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

d. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

e. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}})

f. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

g. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

h. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} |

{5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

i. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

j. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

k. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

l. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

2.5. Funktionen mit w = $(1.1_{1,3,4})$

a. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

b. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

c. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

d. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

e. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

f. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

g. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

h. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

i. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

j. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

k. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},}$)

l. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

m. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

n. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

o. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

p. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

q. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

r. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

s. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

t. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}_{{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}_{{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}_{{{2} | {{{3} | {5}}}},{{3} | {5}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}_{{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}}})

u. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}_{{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}_{{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}_{{{2} | {{{3} | {5}}}},{{3} | {5}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}_{{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}})

v. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}_{{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}_{{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}_{{{2} | {{{3} | {5}}}},{{3} | {5}}}, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}_{{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}})

w. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}_{{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}_{{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}_{{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}})

x. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}_{{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}_{{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}_{{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}_{{{2} | {{{3} | {5}}}},{{3} | {5}}})

y. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}_{{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}_{{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}}, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}_{{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}})

z. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

aa. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

bb. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

cc. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

dd. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

ee. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}})$

ff. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

gg. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$)

hh. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

ii. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

jj. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

kk. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

ll. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

mm. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

nn. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

oo. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

pp. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

qq. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-

{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$)

rr. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

ss. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

tt. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

uu. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

vv. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

ww. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} |

{{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$)

xx. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

yy. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

zz. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

aaa. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

bbb. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

ccc. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} |

{5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

ddd. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

eee. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$)

fff. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

ggg. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

hhh. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

iii. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

jjj. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

kkk. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$)

lll. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

2.6. Funktionen mit w = (1.2.$_{1,4}$)

a. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

b. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

c. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} |

{{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

d. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

e. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

f. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

g. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

h. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

i. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} |

{{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$)

j. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$)

k. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$)

l. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$)

m. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$)

n. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$)

o. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} |}$

$_{\{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

p. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

q. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

r. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

s. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

t. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

u. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

v. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

w. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

x. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

y. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

z. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

aa. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

bb. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

cc. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}})$

dd. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

ee. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}})$

ff. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

gg. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

hh. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

ii. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

jj. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

kk. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

ll. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

mm. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$)

nn. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

oo. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}})$

pp. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

qq. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

rr. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}})$

ss. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

tt. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

uu. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

vv. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

ww. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

xx. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

yy. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

zz. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2}

| {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}} {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

aaa. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

bbb. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}})

ccc. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

ddd. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

eee. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}})

fff. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} |

{{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\},\{\{3\} | \{5\}\}}$)

ggg. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

hhh. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\},\{\{3\} | \{5\}\}}$)

iii. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\},\{\{3\} | \{5\}\}}$)

jjj. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$)

kkk. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

lll. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-

{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$)

mmm. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

nnn. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

ooo. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

ppp. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

qqq. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$)

rrr. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} |

{5}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

sss. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$)

ttt. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

uuu. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

vvv. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

www. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

xxx. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\}}$

| {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

yyy. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

zzz. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}})

aaaa. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}})

bbbb. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

cccc. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}})

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llll. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

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{5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

rrrr. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}})

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wwww. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

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zzzz. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

aaaaa. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$)

bbbbb. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

ccccc. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

ddddd. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

eeeee. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}})

fffff. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

ggggg. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}})

hhhhh. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

iiiii. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

jjjjj. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0}

| {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}})

kkkkk. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}})

2.7. Funktionen mit w = $(1.3_{3,4})$

a. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

b. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

c. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

d. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

e. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

f. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

g. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

h. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

i. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

j. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

k. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

l. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

m. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

n. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

o. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

p. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

q. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

r. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{2} | {{{3}

| {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

s. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

t. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

u. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

v. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

w. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

x. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} |

{5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

y. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

z. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}})

aa. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}})

b. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}})

c. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}})

d. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}{{1} | {{{2}

| {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

e. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}})

f. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}})

g. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}})

h. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}})

i. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}})

j. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}}, {{2}

| {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

k. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$)

l. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

m. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

n. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

o. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$)

p. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0}

| {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

q. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

r. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

s. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

t. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

u. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

v. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} |

{{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}})

w. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}})

x. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

y. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

z. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}})

aa. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

bb. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} |

{{{2} | {{{3} | {5}}}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

cc. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

dd. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

ee. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

ff. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

gg. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}})

hh. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2}

| {{{3} | {5}}}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

ii. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

jj. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$)

kk. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

ll. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$)

mm. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

nn. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} |}$

$_{\{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

oo. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

pp. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$)

qq. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

rr. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

ss. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

tt. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\}}$

$_{| \{\{\{3\} | \{5\}\}\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

uu. $(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}})$

vv. $(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}})$

ww. $(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}})$

xx. $(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}})$

yy. $(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

zz. $(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}})$

aaa. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

bbb. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

ccc. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}})$

ddd. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

eee. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

fff. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

ggg. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

hhh. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

iii. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

jjj. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

kkk. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

lll. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

mmm. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid}$

{5}}}}}},{{3} | {5}}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}_{{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}})

nnn. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}_{{{2} | {{{3} | {5}}}},{{3} | {5}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}_{{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}_{{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}_{{{2} | {{{3} | {5}}}},{{3} | {5}}})

ooo. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}_{{{2} | {{{3} | {5}}}},{{3} | {5}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}_{{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}_{{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}}, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}_{{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}})

ppp. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}_{{{2} | {{{3} | {5}}}},{{3} | {5}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}_{{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}_{{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}}, {{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}_{{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}}})

qqq. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}_{{{2} | {{{3} | {5}}}},{{3} | {5}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}_{{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}_{{{2} | {{{3} | {5}}}},{{3} | {5}}})

rrr. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}_{{{2} | {{{3} | {5}}}},{{3} | {5}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}_{{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}_{{{2} | {{{3} | {5}}}},{{3} | {5}}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}_{{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}})

sss. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}_{{{2} | {{{3} | {5}}}},{{3} | {5}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}_{{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}}, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}_{{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}})

ttt. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}_{{{2} | {{{3} | {5}}}},{{3} | {5}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}_{{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}}, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}_{{{1} | {{{2} | {{{3} | {5}}}}}},{{3}

$_{\mid \{5\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

uuu. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

vvv. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

www. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

xxx. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

yyy. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

zzz. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

aaaa. $(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

bbbb. $(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}})$

cccc. $(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

dddd. $(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

eeee. $(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

ffff. $(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

gggg. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

hhhh. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

iiii. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

jjjj. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

kkkk. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

llll. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

mmmm. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0}

| {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}_{{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}}, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}_{{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}})

nnnn. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}_{{{2} | {{{3} | {5}}}},{{3} | {5}}}) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}_{{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}}, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}_{{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}_{{{2} | {{{3} | {5}}}},{{3} | {5}}})

oooo. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}_{{{2} | {{{3} | {5}}}},{{3} | {5}}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}_{{{2} | {{{3} | {5}}}},{{3} | {5}}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}_{{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}})

pppp. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}_{{{2} | {{{3} | {5}}}},{{3} | {5}}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}_{{{2} | {{{3} | {5}}}},{{3} | {5}}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}_{{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}_{{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}})

qqqq. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}_{{{2} | {{{3} | {5}}}},{{3} | {5}}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}_{{{2} | {{{3} | {5}}}},{{3} | {5}}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}_{{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}_{{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}})

rrrr. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}_{{{2} | {{{3} | {5}}}},{{3} | {5}}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}_{{{2} | {{{3} | {5}}}},{{3} | {5}}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}_{{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}_{{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}})

ssss. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}_{{{2} | {{{3} | {5}}}},{{3} | {5}}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}_{{{2} | {{{3} | {5}}}},{{3} | {5}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}._{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}})

tttt. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}_{{{2} | {{{3} | {5}}}},{{3} | {5}}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}_{{{2} | {{{3} | {5}}}},{{3} | {5}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}._{{0} |

{{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}})

uuuu. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

vvvv. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}})

wwww. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

xxxx. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}})

yyyy. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}})

zzzz. ({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}) aaaaa. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

bbbbb. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

ccccc. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

ddddd. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

eeeee. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

fffff. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

ggggg. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

hhhhh. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}})$

iiiii. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}})$

jjjjj. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}})$

kkkkk. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}})$

lllll. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}})$

mmmmm. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}})$

nnnnn. $(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}})$

ooooo. $(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}})$

ppppp. $(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}})$

qqqqq. $(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}})$

rrrrr. $(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}})$

sssss. $(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}, \{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}})$

ttttt. $(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}, \{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}})$

uuuuu. $(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}})$

vvvvv. $(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} |}$

$_{\{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

2.8. Funktionen mit w = $(2.0_{1,2})$

a. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

b. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

c. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

d. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

e. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

f. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} |}$

$_{\{5\}\}\}\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{2\} | \{\{\{3\} | \{5\}\}\}\},\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{3\} | \{5\}\}}$)

g. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{3\} | \{5\}\}}$)

h. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{2\} | \{\{\{3\} | \{5\}\}\}\},\{3\} | \{5\}\}}$)

i. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{3\} | \{5\}\}}$)

j. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{3\} | \{5\}\}}$)

k. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{2\} | \{\{\{3\} | \{5\}\}\}\},\{3\} | \{5\}\}}$)

l. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{1\} | \{\{\{2\} | \{\{\{3\} |}$

{5}}}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

m. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

n. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}})

o. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

p. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

q. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

r. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} |

$_{\{5\}\}\}\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

s. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

t. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

u. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

v. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

w. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

x. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid}$

$_{\{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

y. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

z. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

aa. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

bb. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

cc. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

dd. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid}$

{{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

ee. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

ff. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

gg. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

hh. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

ii. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

jj. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} |

${}_{\{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

kk. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

ll. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

mm. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

nn. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

oo. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

2.9. Funktionen mit $w = (2.1_{1,4})$

a. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}})$

b. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}})$

c. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}})$

d. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}})$

e. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

f. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} |}$

{5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

g. ({{{1} | {{{2} | {{{3} | {5}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

h. ({{{1} | {{{2} | {{{3} | {5}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}})

i. ({{{1} | {{{2} | {{{3} | {5}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

j. ({{{1} | {{{2} | {{{3} | {5}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}})

k. ({{{1} | {{{2} | {{{3} | {5}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

l. ({{{1} | {{{2} | {{{3} | {5}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} |

$\{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

m. $(\{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

n. $(\{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

o. $(\{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

p. $(\{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

q. $(\{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

r. $(\{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}},$

{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

s. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

t. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

u. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

v. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$)

w. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

x. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

y. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

z. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

aa. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

bb. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$)

cc. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

dd. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$)

ee. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

ff. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

gg. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

hh. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

ii. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

jj. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$)

kk. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$)

ll. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

mm. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

nn. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

oo. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

pp. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$)

qq. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2}

| {{{3} | {5}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

rr. ({{{1} | {{{2} | {{{3} | {5}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

ss. ({{{1} | {{{2} | {{{3} | {5}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

tt. ({{{1} | {{{2} | {{{3} | {5}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

uu. ({{{1} | {{{2} | {{{3} | {5}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$)

vv. ({{{1} | {{{2} | {{{3} | {5}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

ww. ({{{1} | {{{2} | {{{3} | {5}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$)

xx. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

yy. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

zz. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

aaa. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$)

bbb. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

ccc. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

ddd. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

eee. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

fff. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

ggg. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

hhh. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

iii. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

jjj. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}})$

kkk. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}._{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

lll. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

mmm. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

nnn. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}})$

ooo. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

ppp. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

qqq. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

rrr. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}})$

sss. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}})$

ttt. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}})$

uuu. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}})$

vvv. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} |$

{5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

www. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

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dddd. ({{1} | {{{2} | {{{3} | {5}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$)

eeee. ({{1} | {{{2} | {{{3} | {5}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

ffff. ({{1} | {{{2} | {{{3} | {5}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

gggg. ({{1} | {{{2} | {{{3} | {5}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$)

hhhh. ({{1} | {{{2} | {{{3} | {5}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} |

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iiii. ({{{1} | {{{2} | {{{3} | {5}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{{1} | {{{2} | {{{3} | {5}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}})

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llll. ({{{1} | {{{2} | {{{3} | {5}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

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rrrr. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

ssss. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

tttt. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}) uuuu. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{2} | {{{3} | {5}}}})

vvvv. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{2} | {{{3} | {5}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}})

wwww. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}})

xxxx. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}})

yyyy. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}})

zzzz. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) aaaaa. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

bbbbb. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

ccccc. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

ddddd. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

eeeee. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}})$

fffff. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}})$

ggggg. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

hhhhh. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

iiiii. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}})$

jjjjj. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

kkkkk. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

lllll. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

2.10. Funktionen mit w = ($2.2_{1,2,4}$)

a. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

b. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

c. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

d. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

e. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

f. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\}}$

$_{\mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

g. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

h. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

i. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

j. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

k. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

l. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\}}$

_| {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}_{{2} | {{{3} | {5}}}},{{3} | {5}}})

m. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}_{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}}) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}_{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}_{{2} | {{{3} | {5}}}},{{3} | {5}}}, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}_{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}})

n. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}_{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}}) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}_{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}_{{2} | {{{3} | {5}}}},{{3} | {5}}})

o. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}_{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}}) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}_{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}_{{2} | {{{3} | {5}}}},{{3} | {5}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}._{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}})

p. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}_{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}}) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}_{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}_{{2} | {{{3} | {5}}}},{{3} | {5}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}_{{2} | {{{3} | {5}}}},{{3} | {5}}})

q. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}_{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}}) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}_{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}}, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}_{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}})

r. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}_{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}}) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}_{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}}, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}_{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}._{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}})

s. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

t. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$)

u. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

v. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

w. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

x. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

y. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

z. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}})$

aa. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}})$

bb. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}})$

cc. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

dd. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}})$

ee. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} |$

{5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

ff. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

gg. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

hh. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

ii. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$)

jj. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

kk. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} |

{5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

ll. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$)

mm. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

nn. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

oo. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

pp. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

qq. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} |

{5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

rr. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

ss. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

tt. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

uu. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

vv. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

ww. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3}

| {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

xx. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

yy. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

zz. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

aaa. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

bbb. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

ccc. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

ddd. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}})$

eee. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}})$

fff. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}})$

ggg. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}})$

hhh. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

iii. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

jjj. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}})$

kkk. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

lll. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

mmm. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

nnn. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

ooo. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}})$

ppp. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

qqq. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

rrr. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}})$

sss. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

ttt. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

uuu. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

vvv. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

www. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

xxx. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

yyy. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

zzz. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

aaaa. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

bbbb. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\}}$

| {{{3} | {5}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

cccc. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

dddd. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

eeee. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

ffff. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

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iiii. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

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llll. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

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nnnn. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} |

$_{\{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

oooo. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

pppp. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

qqqq. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$)

rrrr. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

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tttt. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

uuuu. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}})$

vvvv. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

wwww. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}})$

xxxx. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}})$

yyyy. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}})$

zzzz. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

aaaaa. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} |$

{5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$)

bbbbb. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

ccccc. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

ddddd. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

eeeee. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

fffff. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

ggggg. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} |

{5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$)

hhhhh. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

iiiii. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

jjjjj. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

2.11. Funktionen mit w = ($2.3_{2,4}$)

a. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

b. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

c. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

d. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

e. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

f. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

g. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

h. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

i. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

j. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

k. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} |}$

$_{\{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

l. ({{1} | {{{2} | {{{3} | {5}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

m. ({{1} | {{{2} | {{{3} | {5}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

n. ({{1} | {{{2} | {{{3} | {5}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

o. ({{1} | {{{2} | {{{3} | {5}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

p. ({{1} | {{{2} | {{{3} | {5}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

q. ({{1} | {{{2} | {{{3} | {5}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

r. ({{1} | {{{2} | {{{3} | {5}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

s. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}\}})$

t. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

u. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

v. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

w. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

x. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

y. $(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid$

{5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$)

z. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

aa. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

bb. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$)

cc. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

dd. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

ee. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid}$

$_{\{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}$)

ff. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$)

gg. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}$)

hh. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}$)

ii. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$)

jj. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

kk. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

ll. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

mm. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}})$

nn. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}})$

oo. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}})$

pp. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

qq. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

rr. $(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} |$

{5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

ss. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

tt. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

uu. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

vv. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

ww. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

xx. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

yy. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1}

| {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

zz. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

aaa. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

bbb. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

ccc. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

ddd. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

eee. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

fff. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

ggg. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$)

hhh. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$)

iii. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$)

jjj. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}}$)

kkk. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}}$)

lll. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$)

mmm. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$)

nnn. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

ooo. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

ppp. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

qqq. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

rrr. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

sss. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

ttt. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

uuu. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

vvv. ({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} |}$

$_{\{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

2.12. Funktionen mit w = ($3.0_{2,3}$)

a. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

b. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

c. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

d. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

e. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

f. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

g. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

h. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

i. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

j. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

k. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

l. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} |

{{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

m. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

n. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

o. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

p. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

q. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

r. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0}

| {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

s. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

t. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

u. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

v. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

w. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

x. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

y. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

z. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

aa. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

bb. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

cc. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

dd. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

ee. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{0} | {{{1} |

{{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

ff. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

gg. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

hh. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

ii. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

jj. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

kk. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

ll. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

mm. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

nn. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

oo. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

pp. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

qq. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

rr. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

ss. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

tt. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

uu. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

vv. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

ww. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

xx. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

yy. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

zz. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

aaa. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

bbb. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

ccc. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

ddd. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{1} | {{{2} |

{{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

eee. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

fff. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

ggg. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

hhh. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

iii. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

jjj. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{2} | {{{3} |

{5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

kkk. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

lll. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

mmm. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

nnn. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

ooo. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

ppp. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

qqq. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} |

{5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

rrr. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

sss. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

ttt. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

uuu. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

vvv. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

www. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

xxx. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} |

{5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

yyy. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

zzz. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

aaaa. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

bbbb. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

cccc. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

dddd. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

eeee. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{2} | {{{3} |

{5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

ffff. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

gggg. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

hhhh. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

iiii. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

jjjj. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

kkkk. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

llll. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$) = f({{2} | {{{3} |

{5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

mmmm. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

nnnn. ({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

2.13. Funktionen mit w = $(3.1_{3,4})$

a. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

b. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

c. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

d. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} |

{{{3} | {5}}}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

e. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

f. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

g. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

h. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

i. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

j. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

k. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

l. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

m. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

n. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

o. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

p. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

q. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

r. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

s. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

t. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

u. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

v. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} |

{{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

w. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

x. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

y. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{2} | {{{3} | {5}}}},{{3} | {5}})

z. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

aa. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

bb. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{1} | {{{2} |

| {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

cc. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

dd. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

ee. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

ff. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

gg. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

hh. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

ii. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$)

jj. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

kk. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

ll. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

mm. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

nn. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

oo. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$)

pp. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

qq. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$)

rr. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$)

ss. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$)

tt. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} |

{{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}}$)

uu. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$)

vv. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$)

ww. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$)

xx. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}}$)

yy. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$)

zz. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$)

aaa. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

bbb. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

ccc. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}})$

ddd. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

eee. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

fff. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

ggg. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}})$

hhh. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

iii. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

jjj. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

kkk. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

lll. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}})$

mmm. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

nnn. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

ooo. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

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{5}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}})

uuu. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}},{{3} | {5}})

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yyy. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}})

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aaaa. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} |

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mmmm. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

nnnn. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$)

oooo. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

pppp. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

qqqq. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

rrrr. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$)

ssss. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

tttt. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}})

uuuu. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

vvvv. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

wwww. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

xxxx. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}) yyyy. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}})$

zzzz. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}})$

aaaaa. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}})$

bbbbb. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}})$

ccccc. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

ddddd. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}})$

eeeee. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{0\} | \{\{\{1\} |}$

$_{\{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

fffff. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

ggggg. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

hhhhh. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

iiiii. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

jjjjj. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

kkkkk. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

lllll. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

mmmmm. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

nnnnn. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

ooooo. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

ppppp. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

qqqqq. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

rrrrr. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

sssss. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

ttttt. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

uuuuu. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

vvvvv. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

wwwww. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

xxxxx. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

yyyyy. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{1\} \mid \{\{\{2\} \mid$

{{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

zzzzz. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

aaaaaa. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

bbbbbb. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

cccccc. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

dddddd. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

eeeeee. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

ffffff. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

gggggg. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

hhhhhh. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

iiiii. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

jjjjj. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

kkkkkk. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

llllll. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

mmmmmm. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$)

nnnnnn. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}}$)

oooooo. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}}$)

pppppp. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$)

qqqqqq. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$)

rrrrrr. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$)

ssssss. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$)

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$_{\{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

uuuuuu. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

vvvvvv. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

wwwwww. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

xxxxxx. ({{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

2.14. Funktionen mit w = (3.2$_{2,4}$)

a. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

b. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\}}$

| {5}}}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

c. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

d. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

e. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}})

f. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

g. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}})

h. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}}) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}{{1} | {{{2} | {{{3} | {5}}}}}},{{2} | {{{3} | {5}}}}, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}{{2} | {{{3} | {5}}}},{{3} | {5}}, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}},{{1} | {{{2} | {{{3} | {5}}}}}},{{3} | {5}})

i. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

j. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

k. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

l. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

m. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

n. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

o. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} |

{{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$)

p. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

q. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

r. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

s. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

t. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}}$)

u. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

v. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

w. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

x. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

y. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

z. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

aa. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

bb. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

cc. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

dd. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

ee. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

ff. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

gg. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$)

hh. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}.$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

ii. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\},\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

jj. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

kk. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

ll. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

mm. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}})$

nn. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}._{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

oo. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

pp. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\},\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\}$

| {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

qq. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

rr. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

ss. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

tt. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

uu. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

vv. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

ww. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

xx. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

yy. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

zz. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

aaa. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$)

bbb. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

ccc. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

ddd. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}$)

eee. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}} | {{{0} | {{{1}

$| \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

fff. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}})$

ggg. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}})$

hhh. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}})$

iii. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}})$

jjj. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}})$

kkk. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

lll. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}})$

mmm. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

nnn. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

ooo. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}})$

ppp. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

qqq. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

rrr. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

sss. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

ttt. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid$

{{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

uuu. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

vvv. ({{2} | {{{3} | {5}}}}.{{1} | {{{2} | {{{3} | {5}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{2} | {{{3} | {5}}}}.{{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$)

2.15. Funktionen mit w = (3.3$_{2,3,4}$)

a. ({{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

b. ({{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

c. ({{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$)

d. ({{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$)

e. ({{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} \mid \{\{\{3\} \mid}$

$_{\{5\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

f. ({{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$)

g. ({{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$)

h. ({{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

i. ({{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

j. ({{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$)

k. ({{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$)

l. ({{2} | {{{3} | {5}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$) = f({{1} | {{{2} | {{{3} | {5}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}\}}$, {{-{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}} | {{{0} | {{{1} | {{{2} | {{{3} | {5}}}}}}}}}}}.{{2} | {{{3} | {5}}}}$_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}$)

m. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}})$

n. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

o. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

p. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}})$

q. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}})$

r. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{-\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\} | \{\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

s. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} | \{5\}\}})$

t. $(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{2\} | \{\{\{3\} | \{5\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}) = f(\{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{0\} | \{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}\}\}_{\{\{2\} | \{\{\{3\} | \{5\}\}\}\},\{\{3\} | \{5\}\}}, \{\{2\} | \{\{\{3\} | \{5\}\}\}\}.\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\}_{\{\{1\} | \{\{\{2\} | \{\{\{3\} | \{5\}\}\}\}\}\},\{\{3\} |}$

$_{\{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

u. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

v. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

w. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}})$

x. $(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}) = f(\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}_{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\},\{\{3\} \mid \{5\}\}}, \{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}.\{\{-\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\} \mid \{\{\{0\} \mid \{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\}\}\}\}\}_{\{\{1\} \mid \{\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}\}\},\{\{2\} \mid \{\{\{3\} \mid \{5\}\}\}\}})$

3. Die Liste der 1´162 kontexturierten surrealen semiotischen Funktionen ist erschöpfend für die doppel-dyadisch tetravalente 4-kontexturale Semiotik. Allerdings ist zu bedenken, dass Funktionen, die mehr als eine Kontexturenzahl haben, „aufgesplittert" werden können in mehrere Teilfunktionen und ihre Kombinationen. Als Beispiel stehe die Funktion

$(3.3_{2,3,4}) \quad = \quad f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$,

die aufgespalten werden kann in

$(3.3_{2}) \quad = \quad f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$

$(3.3_{3}) \quad = \quad f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$

$(3.3_{4}) \quad = \quad f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$

$(3.3_{2,3}) \quad = \quad f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$

$(3.3_{2,4}) \quad = \quad f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$

$(3.3_{3,4}) \quad = \quad f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$

$(3.3_{2,3,4}) \quad = \quad f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$

$(3.3_{2,4,3}) \quad = \quad f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$

$(3.3_{3,2,4}) \quad = \quad f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$

$(3.3_{3,4,2}) \quad = \quad f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$

$(3.3_{4,2,3}) \quad = \quad f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$

$(3.3_{4,3,2}) \quad = \quad f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$, usw.

Eine weitere Quelle gewaltigen Anwachsens semiotischer Funktionen liegt in der Möglichkeit, die Ordnung der Kontexturenzahlen zu permutieren.

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Zwischen aussen und innen: dyadisch-tetravalentes Zeichenmodell

1. In Toth (2011) hatte ich den Versuch gemacht, zu einem dyadischen Zeichenmodell zurückzukehren, aber die Peircesche Trivalenz beizubehalten. Diese Unbalanciertheit zwischen der Stelligkeit (Valenz) der Relation und der Anzahl zur Verfügung stehender Werte führte in der Folge zu einigen bemerkenswerten Ergebnissen, die in meinem „Electronic Journal" publiziert sind. Ein dyadisches anstatt triadisches Zeichenmodell ergibt sich natürlich aus dem dichotomischen Charakter des Grossteils der Zeichen: So ist z.B. eine Grammatik eine Zuordnung von Ausdruck und Inhalt, d.h. zwischen Mittel und Objekt, und es ist also sinnlos und falsch, ein Drittes, angeblich Vermittelndes (Arbitraritätsgesetz!), hinzuzuhalluzinieren, nur weil das triadische Zeichenmodell eben noch einen Interpretantenbezug besitzt. Dyadisch ist auch die landläufige Vorstellung dessen, was ein Zeichen ist: Ein Etwas, das für ein Anderes steht (bzw. auf es zeigt, hinweist, es ersetzt, substituiert, repräsentiert, usw.).

2. Damit dürfte auch sogleich klar sein, dass weder das bezeichnete Objekt noch der Zeichensetzer, -interpret, -sender, -empfänger usw. in der Zeichenrelation stehen, denn sonst wäre das Zeichen entweder überflüssig (wenn das Objekt neben dem Zeichen steht) oder es wäre nicht von einem Kommunikationsschema unterscheidbar (was keiner mir bekannten Zeichendefinition entspricht). Auch wenn also Objekt und Interpret als ontologische Grössen (bzw. 0-stellige Relationen) natürlich keinen Platz in der triadischen Zeichenrelation als „verschachtelter" Relation über einer triadischen, einer dyadischen und einer monadischen Relation (Bense 1979, S. 53) haben, müssen sie mindestens als semiotische „Mitführungen" (Bense 1979, S. 43 ff.) in der Zeichenrelaiton präsent sein. In meiner dyadischen Semiotik erscheinen sie daher nicht als Kategorien (Relationen), sondern als Werte (Valenzen).

3. Allerdings ist die in Toth (2011) eingeführte dyadische Semiotik wie diejenige von Peirce, wo der sie abstrahiert ist, trivalent. Genau besehen, ist ein solches Konzept jedoch defektiv, denn in einer aristotelischen Hierarchie von der Hyle zum Logos haben wir zwei und nicht nur eine Vermittlungsstufe („verschmierte Kategorien"):

sS λόγος (Bewusstsein, β)

↑

oS

↑ Vermittlung: ZR = f(ω, β)

sO

↑

oO ὕλη (Welt, ω)

Zu ZR = f (ω, β) vgl. Bense (1975, S. 16), wo das Zeichen ebenfalls dyadisch definiert wird.

Das logisch-epistemologische Intervall [oO, sO, oS, sS] stellt somit die maximale Reichweite der Dichtotomie von Subjekt und Objekt und damit von logisch-ontologischer Monokontexturalität dar. Wie Kaehr (2011) korrekt gesehen hat, sind die logisch-epistemologisch-semiotischen Entsprechungen:

oO ↔ O (.2.)

sO ↔ M (.1.)

oS ↔ Q (.0.)

sS ↔ I (.3.).

4. Kaehr geht nun aber einen wesentlichen Schritt über diese Basistheorie hinaus, und zwar mit einer Definition eines Paares von dichtomischen Kenogrammschemata, die sehr nahe jener modernen Auffassung kommen, nach der praktisch kein Unterschied zwischen Zahl und Spiel mehr besteht (vgl. z.B. Conway 1976). Ich stelle diesen Prozess wie folgt dar:

Innen ⌈ Aussen

↓

[○⌈□]⌈[□⌈○]

$\downarrow$

$[sS \lceil oS] \lceil [oO \lceil sO]$

$\downarrow$

$[I \lceil Q] \lceil [O \lceil M]$

$\downarrow$

$[(3.a \lceil 0.d] \lceil [2.b \lceil 1.c]$ mit $a, b, c, d \in \{0, 1, 2, 3\}$.

Die dyadische Grundstruktur des Zeichens besteht also aus einer Subjekt- und einer Objektseite mit je 2 Positionen sowie 4 Werten. Das einfachste Schema ist somit:

Zei = [[Subjekt] $\lceil$ [Objekt]].

Die Subjektseite besteht aus dem Interpretanten und der Qualität, diese ist semiotisch die Mitführung des zum Zeichen erklärten Objekts. Die Objektseite besteht aus dem Objektbezug und dem Mittel oder aus Inhalt und Ausdruck. Somit entspricht also das Saussuresche Zeichenmodell nicht etwa unserem Zeichen Zei, sondern nur dessen Objektseite!

Sieht man von Permutationen der Positionen ab, so können also für alle 4 Positionen je 4 Werte in a, ..., d eingesetzt werden, wodurch wir $4^4 = 256$ dyadische Zeichenrelationen, {Zei}, bekommen. Dabei ist höchst bemerkenswert, dass, im Falle dass wir die Bensesche Dualisation zusammenlassen, jede Kategorie mit jeder anderen in einer Austauschrelation steht, vgl. z.B.

$\times(3.0) = (0.3)$, d.h. $I \rightarrow Q$

$\times(3.1) = (1.3)$, d.h. $I \rightarrow M$

$\times(3.2) = (2.3)$, d.h. $I \rightarrow O$

$\times(3.3) = (3.3)$, d.h. $I \rightarrow I$ (Selbstdualität).

In anderen Worten: Dualisation führt bei Zei nicht nur zum Austausch von Kategorien, sondern von Kategorien und Werten:

Cat ↔ Val.

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Subjektivität und Objektivität des architektonischen Objektes

1. Die Festellung, dass die Peircesche Zeichenklasse der allgemeinen Form

Zkl = (3.a 2.b 1.c)

jeweils in den Triaden den Subjekts- und in den Trichotomien den Objektspol der „verdoppelten Repräsentation“ (Bense) thematisiert

Zkl = [[S, O], [S, O], [S, O]]

Rth = ×Zkl = [[O, S], [O, S], [O, S]],

habe ich zuerst in Toth (2007a) publiziert. Sie stellt eine Verallgemeinerung der Feststellung Gfessers dar, dass im Zeichen sowohl die subjektive als auch die objektive Komponente des erkenntnistheoretischen Subjekt-Objekt-Schemas repräsentiert sind (Gfesser 1990, S. 133). Diese Tatsache wiederum gründet in einem Satz Benses, dass das Zeichen, aufgefasst als Funktion, die „Disjunktion zwischen Welt und Bewusstsein“ vermittle (Bense 1975, S. 16).

2. Nun hatte ich ebenfalls bereits in Toth (2007b, S. 64) gezeigt, dass von den 4 Kombinationsmöglichkeiten des Subjekt-Objekt-Schemas (objektives und subjektives Subjekt, subjektives und objektives Objekt) in der triadischen Semiotik nur 3 realisiert sind und dass die Peircesche Semiotik daher defektiv ist. Die fehlende Kategorie des objektiven Subjektes korrespondiert mit der Kategorie der „Qualität“, die bei Bense bestenfalls durch die mysteriöse Operation der „Mitführung“ (vgl. z.B. Bense 1979, S. 43) vage durchschimmert, doch entspricht sie der von Bense (1975, S. 41 ff., 65 f.) selbst eingeführten (und später in mehreren Arbeiten v.a. von Hans Michael Stiebing behandelten) Ebene der „Nullheit“ bzw. dem „ontologischen Raum“ (im Gegensatz zum semiotischen Raum). Zusammenfassend ergeben sich folgende epistemologisch-logisch-semiotische Korrespondenzen:

oS ↔ Q (.0.)

sO ↔ M (.1.)

oO ↔ O (.2.)

sS $\leftrightarrow$ I (.3.).

3. Kaehr (2011) geht nun aber einen wesentlichen Schritt über diese Basistheorie hinaus, und zwar mit einer Definition eines Paares von dichtomischen Kenogrammschemata, die sehr nahe jener Auffassung kommen, nach der praktisch kein Unterschied zwischen Zahl und Spiel mehr besteht (vgl. z.B. Conway 1976). Ich stelle diesen Prozess wie folgt dar:

Innen ⌈ Aussen

↓

[○ ⌈ □] ⌈ [□ ⌈ ○]

↓

[sS ⌈ oS] ⌈ [oO ⌈ sO]

↓

[I ⌈ Q] ⌈ [O ⌈ M]

↓

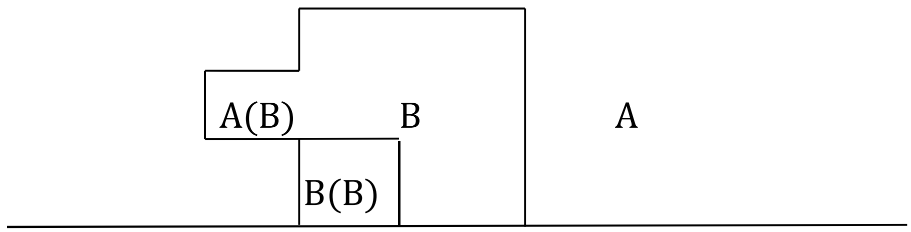
[(3.a ⌈ 0.d] ⌈ [2.b ⌈ 1.c] (a, b, c, d $\in$ {0, 1, 2, 3})

Im Grunde genommen kommen wir damit zwar nicht über meine bereits 2007 eingeführte Aufspaltung von Zeichenklassen in subjektive und objektale Pole der dyadischen Subzeichen hinaus:

Zei = [[Subjekt] ⌈ [Objekt]],

aber die in Verbindung mit der Subjekt- und Objektsseite der Erkenntnisrelation nun möglichen Austauschrelationen zwischen dem Innen (System) von Objekten oder Zeichen und ihrem Aussen (Umgebung) erlaubt eine interessante Mehrfachklassifikation, die wir hier an einem sich fast aufdrängenden Beispiel eines elementaren architektonischen Objektes untersuchen wollen.

4. Nehmen wir an, ein Haus B werde in eine Landschaft A gebaut:



Dann ist relativ zu A - B „innen“ und relativ von B - A „aussen“. Das Zimmer imm Haus ist „innen von innen“, aber der Balkon, der aussen am Haus ange bracht ist, ist „aussen von innen“. Wir haben also

A = A(A) = oO = (2.b)

B = B(B) = sS = (3.a)

A(B) = oS = (0.d)

B(B) = sO = (1.c),

wenn wir, wie in Toth (2007c) vorgeschlagen, die Kategorie der Nullheit in der erweiterten Zeichenklassen-Definition mit (0.d) bezeichnen:

ZR* = (3.a 2.b 1.c 0.d).

Dass man noch weitergehen, d.h. mehrfache Iterationen einführen kann, sei anhand der sog. „eigesperrten Räume“ gezeigt, d.h. Zimmer, die man nur von anderen Zimmern aus betreten kann (z.B. bei separaten Badezimmern, die nur vom Elternschlafzimmer aus erreichbar sind oder „Chaminadas“, Vorratskammern, die in einer Nische zwischen Küche und Aussenmauer des Hauses angebracht sind: man müsste sie als B(B(B)) = I(I(I)) bzw. ssS oder 3.(3.a) = (3.a)´ (Iterationszeichen) bezeichnen.

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Oneness

1. In einer 2-wertigen Logik hat es nur Platz für 1 Subjekt, denn der 2. Platz ist für das ewig unveränderliche Objekt als „factum brutum" reserviert. Da Objekte nicht iteriert werden können, hat auch eine Logik mit mehr als 2 Werten immer nur 1 Objekt. Das bedeutet, dass also eine n-wertige Logik immer n-1 Subjekte hat. Die natürlichen Sprachen sind der Logik, auf der sie doch beruhen, immerhin soweit voraus, als dass viele von Ihnen 6 „Personen" oder Subjekte besitzen. Allerdings findet grammatisch in der 3. Person, dem Er-/Sie-Subjekt (das selbst genusspezifisch koinzidiert) eine Koinzidenz mit dem Objekt statt: Für die Grammatik aller bekannten Sprache der Erde ist offenbar das ausserhalb von aller Subjektivität definierte Objekt funktionell gleich mit der besprochenen Person! Gewisse (sprachspezifisch unterschiedlich grammatikalisierte) Konstruktionen drücken dies dadurch aus, dass der personelle Subjekt-Objekt-Austausch meistens mit Hilfe sehr einfacher Transformationen geschieht, sog. Diathesen. Die bekannteste in den europäischen Sprachen ist die Aktiv-Passiv-Shift-Konstruktion:

1. Hans schlägt Fritz.

2. Fritz wird von Hans geschlagen.

2. One-ness kann nun zweierlei bedeuten: Entweder dass logische Subjekte koexistent oder dass sie koinzident addiert werden. Im ersten Fall liegt also syntaktisch eine Adjunktion vor:

$S_1 + S_2 + S_3 = (S_1, S_2, S_3)$,

im zweiten Falle eine Absorption:

$S_1 + S_2 + S_3 = S_4$.

Die besonders indische Idee der „Verschmelzung" von „Teilen" zu einem „Ganzen", mit der angeblich daraus resultierenden „universellen Wahrnehmung", die somit subjektübergreifend ist und eine mehrwertige Logik voraussetzt, gehört klar zum zweiten Falle. Behandeln wir trotzdem beide, denn wie man erkennen wird, laufen sie beide auf denselben Widerspruch hinaus.

2.1. Koexistente Addition. Da das Objekt jeder n-wertigen Logik konstant bleibt, können nur die Subjekte addiert werden. Das entspricht also etwa der hinter dem Diktum „quot homines tot sententiae“ stehenden Vorstellung. Sie führt also zwangsläufig zu einer Pluralität der Subjekt und damit nicht zu „Oneness“.

2.2. Koinzidente Addition. Die Absorption oder Amalgamierung mehrerer vorauszusetzender Subjekte in einer Art von (meist mit göttlichen Eigenschaften) ausgestattetem „Makro-Subjekt“ würde zu einer bei E.T.H. Hoffmann (vgl. Toth 2003a) und Oskar Panizza (vgl. Toth 2003b) eindrücklich geschilderten Situation führen, wo die Persönlichkeit der Schauplatz sich miteinander in Wettstreit befindlicher „Teil-Subjekte“ sind – von Hoffmann mit den Scherben im Prisma eines Kaleidoskops, von Panizza mit den Stimmen eines Schizophrenen verglichen. Was aber äusserlich wie die Situation einer 2-wertigen Logik mit 1 Subjekt ausschaut, ist in Wahrheit ein hochkomplexes Gebilde, in dem jedes Paar logisch immer noch geschiedener Subjekte eines weiteres Subjektes bedarf, das zur Vermittlung fungiert. Da also n Subjekte (n· (n+1))/ 2 Paare besitzen, hat und jedes Paar eines vermittelnden Subjektes bedarf, benötigt also bereits eine 3-wertige Logik (6-1) = 5 vermittelnden Subjekten. Bei 4 Werten sind es bereits 9, bei 5 Werten 14 und bei 99 logischen Werten sind es 4´949 vermittelnde Subjekte. Dass hier keine Spur von Unizität oder One-Ness sein kann, versteht sich also von selbst.

Man kann das Problem noch einfacher ausdrücken: Gehen wir aus von der 2-wertigen Logik, auf der all unser Denken, Fühlen und Handeln beruht, dann führt die koexistente Addition direkt zu einer Pluralität von Subjekten und steht damit in Widerspruch zum 1 Subjekt der 2-wertigen Logik. Geht man hingegen aus von der absorptiven Addition, dann wird zwar vordergründig die Struktur 1 Subjekt + 1 Objekt mit Hilfe eines zu hypostasierenden Makro-Subjektes wiederhergestellt, aber dieses weist eine innere Struktur auf mit erstens einer Pluralität vorgegebener Subjekte und zweitens mit einer schnell wachsenden Pluralität von zwischen Paaren dieser Subjekte vermittelnden Subjekten und führt somit noch höher in die Regionen mehrwertiger Logiken, die also unserer 2-wertigen Logik ebenfalls widersprechen.

„One-ness“ ist somit ein typischer Fall eines metaphysischen Denkfehlers, ein Denkfehler, der darauf beruht, dass bedenkenlos Austauschrelationen und Mehrsubjektivität im Rahmen unserer 2-wertigen Logik angenommen werden, die es hier nicht geben kann.

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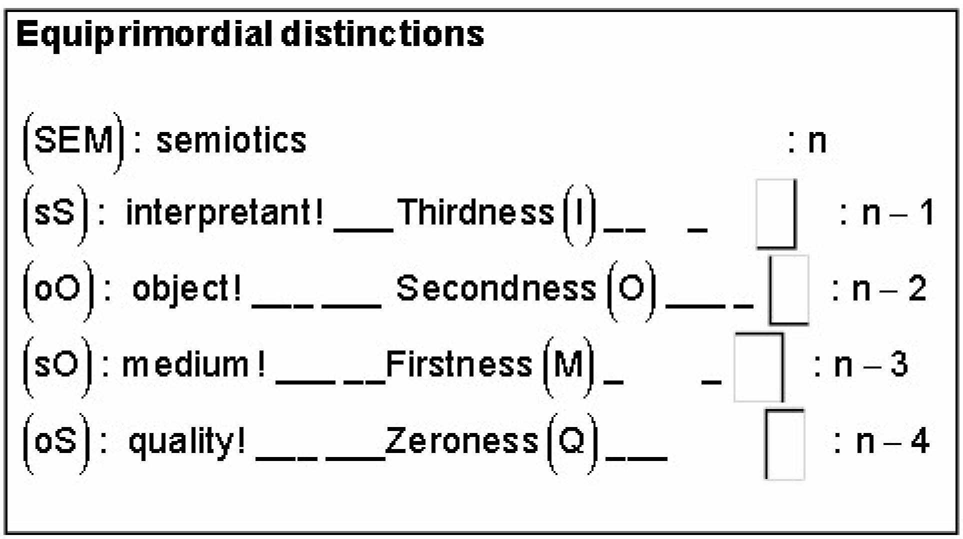
Systemtheoretische Satzperspektive

1. Sätze können entweder syntaktisch nach der Subjekt-Prädikat-Struktur, semantisch nach ihrer Agent-Aktions-Struktur (thematische Rollen), pragmatisch nach ihrer Topic-Comment/Thema-Rhema-Struktur (Informationsstruktur) oder schliesslich, was allen diesen grammatischen Modellen zugrunde liegt, logisch nach ihrer Subjekt-Prädikat-Struktur (Prädikationsstruktur) analysiert werden. Eine neuere Variante der Topic-Comment-Struktur, die mit dieser oft zu nicht kongruenten Ergebnissen führt, ist die Vordergrund-Hintergrund-Strategie (vgl. Toth 2010).

2. Im folgenden sei als weitere und viel abstraktere Grundlage die Korrespondenz logisch-epistemologischer, semiotischer und systemtheoretischer Funktionen zugrunde gelegt, die in Kaehr (2011, S. 7) wie folgt tabelliert sind:



Wir gehen aus von den folgenden zwei Satzpaaren:

1.a) Das Bild hängt an der Wand.

1.b) *Die Wand hängt an dem Bild.

2.a) Das Fahrrad steht neben der Garage.

2.b) ?Die Garage steht neben dem Fahrrad.

Die Ungrammatizität (*) bzw. Fragwürdigkeit (?) der b)-Sätze liegt an dem Austausch von Vordergrund und Hintergrund, sofern dieser eine Rolle spielt, vgl. noch

3.a) Der Wagen steht neben dem Fahrrad.

3.b) Das Fahrrad steht neben dem Wagen.

4.a) Der Kasten steht auf einem Podest.

4.b) Das Podest steht auf einem Kasten.

In den letzten vier Fällen spielt die Vorder-/Hintergrund-Unterscheidung keine Rolle.

3. Das Bild befindet sich normalerweise an einer Wand, an der es hängt. Es ist somit ein innerer Teil der Wand, die demnach vom Bild aus als äusserer Teil erscheint, d.h. es liegt in (1.a) die Relation IO vor. Die Wand selbst steht allerdings ebenfalls in der Relation IO, wobei von ihr aus gesehen das Hand, in dem sie sich befindet, das sie stützt und in dem sie Zimmer voneinander abtrennt, das Äussere ist. Die systemtheoretische Struktur der 1. Satzespaares ist somit

1.a) $I(IO) \rightarrow IO$

1.b) $*IO \rightarrow I(IO)$.

Beim 2. Satzpaar ist das Fahrrad relativ zur Garage das Äussere, diese selbst das Innere. Kurz gesagt, haben wir hier also den zum 1. Satzpaar dualen Fall vor uns:

2.a) $O(OI) \rightarrow OI$

2.b) $*OI \rightarrow O(OI)$.

Wie man erkennt, kann man die Umkehrung der Vorder-/Hintergrundstragie auf die Konversion der systemtheoretischen Relationen zurückführen.

4. Zusätzliche Hinweise und zugleich Bestätigung für die Richtigkeit der hier präsentierten Analyse erhält man, wenn man die Innen/Aussen-Distinktion iteriert, vgl.

5.a) *Die Leinwand des Bildes hängt an der Wand.

5.a) Das Kunstwerk hängt an der Wand.

Hier ist die Leinwand ein Teil des Bild, während das Bild ein Teil der Kunstwerke ist. Es liegen also (i.d.Reihenfolge) die Relationen

6.a) *I(I(IO)) → IO

6.b) I(IO)) → IO

Wiederum genau die dualen Relationen zum 5. Satzpaar erhält man, wenn man nun die folgenden Sätze konstruiert:

7.a) *Die Speichen des Fahrrades stehen neben der Garage.

7.b) Das Gefährt steht neben der Garage.

Wiederum stellt das Element des a)-Satzes „Speichen" einen Teil dar, während im b)-Satz das Fahrrad nun selbst als Teil erscheint, d.h. durch einen ihm übergeordneten Begriff ersetzt ist. Wir haben somit die Relationen

8.a) *O(O(OI)) → OI

8.b) O(OI) → OI.

5. Zusammenfassung: Zu Ungrammatizität bzw. borderline-Akzeptanz führen die Strukturen

1.b) *IO → I(IO).

2.b) *OI → O(OI)

6.a) *I(I(IO)) → IO

8.a) *O(O(OI)) → OI,

d.h. es gibt zwei systemtheoretische Prinzipien, welche Sätze nicht verletzen dürfen. Das erste lautet: Iterierte Relationen müssen nach rechts serialisiert werden (b-Sätze). Das zweite lautet: Mehrfach iterierte Relationen sind nur dann erlaubt, wenn auch ihre entsprechenden einfach iterierten manifest sind. Zum letzteren Punkt schulden wir noch Beispiele:

9.a) Das Fahrrad einschliesslich aller seiner Teile stand neben der Garage. (= Das Fahrrad stand unbeschädigt neben der Garage).

9.b) Das Bild, das auf Leinwand gemalt war, hing an der Wand.

Wie man erkennt, gibt es sehr viele Konstruktionen, um die fehlenden Mittelglieder in den systemtheoretischen Ketten

$*I(I(IO)) \rightarrow I(IO) \rightarrow IO$

$*O(O(OI)) \rightarrow O(OI) \rightarrow OI$

zu ergänzen.

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Logisch-epistemologische Ordnung der Fundamentalkategorien

1. Nach Toth (2007, S. 61 ff.) ist die Korrespondenz zwischen den logisch-epistemologischen Funktionen und den semiotischen Fundamentalkategorien wie folgt festgesetzt:

LER = (OI, IO, OO, II)

4ZR = (.0., .1., .2., .3.)

Hinzu tritt als korrespondierende systemtheoretische Relation (Kaehr 2011):

SZR = (⌞, ⌟ , ⌜, ⌝).

Es gilt also für alle drei Typen von Relationen die Ordnung

OI < IO < OO < II

.0. < .1. < .2. < .3.

⌞ < ⌟ < ⌜ < ⌝

2. Die Frage ist nur, ob das richtig ist. Auch wenn wir hier 4 logisch-epistemologische Funktionen haben, so ist deren kombinatorische Basis doch die zweiwertige Dichotomie von Subjekt und Objekt. Die klassische, übrigens bereits vorsokratische Pyramide führt vom Objekt zum Subjekt, von der ungeformten Materie bis hinauf zur entlösten Hyle, der reinen Form. Kombinationen, d.h. Mischformen zwischen Materie und Form sind also Zwischenprodukte innerhalb und nicht ausserhalb der Dichotomie, und zwar muss, in unsere Terminologie übertragen, IO vor OI gelten, da ersteres noch ein Objekt ist, letzteres jedoch bereits ein Subjekt. Damit kommen wir nun zu einer von der obigen ganz verschiedenen Ordnung:

OO < IO > OI > II

.2. < .1. < .0. < .3.

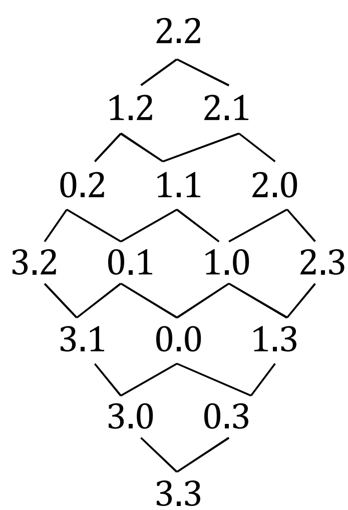
⌜ < ⌟ < ⌞ < ⌝ .

Die Semiose startet also in Einklang mit Bense (1967, S. 9) mit dem thetisch einzuführenden Objekt und endet mit Meta-Objekt, denn dieses ist als Zeichen selber drittheitlich, weil der Interpretant das Zeichen im Zeichen ist, das es ermöglicht, die Zeichenrelation als („verschachelte") „Relation über Relationen" zu definieren (vgl. Bense 1979, S. 53). Die obige Ordnung ist also eine nach der zunehmenden Semiotizität und damit der abnehmenden Ontizität (vgl. Bense 1976, S. 60 ff.).

Damit erhalten wir natürlich eine ebenfalls ganz verschiedene 4 × 4-Matrix:

	.2	.1	.0	.3
2.	2.0	2.1	2.0	2.3
1.	1.2	1.1	1.0	1.3
0.	0.2	0.1	0.0	0.3
3.	3.2	3.1	3.0	3.3

und im Anschluss an Toth (2011) einen völlig verschiedenen semiotischen Verband:



einen völlig verschiedenen systemtheoretischen Verband in der „I-O-Notation":

OO,OO

IO,OO OO,IO

OI,OO IO,IO OO,OI

II,OO OI,IO IO,OI OO,SS

SS,IO OI,OI IO,SS

SS,OI OI,SS

SS,SS

sowie einen ganze neuen Verband in der symbolischen Notationsweise:

┌ ┌

┘ ┌   ┌┘

└ ┌ ┘ ┘   ┌ └

┐ ┌ └┘ ┘ └ ┌┐

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Der semiotische und der Kaehrsche quadralektische Diamant

1. In Toth (2007, S. 61 ff.) hatte ich die Günthersche Idee der binären Subklassifikation der logisch-epistemologischen Dichotomie von Subjekt und Objekt, d.h. die Subkategorien objektives und subjektives Subjekt sowie subjektives und objektives Objekt mit der Benseschen Semiotik zusammengebracht und daraus ein tetradisches Zeichenmodell konstruiert, das ich präsemiotisch nannte, weil in sie das bezeichnete Objekt als Qualität des kategorialen Objektes (vgl. Bense 1975, S. 65 f.) neben der triadischen Peirceschen Zeichenrelation eingebettet ist. Rudolf Kaehr (2011) hat nun eine regelrechte polykontexturale Semiotik daraus entwickelt, wobei er sich auf meinen Entwurf einer präsemiotisch-handlungstheoretischen Semiotik (Toth 2010), die sog. „Theorie der Nacht", gestützt hatte. Er geht aus von der folgenden fundamentalen Diamantenstruktur (Kaehr 2011, S. 8):

Internal structure of the epistemological distinction system SEM = $[\mathrm{sS},\ \mathrm{oO},\ \mathrm{sO},\ \mathrm{oS}]$:

Diamond structure of $[\mathbf{sS},\ \mathbf{oO},\ \mathbf{sO},\ \mathbf{oS}]$

$$\begin{array}{ccccccc}
(\mathrm{sS})_3 & \cong & (\mathrm{sS})_1 & \longrightarrow & (\mathrm{oO})_1 & \cong & (\mathrm{oO})_4 \\
\downarrow & & \updownarrow & X & \updownarrow & & \uparrow \\
(\mathrm{sO})_3 & \cong & (\mathrm{sO})_2 & \longleftarrow & (\mathrm{oS})_2 & \cong & (\mathrm{oS})_4
\end{array}$$

$$(\mathrm{sO}) < (\mathrm{oS}) < (\mathrm{oO}) < (\mathrm{sS})$$

2. Nun hatten wir in Toth (2011) argumentiert, dass die tetradische Zeichenrelation, wenn ihre Relata epistemologisch nach ansteigender Semiotizität angeordnet sind, d.h. die Ordnung

LER = (OO < IO < OI < II)

und somit die semiotisch-fundamentalkategoriale Ordnung

4ZR = (.2. < .1. < .0. < .3.)

bzw.

rZR = (2.a 1.b 0.c 3.d)

aufweist, der Aufspaltung der Materie-Hyle-Pyramide korrespondiert, die sich aus der puren Materialität des Objektes über ihre beiden Haupt-„Mischungen“ bis hinauf zur Hyle läutert. Wir sprechen als von einer „natürlichen“ ordnung

4ZR = (.2. < .1. < .0. < .3.),

die insofern auch der Ordnung der Semiose entspricht, die sich zwischen den beiden Polen des vorgegebenen, vorthetischen Objektes (.2.) und dem intentionalen, thetischen Interpretanten (.3.) so abspielt, dass das für das Objekt bei der Metaobjektivierung gewählte materiale Substrant (.1.) das bezeichnete Objekt als kategoriales in dessen Qualität mitführt (.0.).

Falls es sich so verhält, dann erkennt man anhand der folgenden Skizzen, dass sich der abstrakte, Kaehrs polykontexturaler Semiotik zugrunde liegende Diamant und mein präsemiotischer Diamant nur durch Umkehrung der Abbildungsrichtungen bei den Subkategorien unterscheiden:

$$
\begin{tikzcd}
\underline{sS} \arrow[r] \arrow[d] & oO \\
sO & oS \arrow[u] \arrow[l]
\end{tikzcd}
\qquad
\begin{tikzcd}
\underline{oO} \arrow[r] & sO \arrow[d] \\
sS \arrow[u] & \underline{oS} \arrow[l]
\end{tikzcd}
$$

Gerüst des Kaehrschen Diamanten   Gerüst des präsemiotischen Diamanten

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Das Zeichen als Funktion seiner logisch-epistemologischen Funktionen

1. Nach Toth (2008, S. 36 ff.) sowie Kaehr (2011) besteht folgende Korrespondenz zwischen den Primzeichen und den vier aus der Dichotomie von Subjekt und Objekt kombinierbaren logisch-epistemologischen Funktionen:

(.0.) ↔ oS

(.1.) ↔ sO

(.2.) ↔ oO

(.3.) ↔ sS

Daraus folgt, dass das Zeichen als Funktion seiner logisch-epistemologischen Funktion geschrieben werden kann:

ZR = (.0., .1., .2., .3.) = (oS, sO, oO, sS).

2. Da ferner in einer 4-wertigen Logik folgende Korrespondenz besteht

oS ↔ Wir

sO ↔ Du

oO ↔ Es

sS ↔ Ich,

scheint es naheliegend, die obige logisch-epistemologische Zeichendefinition dahingehend zu verallgemeinern, dass man zulässt, dass jedes Primzeichen seine in der Monokontexturalität festgesetzte Kontextur verlassen und zwischen den Kontexturen wechseln (evtl. auch gleichzeitig in mehreren Kontexturen aufscheinen) kann, so dass also jedem Primnzeichen alle 4 logisch-epistemologischen Funktion zugeschrieben werden können. Das bedeutet also

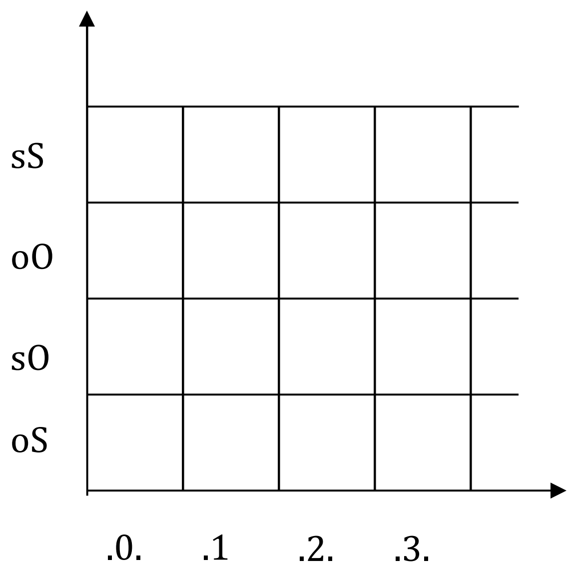
(.0.) = f(oS, sO, oO, sS)

(.1.) = f(oS, sO, oO, sS)

(.2.) = f(oS, sO, oO, sS)

(.3.) = f(oS, sO, oO, sS),

und man kann dann im Anschluss an Bense (1976, S. 60), wo ein Koordinatensystem auf der Basis der Abhängigkeit des Zeichens von seiner Ontizität und seiner Semiotizität entwickelt wurde, ein neues Koordinatensystem konstruieren, dessen Abszisse die Fundamentalkategorien und dessen Ordinate die logisch-epistemologischen Funktionen enthält:



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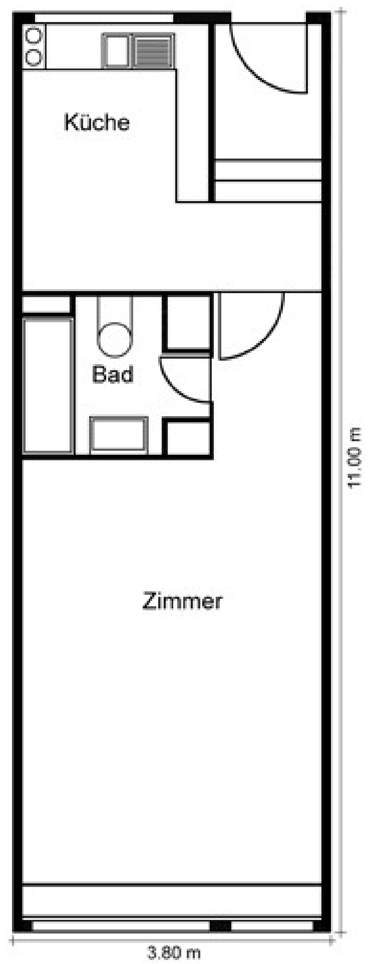
Gefangene Räume

1. Unter gefangenen Räumen werden in der Architektur solche Räume verstanden, die nur von anderen Räumen aus betreten werden können. Die bekanntesten Beispiele sind zusätzliche Badezimmer und Toiletten, die z.B. nur vom Elternschlafzimmer aus zugänglich sind. Ein im einzigen Zimmer einer Wohnung gefangenes Badezimmer zeigt der folgende Grundriss



„Gefangenes Bad/WC“, 1-Zimmer-Wohnung, Englischviertelstrasse 71,

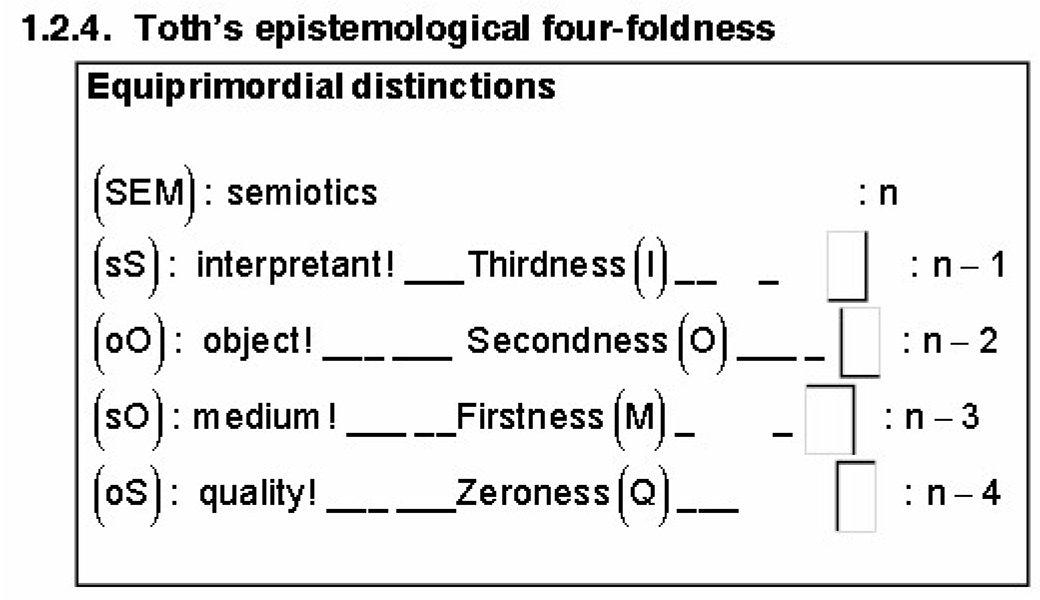
8032 Zürich, erbaut 1958.

Gefangene Badezimmer und Toiletten sind meist sekundäre Räume, da die primären vom Flur der Wohnung aus zugänglich sind, damit Gäste nicht gezwungen sind, in die Privatsphäre der Wohnung einzudringen. Falls man Speisekammern als Räume rechnet, sind sie immer gefangen, denn sie können nur von der Küche aus erreicht werden. Einen Grenzfall von architektonischer Gefangenheit stellen Liftkabinen, als Räume verstanden, dar, wenn man aus ihnen direkt in die Wohnungen eintreten kann. Von sekundärer Gefangenschaft könnte man dort sprechen, wo Trennwände zwischen Zimmern entfernt wurden, um grössere Räume, v.a. Stuben, zu bekommen, oder im ehemaligen Ostblock, um die Familien zustehenden Zimmerzahlen von Wohnungen zu reduzieren.

2. Systemtheoretisch stellen gefangene Räume ein Innen des Aussen, bezogen auf das gefangen haltende Zimmer dar, das aber selbst ein Innen des Aussen ist, bezogen auf die Wohnung. Da sich diese in der Regel innerhalb eines Hauses befindet (d.h. dass Haus und Wohnung nicht zusammenfallen), haben wir bei gefangenen Räumen also

O	→	IO	→	I(IO)	→	I(I(IO))	→	I(I(I(IO)))
Umgebung	→	Haus	→	Wohnung	→	Zimmer	→	gef. Raum
1		2		3		4		5

Gefangene Räume sind stehen also, bezogen auf die Umwelt, in welche das Haus gebaut ist, bereits Iterationen 5. Grades dar, d.h. die Innen-heit ist 4 Mal in die Aussenheit eingebettet. Da man wir von den folgenden Korrespondenzen ausgehen dürfen (Kaehr 2011, S. 7):

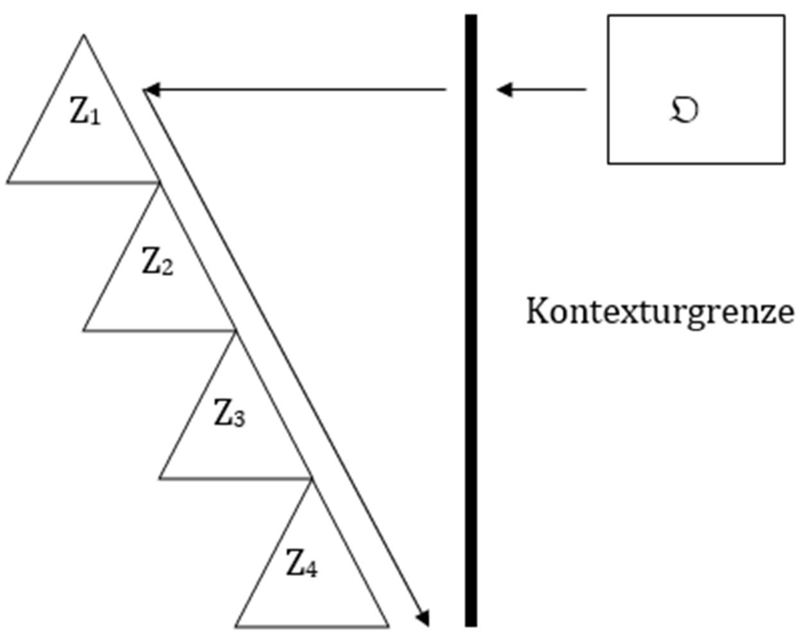


d.h. wir bekommen

$O \rightarrow IO \rightarrow I(IO) \rightarrow I(I(IO)) \rightarrow I(I(I(IO))) \Rightarrow$

$O \rightarrow Z(O) \rightarrow Z(Z(O)) \rightarrow Z(Z(Z(O))) \rightarrow Z(Z(Z(Z(O)))$

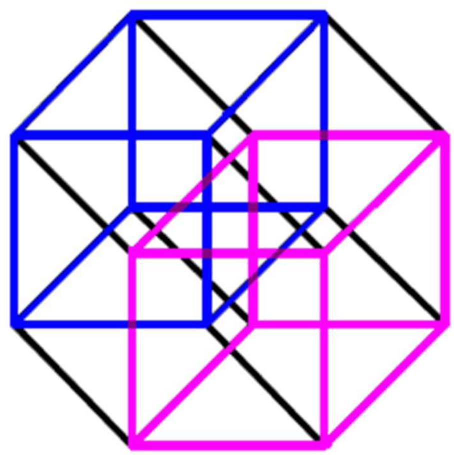
oder graphisch



Gefangene Räume sind also semiotische Funktionen mit 4 Variablen, davon 3 Zeichenrelationen (Zi) und 1 Objekt ($\mathfrak{O}$):

$Z_1 = f(Z_2, Z_3, Z_4, \mathfrak{O})$.

Zu ihrer Darstellung benötigt man deshalb ein 4-dimensionales Koordinatensystem, wie etwa dasjenige, das dem folgenden 4-dimensionalen euklidischen Würfel zugrunde liegt:



Jeder dieser 4 Kuben kann semiotisch als Stiebing-Kubus (vgl. Stiebing 1978, S. 77) dargestellt werden. Das bedeutet aber, dass die Zeichenfunktion Z_1 keine eindeutige Asymptote (vgl. Toth 2002) mehr zur $\mathfrak{O}$-Achse besitzt. Mit anderen Worten: Z_1 ist wegen der vierfachen Einbettung hinsichtlich seines Objektes ambig. Landläufig entspricht dies der Frage, zu welcher Umgebung des Hauses sich der gefangene Raum noch verhält.

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Hypallage

Unter Hypallage (griech. ὑπαλλαγή "Verwechslung") versteht man die Vertauschung der semantischen Zugehörigkeit von Wörtern. In Sonderheit fällt hierunter die uns hier allein interessierende Verwechslung von Subjekt und Objekt, die, da sie systemtheoretische Relationen verkehrt, von semiotischer Relevanz ist (vgl. z.B. Toth 2011). Ausserhalb der römischen und griechischen Rhetorik sind Beispiele selten, vgl. aus dem St. Gallerdeutsch „en gschtollne Brüeder", wörtl. „ein gestohlener Bruder", d.h. einer, der Wucherpreise verlangt, kurz: ein Wucherer.

Viel häufiger ist jedoch der Einsatz der Hypallage als Stilmittel, z.B. bei der logisch als Subjekt-Objekt-Relation kodierten Eigentümer-Eigentums-Beuziehung (Lausberg 1990, S. 344):

2) die Eigentümer-Eigentum-Beziehung (s. § 568, 1 c) in der syntaktischen Umorientierung des Adjektivs, die modern *enallage adiectivi* (s. Havers p. 281 s.v.; E. Norden, Aeneis VI, 4. Aufl., 1957, p. 112) oder *hypallage adiectivi* (s. H. Menge, Repetitorium..., 11. Aufl., 1953, § 197) genannt wird (zu den beiden Termini s. § 509). Sie tritt beim *genetivus possessivus* auf, und zwar wird entweder das zum Genetiv semantisch passende Adjektiv zum übergeordneten Substantiv gestellt (Liv. 1, 1, 4 *maiora rerum initia* für *maiorum rerum initia;* Aen. 12, 199 *vimque deum infernam* für *vimque deorum infernorum;* Ecl. 9, 46 *antiquos signorum . . . ortus;* Claud. 1, 25 *fulva Leonis ira*), oder das zum übergeordneten Substantiv passende Adjektiv wird zum Genetiv gestellt (Aen. 1, 7 *altae moenia Romae* für *alta moenia Romae;* 3, 411 *angusti . . . claustra Pelori* für *angusta claustra Pelori;* Liv. 1, 4, 4 *ad iusti cursum . . . amnis*). Darüber hinaus kann der Genetiv selbst unterdrückt werden und das zu diesem passende Adjektiv dem allein übrigbleibenden übergeordneten Substantiv zugeordnet werden: Hor. carm. 1, 15, 19 *adulteros crines* ›deine Haare, du Ehebrecher‹; Mart. 1, 35, (36), 8 *stolatum pudorem* ›das einer *stolata* (Matrone) eigene Schamgefühl‹ (s. V. Pöschl, Hermes 84, 1956, p. 79 ss.). Die Erscheinung kommt auch außerhalb des genetivischen Verhältnisses vor, und zwar hier auch als Adjektivtausch zwischen zwei Satzteilen: Serv. Aen. 9, 453 (455) ›*tepidaque recentem / caede locum*‹ *hypallage est:* ›*tepidum locum recenti caede*‹; Aen. 6, 268 *ibant obscuri sola sub nocte* (für *soli obscura*). – In allen Fällen erfährt das Adjektiv durch die syntaktische Bezugsverschiebung eine semantische Verschiebung und Bereicherung (eben die Metonymie), die die Phantasie des Publikums durch Verfremdung (*hypallage;* s. §§ 509, 1237 ss.) anregt, so daß die Rückübersetzung in die normale syntaktische Abhängigkeit (etwa *alta moenia Romae*) als inhaltsschwache Platitüde wirkt.

Weitere Belege findet sich in dem folgenden Ausschnitt aus einem Seneca-Kommentar (Hillen 1989, S. 148 f.):

3.2.4 Vertauschungen von weiteren (aufeinander bezogenen) Ausdrücken

Dieser Vertauschungstyp[751] findet sich bei Seneca in den Formen pleonastischer Sprache, die eine Sache/einen Sachverhalt entweder in nicht-attributivem Verhältnis durch zwei verschiedene Satzteile (3.2.4.1 Subjekt und Akkusativobjekt[752]) oder in adnominaler Zuordnung (3.2.4.2 explikativer Ablativ oder 3.2.4.3 gen. identitatis/explicativus), die weitgehend synonym sind, erfaßt. Bewertungskriterium ist der inhaltliche Aspekt der übergeordneten Begriffs und der semantische der vox propria, d. h. der eigentlichen Bezeichnung oder Vokabel für eine Sache (gegenüber der uneigentlichen wie z. B. Metonymie), beim Ablativ zusätzlich die kontrastierende Struktur im Genetiv (s. 2.1.4 S. 142 ff.).

3.2.4.1 Subjekt und Objekt vertauscht

Die Belege gehören zur Stilistik der Auffächerung (2.1.2 S. 123 ff.).

1) Hf 712 ff. ***alter** (latex) quieto similis (hunc iurant dei)*
*tacente sacram **devehens fluvio Styga***
at hic … rapitur
et saxa fluctu volvit Acheron
statt: *alterum qu. similem / tacente s. devehit fluvio Styx*

2) Phae 701 *(sequor per) amnes, **unda**, quos **torrens rapit***
statt: *amnes, qui undam torrentem rapiunt*

Zu 1) Z. St. s. S. 137. Schon ein Blick auf einen Teil der Auffächerungen, die einen Flußnamen enthalten, zeigt, daß die normale Konstruktion diesen Namen im Nominativ (bzw. in Subjektsfunktion) aufweist, s. z. B. Ov. met. 1,423 (*Nilus*) *sua flumina reddit*; Lucan. 10,252 f. *flumina Nilus / … perfert*; Stat. silv. 1,2,205 (*Alpheos*) *flumina trahit*; Sil. 3,448 *extrahit amnem* (*Rhodanus*), 17,642[753].

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751 Die Literatur zu diesem Thema ist recht spärlich: Lediglich R. Hildebrandt, Beiträge zur Erklärung des Gedichtes Aetna, Programm Leipzig 1900,9 äußert sich zu Aetna 20 *aversumque diem sparsumve in semine dentem* — vorbehaltlich Scaligers Konjektur *in semina* —, statt: *sparsumve in dente semen*. *Semen* ist der uneigentliche Begriff, der den *dentes* im Mythos der Drachensaat erst zukommt. Die dort weiter angeführten Stellen sind, falls überhaupt Vertauschungen, keine Belege für Vertauschungen von Bezeichnungen für die gleiche Sache. — Vgl. noch Langen zu Val. Fl. 1,499 (zu Phoen 250).

752 Diese Vertauschung erfüllt die Bedingungen für 3.2.1, aber der inhaltliche Aspekt der sachlichen Identität des von beiden Satzteilen Beschriebenen hat größeres Gewicht.

753 Vgl. die Auffächerung mit abl.: Hor. c. 2,14,17 f. *flumine languido | Cocytos*; Verg. georg. 4,288 *flumine Nilum*, 7,30, 9,31; Ov. met. 2,251 *suo Tagus amne vehit*; Sil. 4,84, 8,180.454 u. ö.; zu Seneca s. 2.1.1.1 S. 52. Die Genetivkonstruktion ist seltener: Hor. epod. 13,14 *Scamandri flumina*; Verg. Aen. 12,331; Prop. 2,28,18 *Nili flumine*, 3,11,51.

Für die Semiotik bedeutet dies die logisch-epistemologische Konversion der den Subzeichen einer tetradischen Zeichenrelation

4-ZR = (3.a 2.b 1.c 0.d) mit a, ..., d ∈ {0, 1, 2, 3}

entsprechenden systemtheoretischen Relationen, d.h.

subjektives Subjekt → objectives Objekt: (3.a) → (2.b)

objektives Objekt → subjektives Subjekt: (2.b) → (3.a)

subjektives Objekt → objektives Subjekt: (1.c) → (0.d)

objektives Subjekt → subjektives Objekt: (0.d) → (1.c).

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Derricks Theorie. Elemente einer objektalen Spurentheorie

1. Nur Zeichen und Spuren benötigen materiale Träger, sog. Medien. Im Gegensatz zu Zeichen sind jedoch Spuren nicht im Sinne Benses (1967, S. 9) metaobjektiviert, d.h. eben thetisch als Zeichen eingeführt; sie werden oft sogar nicht-intendierter Weise auf ihren nachträglichen Trägersubstanzen hinterlassen. Wenn aber Spuren keine Zeichen sind, so folgt wegen der absoluten Dichotomie von Zeichen und Objekt, daß sie eben Objekte sein müssen. Objekte können deshalb zu Spuren werden bzw. als Spuren interpretiert werden, ohne daß diese Identifizierung eine Zeichensetzung bedeutet. Unter Objekt wird hier im semiotischen Sinne also alles verstanden, was kein Zeichen ist, d.h. auch Vorgänge, Abläufe, Zustände und selbst nur gedachte oder sogar illusionäre Gegen-Stände, d.h. alles, worauf sich das Denken richtet oder von diesem erzeugt wird, und zwar ohne damit zu implizieren, daß diese Gegenstände des Denkens dadurch, daß an sie gedacht wird oder daß sie denkend erzeugt werden, bereits Zeichen sind. Allein die Existenz von Spuren verbietet somit, abstrakte oder apriorische Objekte anzunehmen.

2. Ein Objekt allein kann niemals Spur sein, daraus folgt, daß Spuren als n-tupel von Objekten definiert werden müssen. Als einfachsten Fall haben wir ein Paar von Objekten

$\Omega_i, \Omega_j \rightarrow [\Omega_i, \Omega_j]$,

woraus aber noch nicht folgt, daß entweder Ω_i eine Spur von Ω_j oder Ω_j eine Spur von Ω_i ist. Genauso wie Zeichen verlangen Spuren eine referentielle Beziehung

$[\Omega_i, \Omega_j] \rightarrow [\Omega_{i,a}, \Omega_{j,b}]$,

wobei a, b $\in \{\rightarrow, \leftarrow\}$, d.h. 2 Objekte können, falls sie einem Spuren-Tupel angehören, auf 2! = 4 Möglichkeiten referieren: $[\Omega_{i\rightarrow}, \Omega_{j\rightarrow}]$, $[\Omega_{i\rightarrow}, \Omega_{j\leftarrow}]$, $[\Omega_{i\leftarrow}, \Omega_{j\rightarrow}]$, $[\Omega_{i\leftarrow}, \Omega_{j\leftarrow}]$. Inhaltlich bedeutet das, daß entweder eines der beiden Objekte auf das andere oder eines oder beide auf ein Objekt außerhalb des Spuren-Paars referieren können. Für ein n-Spuren-Tupel gibt es also n! Möglichkeiten.

3. Ferner muß die Referenz qualitativ determinierbar sein. Spuren können material oder immaterial sein, d.h. sie können qualitativ (z.B. Blut), quantitativ (Temperatur) oder referentiell (Täter und Tatort) sein. Ferner kann die Referenz von Spuren iconisch (Haare), indexikalisch (Fingerabdruck) oder symbolisch (aufgezeichnetes Gespräch) sein. Schließlich können Spuren – wie bereits aus ihrer Definition hervorgeht – alle drei Grundformen von Konnexen innerhalb der sich definierenden n-Tupel einnehmen, d.h. sie können rhematisch, dicentisch oder argumentisch ist. Daraus folgt, daß sich Spuren von Zeichen hinsichtlich der neun Partialrelationen des triadisch-trichotomischen Zeichenmodells gar nicht unterscheiden. Damit bekommen wir

$[\Omega_{i,a}, \Omega_{j,b}] \rightarrow [\Omega_{i,A,a}, \Omega_{j,Bb}]$

mit A, B $\in$ {(1.1), ..., (3.3)}, zusammengefaßt also

$[\Omega_i, \Omega_j] \rightarrow [\Omega_{i,a}, \Omega_{j,b}] \rightarrow [\Omega_{i,A,a}, \Omega_{j,Bb}]$,

wobei wir die anfangs zur Unterscheidung zweier Objekte eingeführte Numerierung weglassen können. Damit haben wir

$Sp = [\Omega_{A,a}, \Omega_{Bb}]$

mit A, B $\in$ {(1.1), ..., (3.3)} und a, b $\in$ {$\rightarrow$, $\leftarrow$}. Spuren sind damit nichts anderes als indizierte Objekte, und zwar in doppeltem Sinne: Mathematisch als indizierte Objekte Ω_1, ..., Ω_n, und inhaltlich oder praktisch im Sinne von interpretierbaren, referentiellen Objekten. Die theoretische Gesamtheit referentieller Objekte kann man als

$\underline{\Omega} = \{ \Omega_i, \ldots, \Omega_n\}$,

im Sinne einer Familie von referentiellen Objekten definieren.

4. Eine wesentliche Komplikation ergibt sich nun dadurch, daß die Indexmengen {a, b} und {A, B} miteinander in einer Funktionsbeziehung stehen, so zwar, daß erstens die Entscheidung, ob eine Spur σ auf einer Objekt Ω_i oder ein anderes Objekt Ω_j oder gar nicht referiert (und in diesem letzteren Falle gar keine Spur ist), von der Menge {A, B} abhängt

$\sigma = f(\{A, B\})$,

zweitens aber auch die Entscheidung über die Qualität der Spuren von {a, b}, d.h. von der Referenz abhängt:

$\sigma = f(\{a, b\})$.

Somit ergibt sich die „paradoxe“ (heterarchive und damit die zweiwertige Logik hinter sich lassende) Gleichung

$\sigma = f(\{A, B\}, \{a, b\})$,

die praktisch jedoch dadurch aufgelöst sind, daß entweder weitere Spuren aufgedeckt werden oder sich weitere Referenzen einstellen. Semiotisch bedeutet das, daß es mit der doppelten Indizierung referentieller Objekte nicht getan ist: Diese genügt zwar zur Definition einer Spur, aber nicht für die Darstellung einer Spurentheorie, da die Spuren selbst untereinander ebenfalls referieren, und zwar „in beiden Richtungen“, d.h. zwischen n Spuren $\sigma_1 \dots \sigma_n$ gibt es wieder n! kombinatorische Möglichkeiten, z.B. bei drei Spuren σ_i, σ_j und σk die Fälle $[\sigma_i \rightarrow \sigma_j \rightarrow \sigma_k]$, $[\sigma_i \rightarrow \sigma_j \leftarrow \sigma_k]$, $[\sigma_i \leftarrow \sigma_j \rightarrow \sigma_k]$ und $[\sigma_i \leftarrow \sigma_j \leftarrow \sigma_k]$.

5. Abschließend kann man die Qualität der Referenz einer Spur – die wiederum in heterarchischer Weise einerseits von der Interpretation der referentiellen Objekte abhängt, sie aber gleichzeitig determiniert – mit Hilfe der mereotopologischen Semiotik (vgl. z.B. Toth 2010, 2011a, b) darstellen:

5.1. $O(\Omega_{i\rightarrow}, \Omega_{j\rightarrow}) := \exists \Omega k(P(\Omega_{k\rightarrow}, \Omega_{i\rightarrow}) \wedge P(\Omega_{k\rightarrow}, \Omega_{j\rightarrow}))$

$O(\Omega_{i\leftarrow}, \Omega_{j\leftarrow}) := \exists \Omega k(P(\Omega_{k\leftarrow}, \Omega_{i\leftarrow}) \wedge P(\Omega_{k\leftarrow}, \Omega_{j\leftarrow}))$ Überlappung

5.2. $A(\Omega_i, \Omega_j) := C(\Omega_{i\rightarrow}, \Omega_{j\rightarrow}) \wedge \neg O(\Omega_{i\rightarrow}, \Omega_{j\rightarrow})$

$A(\Omega_{i\leftarrow}, \Omega_{j\leftarrow}) := C(\Omega_{i\leftarrow}, \Omega_{j\leftarrow}) \wedge \neg O(\Omega_{i\leftarrow}, \Omega_{j\leftarrow})$ Angrenzung

5.3. $E(\Omega_i, \Omega_j) := P(\Omega_{i\rightarrow}, \Omega_{j\rightarrow}) \wedge P(\Omega_{j\rightarrow}, \Omega_{i\rightarrow})$

$E(\Omega_i, \Omega_j) := P(\Omega_{i\leftarrow}, \Omega_{j\leftarrow}) \wedge P(\Omega_{j\leftarrow}, \Omega_{i\leftarrow})$ Gleichheit

5.4. $PP(\Omega_i, \Omega_j) := P(\Omega_{i\rightarrow}, \Omega_{j\rightarrow}) \wedge \neg P(\Omega_{j\rightarrow}, \Omega_{i\rightarrow})$

$P(\Omega_{i\leftarrow}, \Omega_{j\leftarrow}) \wedge \neg P(\Omega_{j\leftarrow}, \Omega_{i\leftarrow})$ echter Teil

5.5. $TP(\Omega_i, \Omega_j)$:= $P(\Omega_{i\rightarrow}, \Omega_{j\rightarrow}) \wedge \exists\Omega_{k\rightarrow}(A(\Omega_{k\rightarrow}, \Omega_{i\rightarrow}) \wedge A(\Omega_{k\rightarrow}, \Omega_{j\rightarrow}))$

$P(\Omega_{i\leftarrow}, \Omega_{j\leftarrow}) \wedge \exists\Omega_{k\leftarrow}(A(\Omega_{k\leftarrow}, \Omega_{i\leftarrow}) \wedge A(\Omega_{k\leftarrow}, \Omega_{j\leftarrow}))$

tangentialer Teil

5.6. $O(\Omega_{i\rightleftarrows}, \Omega_{j\rightleftarrows})$:= $\exists\Omega_k(P(\Omega_{k\rightleftarrows}, \Omega_{i\rightleftarrows}) \wedge P(\Omega_{k\rightleftarrows}, \Omega_{j\rightleftarrows}))$

$O(\Omega_{i\leftrightarrows}, \Omega_{j\leftrightarrows})$:= $\exists\Omega_k(P(\Omega_{k\leftrightarrows}, \Omega_{i\leftrightarrows}) \wedge P(\Omega_{k\leftrightarrows}, \Omega_{j\leftrightarrows}))$ Überlappung

5.7. $A(\Omega_i, \Omega_j)$:= $C(\Omega_{i\rightleftarrows}, \Omega_{j\rightleftarrows}) \wedge \neg O(\Omega_{i\rightleftarrows}, \Omega_{j\rightleftarrows})$

$A(\Omega_{i\leftrightarrows}, \Omega_{j\leftrightarrows}) := C(\Omega_{i\leftrightarrows}, \Omega_{j\leftrightarrows}) \wedge \neg O(\Omega_{i\leftrightarrows}, \Omega_{j\leftrightarrows})$

Angrenzung

5.8. $E(\Omega_i, \Omega_j)$:= $P(\Omega_{i\rightleftarrows}, \Omega_{j\rightleftarrows}) \wedge P(\Omega_{j\rightleftarrows}, \Omega_{i\rightleftarrows})$

$E(\Omega_i, \Omega_j)$:= $P(\Omega_{i\leftrightarrows}, \Omega_{j\leftrightarrows}) \wedge P(\Omega_{j\leftrightarrows}, \Omega_{i\leftrightarrows})$ Gleichheit

5.9. $PP(\Omega_i, \Omega_j)$:= $P(\Omega_{i\rightleftarrows}, \Omega_{j\rightleftarrows}) \wedge \neg P(\Omega_{j\rightleftarrows}, \Omega_{i\rightleftarrows})$

$P(\Omega_{i\leftrightarrows}, \Omega_{j\leftrightarrows}) \wedge \neg P(\Omega_{j\leftrightarrows}, \Omega_{i\leftrightarrows})$ echter Teil

5.10. $TP(\Omega_i, \Omega_j)$:= $P(\Omega_{i\rightleftarrows}, \Omega_{j\rightleftarrows}) \wedge \exists\Omega_{k\rightleftarrows}(A(\Omega_{k\rightleftarrows}, \Omega_{i\rightleftarrows}) \wedge A(\Omega_{k\rightleftarrows}, \Omega_{j\rightleftarrows}))$

$P(\Omega_{i\leftrightarrows}, \Omega_{j\leftrightarrows}) \wedge \exists\Omega_{k\leftrightarrows} (A(\Omega_{k\leftrightarrows}, \Omega_{i\leftrightarrows}) \wedge A(\Omega_{k\leftrightarrows}, \Omega_{j\leftrightarrows}))$

tangentialer Teil

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Systemische Zeichenoperationen und Zeichenstrukturen

1. Die Definition der triadisch-trichotomischen Zeichenrelation von Peirce und Bense läßt, mindestens nach der sog. semiotischen „Basistheorie“ keine bedeutenden operativen und strukturellen Variationen zu: Es gibt, grob gesagt, eine Zeichenklasse der Form

$Zkl = (3.a\ 2.b\ 1.c)$

und eine ihr duale Realitätsthematik der Form

$\times Zkl = Rth = (c.1\ b.2\ a.3)$,

d.h. es sind z.B. die Konversionen

$K(3.a\ 2.b\ 1.c) = (1.c\ 2.b\ 3.a)$

$K(c.1\ b.2\ a.3) = (a.3\ b.2\ c.1)$

gar nicht definiert, obwohl erst alle 4 Strukturen zusammen den bereits in der Peirce-Bense-Semiotik angelegten Strukturreichtum ausmachen.

2. Bisher unbekannte operative und strukturelle Komplexität bieten dagegen die in Toth (2012a) eingeführten systemischen Repräsentationsklassen, v.a. wenn man die in Toth (2012b) eingeführten relationalen Einbettungszahlen zu ihrer Darstellung verwendet:

1. $RS = [[[1_{-2}, a], [1_{-1}, b], [1, c]]$

$$\left.\begin{array}{l} 2.\ \times_1 RS = [[c, 1], [b, 1_{-1}], [a, 1_{-2}]]] \\ 3.\ \times_2 RS = [[c, 1], [b, {}_{-1}1], [a, {}_{-2}1]]] \end{array}\right\} \quad \text{Dualisationen}$$

$$\left.\begin{array}{l} 4.\ K_1 RS = [[[c, 1_{-2}], [b, 1_{-1}], [a, 1]] \\ 5.\ K_2 RS = [[[c, {}_{-2}1], [b, {}_{-1}1], [a, 1]] \end{array}\right\} \quad \text{Konversionen}$$

Weiter ergibt sich die Möglichkeit, im Anschluß an Toth (2011), gerichtete REZ einzuführen. Damit erhält man

1. $RS = [[[1_{-2}, a^{\leftrightarrows}], [1_{-1}, b^{\leftrightarrows}], [1, c^{\leftrightarrows}]]$

$$\left.\begin{array}{l} 2.\ \times_1 RS = [[c^{\leftrightarrows}, 1], [b^{\leftrightarrows}, 1_{-1}], [a^{\leftrightarrows}, 1_{-2}]]] \\ 3.\ \times_2 RS = [[c^{\leftrightarrows}, 1], [b^{\leftrightarrows}, {}_{-1}1], [a^{\leftrightarrows}, {}_{-2}1]]] \end{array}\right\} \text{Dualisationen}$$

$$\left.\begin{array}{l} 4.\ K_1 RS = [[[c^{\leftrightarrows}, 1_{-2}], [b^{\leftrightarrows}, 1_{-1}], [a^{\leftrightarrows}, 1]] \\ 5.\ K_2 RS = [[[c^{\leftrightarrows}, {}_{-2}1], [b^{\leftrightarrows}, {}_{-1}1], [a^{\leftrightarrows}, 1]] \end{array}\right\} \text{Konversionen}$$

Hinzu kommen natürlich noch die Permutationen der Partialrelationen, d.h. für die Kategorien zur Peirce-Bense-Semiotik $\underline{P}$ = {(M, O, I), (M, I, O), (O, M, I), (O, I, M), (I, O, M), (I, M, O)}; sie sind für die Semiotik natürlich alle nicht-isomorph zueinander und daher teilweise bereits in der Peirce-Bense-Semiotik definiert.

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Der Austausch von Ich und Du

1. Außer im Werk von Oskar Panizza (vgl. Toth 2006a, 2012a) ist die Aufhebung der zweiwertigen Kontexturgrenzen ein Leitmotiv im Werk von E.T.A. Hoffmann (vgl. Toth 2006b). Während es sich bei Panizza allerdings meist um objektbezogene Kontexturgrenzen handelt, findet man die logisch und semiotisch besonders interessante Fälle des Austausches von subjektiven und objektiven Subjekten zur Hauptsache in Hoffmanns "Klein Zaches, genannt Zinnober" (1819), in dem nicht weniger als dreizehn Fälle gezählt werden können, vgl. z.B.

> “Balthasar griff herab nach dem Kleinen, ihm aufzuhelfen, und berührte dabei unversehens sein Haar. Da stiess der Kleine einen gellenden Schrei aus, dass es im ganzen Saal widerhallte und die Gäste erschrocken auffuhren von ihren Sitzen. Man umringte den Balthasar und fragte durcheinander, warum er denn um des Himmels willen so entsetzlich geschrien” (ed. H. Leber, S. 310).

Obwohl also Klein Zaches schreit, wird der Schrei dem Balthasar angelastet. Doch es kommt noch schöner:

> “Balthasar glaubte, dass der rechte Augenblick gekommen, mit seinem Gedicht von der Liebe der Nachtigall zur Purpurrose hervorzurücken [...]. Sein eignes Werk, das in der Tat aus wahrhaftem Dichtergemüt mit voller Kraft, mit regem Leben hervorgeströmt, begeisterte ihn mehr und mehr. Sein Vortrag, immer leidenschaftlicher steigernd, verriet die innere Glut des liebenden Herzens. Er bebte vor Entzücken, als leise Seufzer – manches leise Ach – der Frauen, mancher Ausruf der Männer: ‘Herrlich – vortrefflich, göttlich!’ ihn überzeugten, dass sein Gedicht alle hinriss. Endlich hatte er geendet. Da riefen alle: ‘Welch ein Gedicht! – Welche Gedanken – welche Phantasie, was für schöne Verse – welcher Wohlklang – Dank – Dank Ihnen, bester Herr Zinnober, für den göttlichen Genuss” (ed. H. Leber, S. 311 ff.).

2. Doch Hoffmann begnügt sich nicht mit dem simplen Austausch von subjektivem Subjekt und Objekt, wie es etwa Oscar Wilde in seinem “Bildnis des Dorian Gray” oder Edgar Allan Poe im “Oval Portrait” getan hatten: Im

folgenden Fall ist Prof. Terpin sogar subjektives und objektives Subjekt zugleich:

> "Als sie eintraten, stand der Professor Mosch Terpin allein in der Mitte, die Instrumente noch in der Hand, womit er irgendein physikalisches Experiment gemacht, starres Staunen im Gesicht. Die ganze Gesellschaft hatte sich um den kleinen Zinnober gesammelt, der, den Stock untergestemmt, auf den Fussspitzen dastand und mit stolzem Blick den Beifall einnahm, der ihm von allen Seiten zuströmte. Man wandte sich wieder zum Professor, der ein anderes sehr artiges Kunststückchen machte. Kaum war er fertig, als wiederum alle, den Kleinen umringend, riefen: 'Herrlich – vortrefflich, lieber Herr Zinnober!'. – Endlich sprang auch Mosch Terpin zu dem Kleinen hin und rief zehnmal stärker als die übrigen: 'Herrlich – vortrefflich, lieber Herr Zinnober!'" (ed. H. Leber, S. 313 f.).

Wie alle angeführten (sowie die hier unterdrückten) Beispiele zeigen, befindet sich von allen Partizipanten der Erzählung offenbar einzig Balthasar in der monokontexturalen Welt und dient somit quasi als "Verbindungsmann" zum ebenfalls in der Monokontexturalität lebenden Lesenden. Umgekehrt sind jedoch diese Austauschungen zwischen subjektivem Subjekt und objektivem Subjekt über die normalerweise zwischen beiden bestehenden Kontexturgrenzen hinweg offenbar auch für jemanden wie Balthasar oder den Leser bemerkbar. In Hoffmanns wie in Panizzas Welt sind eben diese Kontexturgrenzen nicht außerhalb der logischen Abgrenzungen der Realität angesiedelt, sondern laufen mitten durch sie hindurch, vgl. etwa bei Hoffmann:

> "Ungeachtet des weiten Weges bis in die einsame Strasse, in der sich das uralte Haus des Archivarius Lindhorst befand, war der Student Anselmus vor zwölf Uhr an der Haustür. Da stand er und schaute den grossen schönen, bronzenen Türklopfer an; aber als er nun auf den letzten, die Luft mit mächtigem Klange durchbrechenden Schlag der Turmuhr an der Kreuzkirche den Türklopfer ergreifen wollte, da verzog sich das metallene Gesicht im ekelhaften Spiel blauglühender Lichtblicke zum grinsenden Lächeln. Ach! Es war ja das Apfelweib vom Schwarzen Tor! [...] Die

> Klingelschnur senkte sich hinab und wurde zur weissen, durchsichtigen Riesenschlange; sie umwand und drückte ihn, fester und fester ihr Gewinde schnürend, zusammen, dass die mürben zermalmten Glieder knackend zerbröckelten und sein Blut aus den Adern spritzte, eindringend in den durchsichtigen Leib der Schlange und ihn rot färbend (Der Goldne Topf, ed. H. Leber, S. 208).

Und wieder bleibt dabei die von solchen merkwürdigen Kontexturverschiebungen betroffene Person dieser gewahr: " 'Er kann aber auch selbst in Person davongeflogen sein, der Herr Archivarius Lindhorst', sprach der Student Anselmus zu sich selbst, 'denn ich sehe und fühle nun wohl, dass alle die fremden Gestalten aus einer fernen wundervollen Welt, die ich sonst nur in ganz besonders merkwürdigen Träumen schaute, jetzt in ein waches, reges Leben geschritten sind und ihr Spiel mit mir treiben" (ibd., S. 218 f.).

3. Polykontexturale Welten können sich somit jederzeit verändern; sie sind ja nicht wie die eine (vermeintlich) monokontexturale Welt "homogen". Diese Einsicht kommt bei Hoffmann sehr gut zum Ausdruck, als Anselmus den Garten des Archivarius Lindhorst betritt:

> "Anselmus schritt getrost hinter dem Archivarius her; sie kamen aus dem Korridor in einen Saal oder vielmehr in ein herrliches Gewächshaus, denn von beiden Seiten bis an die Decke hinauf standen allerlei seltene wunderbare Blumen, ja grosse Bäume mit sonderbar gestalteten Blättern und Blüten. Ein magisches blendendes Licht verbreitete sich überall, ohne dass man bemerken konnte, wo es herkam, da durchaus kein Fenster zu sehen war. Sowie der Student Anselmus in die Büsche und Bäume hineinblickte, schienen lange Gänge sich in weite Ferne auszudehnen. – Im tiefen Dunkel dicker Zypressenstauden schimmerten Marmorbecken, aus denen sich wunderliche Figuren erhoben, Kristallstrahlen hervorspritzend, die plätschernd niederfielen in leuchtende Lichtkelche [...] (Der Goldne Topf, ed. H. Leber, S. 227 f.).

Kurze Zeit später aber:

> “Als er nun mittags durch den Garten des Archivarius Lindhorst ging, konnte er sich nicht genug wundern, wie ihm das alles sonst so seltsam und wundervoll habe vorkommen können. Er sah nichts als gewöhnliche Scherbenpflanzen, allerlei Geranien, Myrtenstöcke u. dgl. Statt der glänzenden bunten Vögel, die ihn sonst geneckt, flatterten nur einige Sperlinge hin und her, die ein unverständliches unangenehmes Geschrei erhoben, als sie des Anselmus gewahr wurden. Das blaue Zimmer kam ihm auch ganz anders vor, und er begriff nicht, wie ihm das grelle Blau und die unnatürlichen, goldnen Stämme der Palmbäume mit den unförmigen, blinkenden Blättern nur einen Augenblick hatten gefallen können” (ibd., S. 251).

Polykontexturale Welten sind ferner eindeutig-mehrmöglich bzw. multi-ordinal im Sinne Korzybskis (vgl. Kronthaler 1986, S. 60). Als Fabian und Balthasar den Garten des Doktors Prosper Alanus betreten, lesen wir:

> “Fabian bemerkte zwei Frösche von ungewöhnlicher Grösse, die schon von dem Gartentor an zu beiden Seiten der Wandelnden mitgehüpft waren. ‘Schöner Park’, rief Fabian, ‘in dem es solch Ungeziefer gibt!’ und bückte sich nieder, um einen kleinen Stein aufzuheben, mit dem er nach den lustigen Fröschen zu werfen gedachte. Beide sprangen ins Gebüsch und guckten ihn mit glänzenden, menschlichen Augen an. ‘Wartet, wartet!’ rief Fabian, zielte nach dem einen und warf. In dem Augenblick quäkte aber ein kleines hässliches Weib, das am Wege sass: ‘Grobian! Schmeiss Er nicht auf ehrliche Leute, die hier im Garten mit saurer Arbeit ihr bisschen Brot verdienen müssen’” (Klein Zaches, ed. H. Leber, S. 325).

Ob Frosch oder Mensch, ob Einhorn oder Pferd – in polykontexturalen Welten können somit nicht nur subjektive und objektive Subjekte ausgetauscht werden, sondern innerhalb eines solchen Austausches können auch die auszutauschenden Glieder selber ausgetauscht werden.

4. Wenn wir wiederum von unserem in Toth (2012b) skizzierten semiotisch-ontischen Modell ausgehen

$[A \to I]$	$[I \to A]$
$[[A \to I] \to A]$	$[A \to [I \to A]]$
$[[A \to I] \to A] \to I]]$	$[I \to [A \to [I \to A]]$
Zeichen	Objekt

(Z, Ω)-System

mit gestrichelt eingezeichneter Kontexturgrenze, dann betreffen also die Fälle des Austausches von subjektivem und objektivem Subjekt die Menge der Austauschrelationen der allgemeinen (systemtheoretischen) Form

$$[[A \to I] \to A] \to I]] \quad \leftrightarrow \quad [I \to [A \to [I \to A]],$$

und falls die auszutauschenden Relationen selber ausgetauscht werden können, kann man einfach von Mengen von Abbildungen der folgenden Form ausgehen

$$\{[[A \to I] \to A] \to I]]\} \quad \leftrightarrow \quad \{[I \to [A \to [I \to A]]\}.$$

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Das Zeichen als Teil des Objekts

1. In seinen frühen semiotischen Studien zu Kafka stellte Bense fest: "So tritt also das Nichts des Nichtseienden stets implizit auf, es schimmert durch das Sein hindurch, es partizipiert am Sein, wie in Platons mythischer Welt, infolgedessen ist es beständig gegenwärtig wie auch beständig abwesend. Die meontologische Differenz erscheint als ontologische Ambivalenz. Das Nichts ist ein Teil des Seins geworden, sofern sich dieses in jedem Seienden kundgibt" (1952, S. 81). Das deckt sich in den Grundzügen mit Heidegger: "Der alte Satz ex nihilo nihil fit erhält dann einen anderen, das Seinsproblem selbst treffenden Sinn und lautet: ex nihilo omne ens qua ens fit" (1986, S. 40).

2. Wie gesagt, ist Benses Argumentation bereits in der "Theorie Kafkas" – obwohl diese 15 Jahre vor Benses erstem ausschließlich semiotischem Buch geschrieben wurde – und übrigens auch noch zehn Jahre vor E. Walthers Habilitationsvortrag über den Zeichenbegriff bei Peirce (1962) – eine semiotische: "Das Seiende tritt als Zeichen auf, und Zeichen überleben in der rein semiotischen Dimension ihrer Bedeutungen den Verlust der Realität" (Bense 1952, S. 80). Somit sind es bei Bense bereits anfangs der 50er Jahre die Zeichen, welche als "meontologische Differenz" in der Form von "ontologischer Ambivalenz" erscheinen, denn sie verdoppeln ja quasi die Welt, indem sie später von Bense ausdrücklich als "Zuordnungen ... zu etwas (das Objekt sein kann)", d.h. als "Metaobjekte" eingeführt werden (Bense 1967, S. 9). Zu jedem Objekt kommen somit ein oder auch mehrere Metaobjekte, d.h. Zeichen dazu, die Welt der Objekt wird dadurch vervielfacht, und der klassischen Ontologie (und Metaphysik) mit dem Geltungsbereich der positiven Seinsthematik wird die Semiotik mit dem Geltungsbereich einer negativen Seinsthematik gegenübergestellt.

3. Nach klassischer Vorstellung sind Sein und Nichts streng voneinander geschieden, d.h. es ist weder das Sein ein Teil des Nichts noch umgekehrt das Nichts ein Teil des Seins. Trotzdem gibt viele Zeugen für nicht-klassische Positionen. So lesen wir etwa in der negativen Theologie des Dionysios Areopagita (1. Jh. n. Chr.): “Möchten doch – auch wir! – in jenes Dunkel eindringen können, das heller ist als alles Licht” (1956, S. 165). Meister

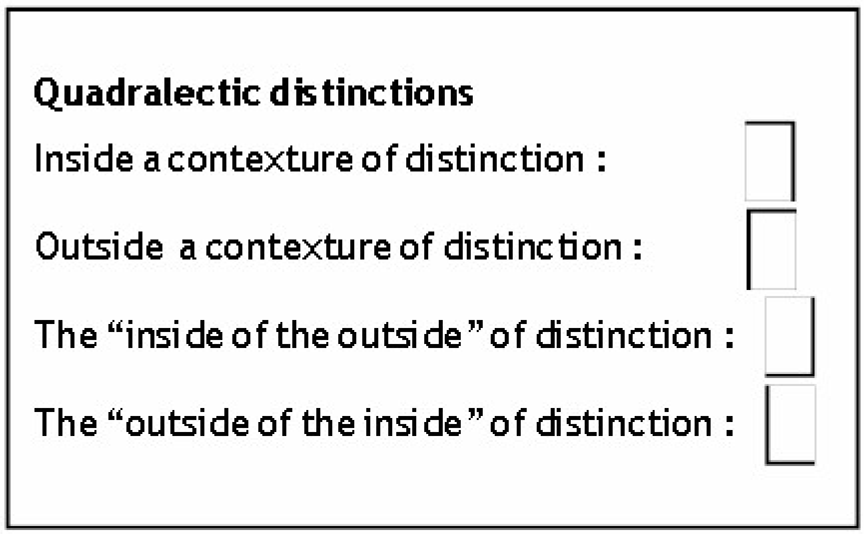
Eckehart (1260-1327): “Es war ein Zeichen dafür, dass er das wahre Licht sah, das da Nichts ist” (ap. Lanczkowski 1988: 207). Quirinus Kuhlmann (1651-1689, wegen seiner Lehren auf Geheiss des Zaren in Moskau verbrannt): “Je dunkler, je mehr lichter: / Je schwärzer alls, je weisser weisst sein Sam. / Ein himmlisch Aug ist Richter: / Kein Irdscher lebt, der was vernahm; / Es glänzt je mehr, je finster es ankam. / Ach Nacht! Und Nacht, die taget! / O Tag, der Nacht vernünftiger Vernunft! / Ach Licht, das Kaine plaget / Und helle strahlt der Abelzunft! / Ich freue mich ob deiner finstern Kunft” (ap. Staiger und Hürlimann 1948, S. 87). Georg Heym (1887-1912): “Tief unten brennt ein Licht, ein rotes Mal / Am schwarzen Leib der Nacht, wo bodenlos / Die Tiefe sinkt” (1947, S. 60). Jakob van Hoddis (1887-1842): “Ist dies der Tod? Sprich, müde Pracht. / Oder werde ich aus Deinen Schächten / Zu lichten nie gekannten Städten steigen / Und jedem Tage seine Donner zeigen?” (1987, S. 86). Die resurrectio mortuorum ist schliesslich das bedeutendste Sakrament der christlichen Kirchen. Beim Kirchenvater Gregor von Nyssa (4. Jh.) liest man: "Wenn demnach der Leib nicht so aufersteht, wie er beschaffen war, als er mit der Erde vermischt wurde, so wird nicht der Verstorbene auferstehen, sondern die Erde wird wiederum zu einem neuen Menschen gebildet werden. Was kümmert mich alsdann die Auferstehung, wenn statt meiner ein anderer auferstehen wird! Und wie soll ich mich als mich selbst anerkennen, wenn ich mich nicht in mir sehe? Denn ich würde tatsächlich nicht ich sein, wenn ich nicht in allen Stücken mit mir selbst identisch wäre” (von Nyssa 1927, S. 321f.)." In meinem Buch "Zwischen den Kontexturen" hatte ich geschrieben (Toth 2007, S. 120 f.):

Besonders phantasievoll werden die Wege ins Jenseits sowie die Grenze zwischen Diesseits und Jenseits ausgemalt, die wir in diesem Buch rein mathematisch behandelt haben. Indonesien: "Auf der Fahrt geht es durchs Nebelmeer, an Mond und Sternen und neidischen Geistern vorbei, für die noch kein Totenfest gehalten wurde und die deshalb den Weg versperren wollen. Das Wegstück durchs Feuermeer erfordert äußerste Konzentration Tempon Telons, der seine Bambusstangen, mit denen er steuert, ständig erneuern muß" (1996: 32). Südostasien: "Der Weg beginnt in der konkreten Landschaft, um sich allmählich in mehr oder weniger imaginären Sphären fortzusetzen. Erste Station der Totenseele ist häufig ein Fluß oder Teich. Dabei handelt es sich um die Grenze zwischen dem Diesseits und dem Jenseits. Die Seele weiß erst, nachdem sie das Wasser überquert oder in ihm gebadet hat, dort drüben, daß sie tot ist […]. Diese trennende Funktion übt die Wächterin des Totenlandes aus, die den neu angekommenen Toten mit einem Bakkenstreich empfängt. Auf einen Schlag löscht die Erinnerung an das irdische Leben aus" (1996: 40). Australien: "Klassisch ist der Bericht der Yirrkalla von einer Totenfahrt, bei der der Erstverstorbene der Menschen, von Delphinen begleitet, die Seele des jeweiligen Toten in einem Rinderkanu in der Richtung des Morgensterns nach der Toteninsel rudert" (1996: 59). Im finnischen Kalevala-Epos ist die Rede von der "gefahrvolle[n] Brücke ins Totenland" (1996: 63). Der nordasiatische Schamane findet "einen See, den man nur über eine Brücke, die aus einem Haar besteht, überqueren kann" (1996: 67). Eskimo: "Nach allem zu urteilen, ist der Weg ins Totenreich, wenigstens teilweise, mit der Milchstraße am Himmel identisch" (1996: 72). "Um in das Land der Toten zu kommen, muß der grönländische Schamane auf den Grund des Meeres hinabfahren, dessen Bereich durch einen Fluß als Grenze zwischen dem Land der Toten und der Lebenden vom Totenreich getrennt ist. Es heißt in einem Bericht: 'Endlich erreichten sie die Grenze zwischen dem Meer und dem Land unter dem Meere, die von einem schäumenden Bach gebildet wurde; um hinüber zu gelangen, mußten sie über große, spitze Steine springen, die ganz von nassen Tanggewächsen bedeckt waren und so glatt schimmerten, daß sich niemand hinüberwagte […]. Durch die Hilfe der Geister springt der Schamane über diese Hindernisse. Die Geister ermuntern ihn und rufen ihm zu: 'Wenn du diesen Sprung nicht wagst und umkehrst, wird du nie das Land der Toten erreichen; an diesen Steinen wird deine Reise immer enden.' Dann wagte der Schamane den Sprung, und zu seinem großen Erstaunen zeigte sich, daß der Tang gar nicht so glatt ist.' Vom gleichen Autor wird von Stufen berichtet, die der Schamane überwinden muß, um in die Totenwelt zu gelangen: 'Der Geisterbeschwörer […] stieß auf eine Treppe mit drei hohen Stufen. Sie waren so hoch, daß er sich mit knapper Not von der einen zur anderen schwingen konnte, und schlüpfrig von Menschenblut, das darüberrieselte. Der Geisterbeschwörer stieg mit Mühe und unter großer Lebensgefahr die schlüpfrigen Stufen hinauf und gelangte zu einer weiten, weiten Ebene, der Himmelsebene.'" (1996: 73f.). Hindukusch: "Regulärer Zugang zur Unterwelt ist möglich durch ein Loch im Boden; man zeigt es nahe dem Zentraltempel in Ushteki. Wer hier hinabschaut, ist augenblicklich des Todes." "Wichtigste Verbindung zwischen diesen beiden Seinsebenen [Diesseits und Jenseits] sind Seen und Teiche. Wer es wagt, sich hineinzustürzen, der hat den Übergang geschafft" (1996: 94). Mesopotamien: Man gibt dem Toten einen Nachen zur Überquerung des Unterweltflusses Chubur mit [...]. Gleich nach dem Tode muß der Verstorbene mit Hilfe eines sturmvogelköpfigen, mit vier Händen und Füßen versehenen Fährmanns namens 'Nimm schnell hinweg' den Unterweltsfluß durchqueren und sieben Tore durchschreiten" (1996: 121). In indischen Texten liest man, "wie die Seele zur Brücke, cinvato, gelangt. Hier wird sie verhört, dann kommt eine von zwei Hunden begleitete schöne Jungfrau und führt die gläubige Seele über die Brücke zu dem Damm oder Wall, der die Grenze der himmlischen Welt ausmacht" (1996: 142). Nordiran: Man gibt dem Toten ein Pferd und eine angemessene Ausrüstung mit. "Bevor der Verstorbene an den Fluß kommt, den er zu überschreiten hat, treten ihm Wächter entgegen; er muß ihnen Hirsekuchen schenken, um weiterziehen zu dürfen. Über den Fluß selbst führt statt einer Brücke nur ein Balken, vor dem eine göttliche Gestalt steht, die ihn zu befragen beginnt" (1996: 146).

Bekannter ist die altgriechische Vorstellung: "Kennzeichen der Unterwelt ist das große Tor, das der Tote durchschreiten muß, um nie mehr zurückzukehren [...]. In der Odyssee wird der Eingang in die Unterwelt jenseits des Okeanos durch Flüsse markiert, den Acheron, in den ein Feuerstrom und ein Klagestrom einmünden, und den Styx mit seinen Wassern des Grauens [...]. Fluß oder See sind die Grenze, über die der Fährmann die Toten auf seinem Schiff ins Jenseits bringt. Zur Sage von Herakles gehört der fünfzigköpfige Hund Kerberos, der das Tor des Hades bewacht" (1996: 191). Einzig die Gnosis, in der ganze Bücher "den Weg der Seele durch unterirdische 'Wachthäuser' oder 'Höllen'" beschreiben, gibt eine Maßzahl für den Weg ins Jenseits: "Nach dem Tode hat die Seele eine lange, 42tägige Reise vor sich" (1996: 252).

4. Wie man also besonders an den letzten Zitaten erkennt, so ist die Vorstellung, das Sein sei ein (wie auch immer gearteter) Teil des Nichts durchaus vorheideggerisch, aber erst Bense (1952) bestimmte die Semiotik als Nichtsthematik im Sinne von meontologischer Metaobjektion durch Zeichen. Nun hatte bereits Bense (1975, S. 16) die Zeichenfunktion als Überbrückung "der Disjunktion zwischen Welt und Bewußtsein" eingeführt. Es bleibt also noch die Frage noch klären, wie man sich die Nahtstelle zwischen Sein und Nichts vorzustellen hat. Hierzu kann man ein Modell benutzen, das erst seit

kurzer Zeit existiert und das von Rudolf Kaehr (2011, S. 12) stammt, und in seinen Grundzügen auf Gotthard Günthers (1976) Unterscheidung der vier möglichen logisch-epistemischen Funktionen in einer 4-wertigen, nicht-klassischen Logik zurückgeht, die ich bereits in Toth (2008, S. 64 ff.) in die Semiotik eingeführt hatte



Nach dem quadralektischen Modell Kaehrs kann man also die "Grundfiguren" quadralektischer Diamanten in dieser Reihenfolge dem Interpretanten-, Objekt- und Mittelbezug des Peirce-Benseseschen Zeichenmodells zuschreiben (vgl. Toth 2011). Also bleibt die semiotische Funktion des Spencer-Brown-Kaehrschen "Outside of the Inside of Distinction" zu klären. Wie bereits die von Kaehr suggestiv gewählten systemtheoretischen Symbole nahelegen, verhalten sich das "Inside of the Outside" und das "Outside of the Inside" so zueinander, dass die horizontalen Striche beider Figuren deckungsgleich werden (⊥), d.h. die beiden systemtheoretischen Funktionen verhalten sich so, wie wenn jemand gleichzeitig z.B. vor und hinter einer Haustür steht. Daraus folgt, daß man als semiotische Funktion des Outside of the Inside (L) die Perspektivierung eines Systems, d.h. die Entscheidung darüber, was jeweils Außen und was jeweils Innen ist, bestimmen kann. Mit anderen Worten: "L" verortet, be-gründet (im Sinne des Heideggschen "zureichenden Grundes" bzw. Kaehrs "anchoring"), das, was hinter der "Tür" steht. Nimmt man nun an, daß das, was von außerhalb der "Tür" betrachtet, innen das Zeichen und daher außerhalb das Objekt ist, dann fundiert also dieser "nullheitliche" Bezug (vgl. Bense 1975, S. 65 f. zur "Zeroness") das Zeichen als triadischer Relation über Erst-, Zweit- und Drittheit. Damit wird aber das Zeichen, aufgefaßt als Menge innerer Punkte, im Sinne der Topologie durch die Koinzidenz von ⊥ durch einen *sowohl äussere wie innere Punkte enthaltenden "Rand"* abgeschlossen. (In dieser systemtheoretisch interpretierten Topologie "partizipiert" also der Rand nicht nur am Nichts, sondern auch am Sein, d.h. genauso, wie es Bense 1952, S. 80, Eingangszitat, sagt). Dagegen wird das Außen im Sinne einer Menge äußerer Punkte, d.h. das Objekt, wiederum von der gleichen Grenze der Menge der Randpunkte, vom Innern abgetrennt. Man könnte diesen Sachverhalt also prägnant wie folgt charakterisieren: DIE SCHNITTSTELLE VON SEIN UND NICHTS, OBJEKT UND ZEICHEN ZEICHNET SICH DADURCH AUS, DAß SIE GEGENSEITIG ANEINANDER PARTIZIPIEREN. Diese "Partizipationsmenge", d.h. die Menge der Randpunkte, ist also nichts anderes als das, was früher auch von mir als das Gebiet der "Prä-semiotik" bezeichnet wurde und von dem weiterhin abzuklären ist, ob es sich hier um eine Liniengrenze oder nicht vielmehr um ein Streifen von "Niemands-land" handelt.

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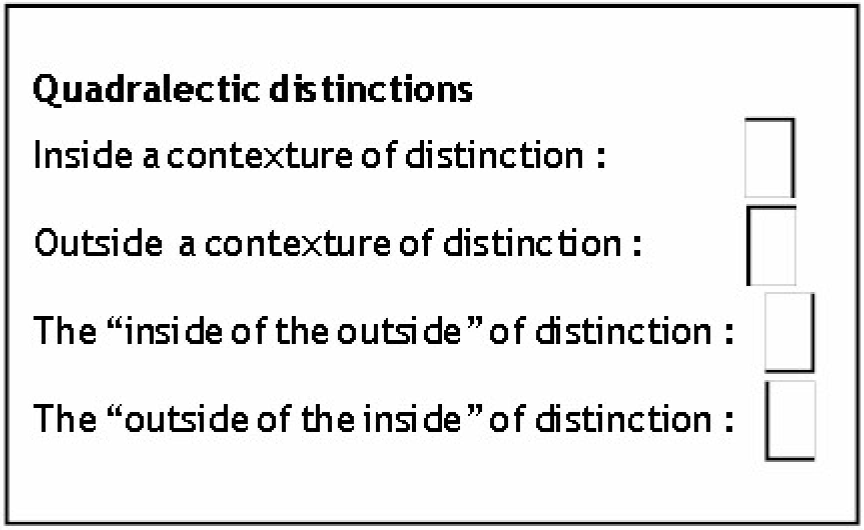
Qualität als Positionierung

> Es war ein Zeichen dafür, daß er das wahre Licht sah, das da Nichts ist.
>
> Meister Eckehart (1260-1327)

1. In seinen frühen semiotischen Studien zu Kafka stellte Bense fest: "So tritt also das Nichts des Nichtseienden stets implizit auf, es schimmert durch das Sein hindurch, es partizipiert am Sein, wie in Platons mythischer Welt, infolgedessen ist es beständig gegenwärtig wie auch beständig abwesend. Die meontologische Differenz erscheint als ontologische Ambivalenz. Das Nichts ist ein Teil des Seins geworden, sofern sich dieses in jedem Seienden kundgibt" (1952, S. 81). Das deckt sich in den Grundzügen mit Heidegger: "Der alte Satz ex nihilo nihil fit erhält dann einen anderen, das Seinsproblem selbst treffenden Sinn und lautet: ex nihilo omne ens qua ens fit" (1986, S. 40).

2. Wie gesagt, ist Benses Argumentation bereits in der "Theorie Kafkas" – obwohl diese 15 Jahre vor Benses erstem ausschließlich semiotischem Buch geschrieben wurde – und übrigens auch noch zehn Jahre vor E. Walthers Habilitationsvortrag über den Zeichenbegriff bei Peirce (1962) – eine semiotische: "Das Seiende tritt als Zeichen auf, und Zeichen überleben in der rein semiotischen Dimension ihrer Bedeutungen den Verlust der Realität" (Bense 1952, S. 80). Somit sind es bei Bense bereits anfangs der 50er Jahre die Zeichen, welche als "meontologische Differenz" in der Form von "ontologischer Ambivalenz" erscheinen, denn sie verdoppeln ja quasi die Welt, indem sie später von Bense ausdrücklich als "Zuordnungen ... zu etwas (das Objekt sein kann)", d.h. als "Metaobjekte" eingeführt werden (Bense 1967, S. 9). Zu jedem Objekt kommen somit ein oder auch mehrere Metaobjekte, d.h. Zeichen dazu, die Welt der Objekte wird dadurch vervielfacht, und der klassischen Ontologie mit dem Geltungsbereich der positiven Seinsthematik wird die Semiotik mit dem Geltungsbereich einer negativen Seinsthematik gegenübergestellt. Nun hatte bereits Bense (1975, S. 16) die Zeichenfunktion als Überbrückung "der Disjunktion zwischen Welt und Bewußtsein" eingeführt. Es bleibt also noch die Frage noch klären, wie man sich die Nahtstelle zwischen Sein und Nichts vorzustellen hat. Hierzu kann man ein Modell benutzen, das erst seit kurzer Zeit

existiert und das von Rudolf Kaehr (2011, S. 12) stammt und in seinen Grundzügen auf Gotthard Günthers (1976) Unterscheidung der vier möglichen logisch-epistemischen Funktionen in einer 4-wertigen, nicht-klassischen Logik zurückgeht, die ich in Toth (2008, S. 64 ff.) in die Semiotik eingeführt hatte



Nach dem quadralektischen Modell Kaehrs kann man also die "Grundfiguren" quadralektischer Diamanten in dieser Reihenfolge dem Interpretanten-, Objekt- und Mittelbezug des Peirce-Benseseschen Zeichenmodells zuschreiben (vgl. Toth 2011):

Mittelbezug (M):	$[A \to I] := I$
Objektbezug (O):	$[[A \to I] \to A := A$
Interpretantenbezug (J):	$[[[A \to I] \to A] \to I := I(A)$
Qualität (Q)	$[A \to I]° = [I \to A] := A(I)$,

und so kann man ferner die systemische Zeichenrelation (Toth 2012a) wie folgt "quadralektisch" umformen

$$ZR_{sys} = [[A \to I], [[[A \to I] \to A], [[[A \to I] \to A] \to I]]] \Rightarrow (I(A), A, I, A(I)).$$

Die entscheidende Frage bleibt jedoch, ob die aus semiotischer Sicht inverse Funktion A(I) bzw. $[I \to A]$ wirklich ihren Platz als 0-stellige Relation INNERHALB der Zeichenrelation hat oder nicht. In Toth (2012b) war allerdings argumentiert worden, daß die beiden Funktion $[A \to I]$ und $[I \to A]$ (die nur formal invertierbar sind!) genau die Menge von Randpunkten der Hülle von

Innen und Außen in einem System ausmachen, d.h. aber, nicht nur [A → I] (vermöge dem Mittelbezug, per definitionem), sondern auch [I → A] muß schon aus strukturellen Gründen Teil von ZR_{sys} = sein, denn das "Inside of the Outside" und das "Outside of the Inside" verhalten sich in der suggestiven Kaehrschen Notation so zueinander, dass die horizontalen Striche beider Figuren deckungsgleich werden (⊥), d.h. die beiden systemtheoretischen Funktionen verhalten sich so, wie wenn jemand gleichzeitig z.B. vor und hinter einer Haustür steht. Daraus folgerten wir bereits in Toth (2012b), daß man als semiotische Funktion des Outside of the Inside (L) die **Perspektivierung eines Systems**, d.h. die Entscheidung darüber, was jeweils Außen und was jeweils Innen ist, bestimmen kann. (Ein Hauseingang z.B. sieht von Innen nicht gleich aus wie von Außen!) Mit anderen Worten: "L" verortet, be-gründet (im Sinne des Heideggschen "zureichenden Grundes" bzw. Kaehrs "anchoring"), das, was hinter der "Tür" steht. Nimmt man nun an, daß das, was von außerhalb der "Tür" betrachtet, innen das Zeichen und daß daher außen das Objekt ist, dann fundiert also dieser "nullheitliche" Bezug (vgl. Bense 1975, S. 65 f. zur "Zeroness") das Zeichen als triadische Relation über Erst-, Zweit- und Drittheit. Damit wird aber das Zeichen, aufgefaßt als Menge innerer Punkte, im Sinne der Topologie durch die Koinzidenz von ⊥ durch einen *sowohl äussere wie innere Punkte enthaltenden "Rand"* abgeschlossen. (In dieser systemtheoretisch interpretierten Topologie "partizipiert" also der Rand nicht nur am Nichts, sondern auch am Sein, d.h. genauso, wie es Bense 1952, S. 80, Eingangszitat, sagt). Dagegen wird das Außen im Sinne einer Menge äußerer Punkte, d.h. das Objekt, wiederum von der gleichen Grenze der Menge der Randpunkte, vom Innern abgetrennt. Man könnte diesen Sachverhalt also prägnant wie folgt charakterisieren: DIE SCHNITTSTELLE VON SEIN UND NICHTS, OBJEKT UND ZEICHEN ZEICHNET SICH DADURCH AUS, DAß SIE GEGENSEITIG ANEINANDER PARTIZIPIEREN. Diese "Partizipationsmenge", d.h. die Menge der Randpunkte, ist also nichts anderes als das, was früher auch von mir als das Gebiet der "Präsemiotik" bezeichnet wurde und von dem weiterhin abzuklären ist, ob es sich hier um eine Liniengrenze oder nicht vielmehr um ein Streifen von "Niemandsland" handelt.

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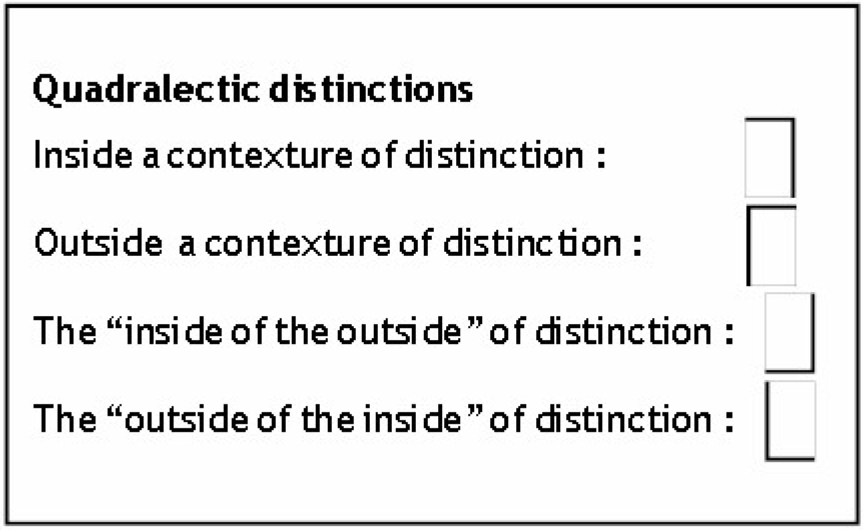
Zum Rand von Zeichen und Objekt

1. Wie in Toth (2012a) gezeigt worden war, kann man die "quadralektischen" systemischen Funktionen in der folgenden Bestimmung von Rudolf Kaehr (2011, S. 12)



nach meinem in Toth (2011) gegebenen Vorschlag wie folgt auf die semiotischen Funktionen (vgl. Walther 1979, S. 113 ff.) abbilden:

Mittelbezug (M):	$[A \rightarrow I] := I$
Objektbezug (O):	$[[A \rightarrow I] \rightarrow A := A$
Interpretantenbezug (J):	$[[[A \rightarrow I] \rightarrow A] \rightarrow I := I(A)$
Qualität (Q)	$[A \rightarrow I]° = [I \rightarrow A] := A(I)$.

Man bemerkt also, daß "Quadralexis" (wie aus dem Namen natürlich nicht anders zu erwarten [auch wenn er korrekt "Tetralexis" lauten müßte!]) eine mindestens 4-stellige Zeichenrelation voraussetzt. Trotzdem ist es natürlich möglich, auch die Peirce-Bensesche triadische Zeichenrelation in quadralektische Notation zu transformieren:

$$ZR_{sys} = [[A \rightarrow I], [[[A \rightarrow I] \rightarrow A], [[[A \rightarrow I] \rightarrow A] \rightarrow I]]] \Rightarrow (I, (A, I(A))$$

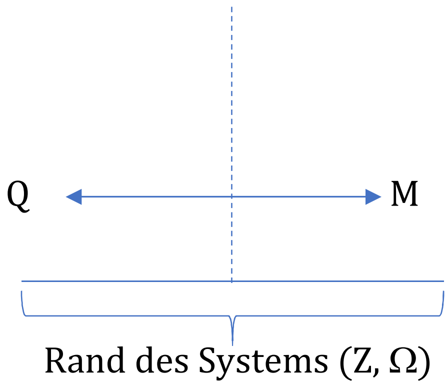
2. In Kaehrs suggestiv gewählten Symbolen machen also die beiden "Distinktionen" I(A) und A(I) den Rand zwischen den inneren und den äußeren Punkten des Zeichen-Objektsystems aus; wenn man die beiden Distinktionen zusammenschreibt, ergibt sich $\perp$, dessen horizontaler Strich die

Kontexturgrenze zwischen Außen und Innen symbolisiert und dessen durchgehender vertikaler "Sockel" symbolisiert, daß Außen und Innen trotz aufweisbarer Kontexturgrenze in Bezug auf den Rand nicht diskret separierbar sind. Und genau dies kommt nun durch die Bestimmung

Mittelbezug (M): $[A \rightarrow I] := I$
Qualität (Q) $[A \rightarrow I]° = [I \rightarrow A] := A(I)$,

d.h. $M° = Q$; $Q° = M$

zum Ausdruck. Bestimmen wir im Einklang mit Bense (1952, S. 80), daß die Nichtsthematik ein Teil der Seinsthematik (und nicht umgekehrt) ist, so bedeutet dies semiotisch (wiederum in Einklang mit Bense, loc. cit.), daß das Zeichen in die Objektwelt eingebettet ist bzw. in abbildungstheoretischer oder funktionaler Abhängigkeit von dieser steht, denn nach Bense (1967, S. 9) ist ein Zeichen ja ein Metaobjekt, d.h. daß das Objekt dem Zeichen vorgegeben sein muß. Somit ist aber die Bestimmung des Zeichens als Menge der inneren Punkte und die Bestimmung des Objekts als Menge der äußeren Punkte des durch den Rand geteilten topologischen Raumes unzureichend: DER RAND PARTIZIPIERT VIELMEHR AN BEIDEN TEILRÄUMEN, und genau diese Partizipation wird durch das Konversionsverhältnis von M und Q bzw. symbolisch durch den "Sockel" in $\perp$ zum Ausdruck gebracht. Es ist somit unzuläßig – wie dies in der Semiotik bisher fast durchwegs geschehen ist –, die qualitative "Nullstufe" bzw. "Zeroness" (vgl. dazu bereits Bense 1975, S. 65 f.) außerhalb des "semiotischen Raumes" und somit innerhalb eines "ontologischen Raumes" anzusiedeln, denn nur eine Konversionsoperation trennt M und Q voneinander – was von innen M ist, ist von außen Q, und was von innen Q ist, ist von außen M – Q gibt nur den Standpunkt des Beobachters des Systems an, oder, was formal dasselbe, ist: die "Verortung" der triadischen Restrelation einer tetradischen semiotischen Relation an (wobei der Begriff "Restrelation" völlig korrekt ist, da die 0-adische Relation nicht in die triadisch-verschachtelte Zeichenrelation eingebettet ist):



Die im obigen Diagramm skizzierte doppelte Abbildung ↔ kann daher als PARTIZIPATIVE AUSTAUSCHRELATION bestimmt werden. Damit ist also gerade auch die nächste Frage beantwortet, welche Werte die "Nullheit" in einer um sie erweiterten semiotischen Relation

$ZR^4 = (0.a, (1.b, (2.c, 3.d)))$

bzw.

$ZR^4_{sys} = [[I \rightarrow A][A \rightarrow I], [[[[A \rightarrow I] \rightarrow A], [[[A \rightarrow I] \rightarrow A] \rightarrow I]]]] \Rightarrow$
$(A(I), (I, (A, I(A)))$

annehmen kann. Da $[A \rightarrow I] := (1.b)$ mit $b \in \{1, 2, 3\}$ ist, ist natürlich wegen $(0.a) = [A \rightarrow I]°$ auch $a \in \{1, 2, 3\}$, d.h. die bereits von Götz (1982, S. 4, 28) vorgeschlagene "trichotomische" Unterteilung der Nullheit (von Götz "Sekanz", "Semanz" und "Selektanz" genannt), ist völlig richtig. Das 3-stufige semiotische Zahlensystem der triadischen Zeichenrelation (vgl. zuletzt Toth 2012b) geht dadurch über in ein 4-stufiges:

3.heit	$[[[A \rightarrow I] \rightarrow A] \rightarrow I]$
2.heit	$[[A \rightarrow I] \rightarrow A]$
1.heit	$[A \rightarrow I]$
0.heit	$[I \rightarrow A]$,

und die zugehörigen numerischen und "quadralektischen" Matrizen sind:

	0	1	2	3
0	0.0	0.1	0.2	0.3
1	1.0	1.1	1.2	1.3
2	2.0	2.1	2.2	2.3
3	3.0	3.1	3.2	3.3

	└	┘	┌	┐
└	└└	└┘	└┌	└┐
┘	┘└	┘┘	┘┌	┘┐
┌	┌└	┌┘	┌┌	┌┐
┐	┐└	┐┘	┐┌	┐┐

Für die Dualisation gilt also:

(× └) = (×.0.) = ┘ = (.1.), d.h. └ × ┘

(× ┌) = (×.2.) = ┐ = (.3.), d.h. ┌ × ┐ ,

das bedeutet jedoch, daß wir also auch innerhalb der Menge der INNEREN Punkte, d.h. in der Nichtsthematik der Semiotik, eine partizipative Austauschrelation haben, und zwar zwischen Objekt- und Interpretantenbezug. Damit stehen also paarweise (Q ↔ M) sowie (O ↔I) in partizipativem Austausch. Wenn wir nun von Benses "verschachtelter" triadischer Zeichenrelation (Bense 1979, S. 53)

ZR = (M → ((M → O) → (O → I)))

ausgehen, so folgt daraus, daß, obwohl Q als Nullheit per se nicht in die triadische Restrelation der tetradischen Zeichenrelation einbettbar ist, Q nun doch, und zwar qua eingebettete Abbildungen der Partialrelationen der triadischen Restrelation, sozusagen durch die Hintertür in der letzteren eingebettet wird; das folgt direkt aus den partizipativen Austauschrelationen sowie aus der Transitivität der triadischen Abbildungen. Daraus folgt allerdings nicht, daß die Nullheit damit sozusagen am Anfang einer hierarchischen Verschachtelung steht, oder anders gesagt: die tetradische Zeichenrelation läßt nicht, oder wenigstens nicht ohne weiteres, auf die Peano-Zahlenfolge (0, 1, 2, 3) abbilden, da diese, tetradisch-semiotisch interpretiert, auch (1, 0, 2, 3), (1, 2, 0, 3) oder (1, 2, 3, 0) sein könnte.

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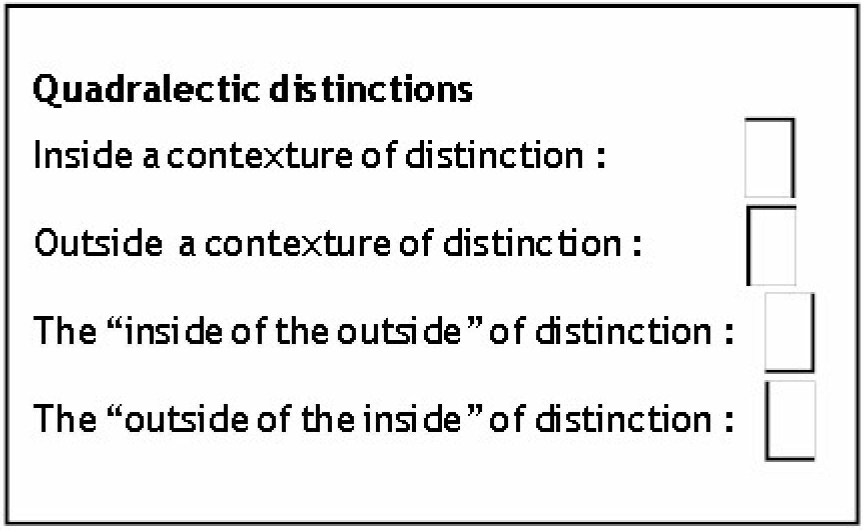
Systemische Austauschrelation zwischen Objekt- und Interpretantenbezug

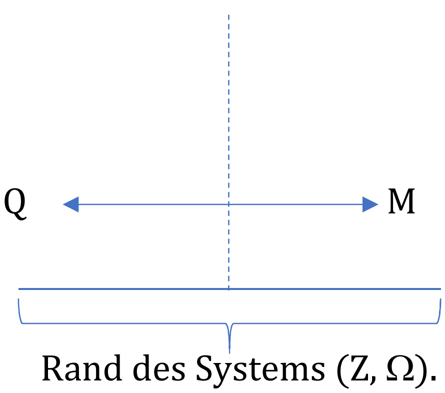
1. Gehen wir wiederum von Kaehrs "quadralektischem" (besser: tetralektischem) systemischem Modell der vier möglichen logisch-epistemischen Basisfunktionen (über Subjekt und Objekt) aus:



und nehmen wir die in Toth (2011) gegebenen Zuordnungen semiotischer Funktionen vor:

Mittelbezug (M):	$[A \rightarrow I] := I$
Objektbezug (O):	$[[A \rightarrow I] \rightarrow A := A$
Interpretantenbezug (J):	$[[[A \rightarrow I] \rightarrow A] \rightarrow I := I(A)$
Qualität (Q)	$[A \rightarrow I]° = [I \rightarrow A] := A(I)$,

so kann man, wie bereits in Toth (2012a), in einem Zeichen-Objekt-System zwischen den äußeren und inneren Punkten sowie dem Rand unterscheiden:



2. Nach Toth (2012b) ist die partizipative Austauschfunktion lediglich im absoluten Nullpunkt nicht definiert, d.h. es handelt sich entweder um finite partielle oder um infinitesial-asymptotische Funktionen. Somit erhält man Q und M aus der wechselweisen Dualisierung der ebenfalls in Toth (2012b) behandelten (r, k)-Gebilde, worin r die Relationszahl und k die Kategorialzahl angibt, durch die jede Subzeichenrelation hinreichend charakterisiert ist:

$\times(0.a) = (a.0)$ mit $a \in \{1, 2, 3\}$.

Was also von außen ein Q / M ist, ist von innen ein M / Q, d.h. der Rand des Zeichen-Objekt-Systems ist keine Liniengranze diskreter Punkte, sondern ein Niemandsland, das sowohl Teilmenge des Außen, d.h. des "ontischen Raumes", als auch dessen Innen, d.h. des "semiotischen Raumes", ist (vgl. dazu Bense 1975, S. 65 f.). Q sind in Benses (1975, S. 45 f.) Terminologie also disponible, d.h. kategoriale Objekte, während M disponible Mittel sind: Ein kategoriales Objekt ist sozusagen der qualitative Pool, aus dem solche Mittel selektiert werden, die allenfalls zu Mittelbezügen werden, d.h. innerhalb einer triadischen Zeichenrelation fungieren.

3. Nun koinzidieren aber nicht nur Q und M, d.h.

$\rceil, \lfloor \Rightarrow \perp,$

sondern auch O und I, d.h.

$\rceil, \lceil \Rightarrow \top,$

d.h. es stehen auch O und J in einer "partizipativen" Austauschrelation. Das bedeutet also, daß wir nicht nur in der Menge der Randpunkte des (Z, Ω)-Systems, sondern auch in der Menge seiner inneren Punkte mit mereotopologisch überlappende Menge vor uns haben. Systemisch gesprochen, gilt also im Randgebiet

$Q \leftrightarrow M \Leftrightarrow (0.a) \leftrightarrow (1.b) \Leftrightarrow [A \rightarrow I] \Leftrightarrow [I \rightarrow A]$

und im Gebiet der inneren Punkte

$$O \leftrightarrow J \Leftrightarrow (2.c) \leftrightarrow (3.d) \Leftrightarrow [[[A \to I] \to A] \to I] \Leftrightarrow [I \to [A \to [I \to A]]].$$

Wenn wir diese konverse Abbildung von Semiosen und Retrosemiosen betrachten:

$$[[[A \to I] \to A] \to \boxed{I] \Leftrightarrow [I} \to [A \to [I \to A]]]$$

$$[[A \to I] \to A],$$

so sehen wir, daß diese zweite Form partizipativer Austauschrelationen im Gegensatz zur ersten selber vermittelt ist, und zwar fungiert als Vermittlung die Menge der inneren Punkte selber. Semiotisch entspricht diese Vermittlung genau derjenigen des Interpretantenbezuges innerhalb der Peirce-Benseschen Zeichenrelation, der ja einerseits semiosisch auf den Objektbezug innerhalb der verschachtelten Hierarchie der drei Zeichenbezüge folgt, andererseits aber als drittheitliche Relation das vermittelnde Zeichen im Zeichen selber darstellt (weshalb das Peirce-Bensesche Zeichen ja autoreproduktiv ist).

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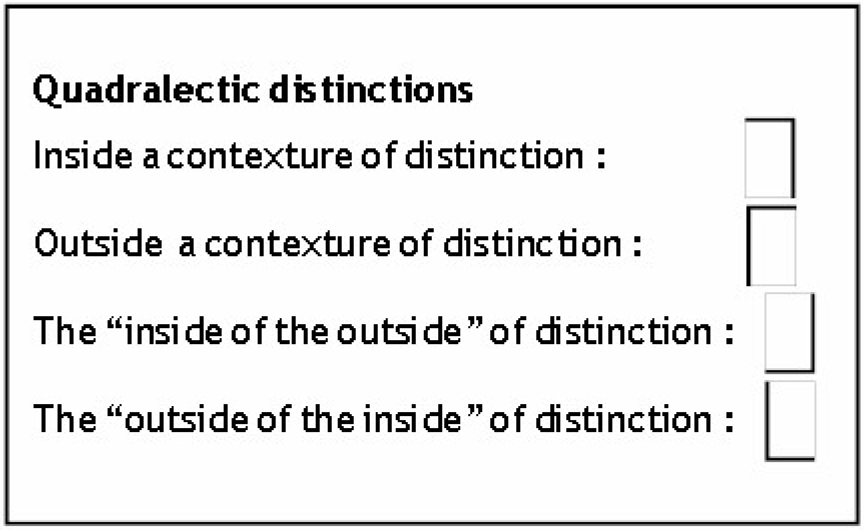
Die Orthogonalität von Außen und Innen

1. Betrachten wir wiederum tetralektischen Distinktionen, die Kaehr (2011, S. 12) vorgeschlagen hatte



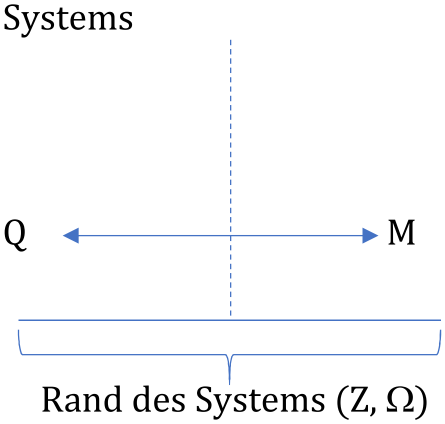
zusammen mit meinen in Toth (2011) gegebenen Zuschreibnungen

Mittelbezug (M):	$[A \rightarrow I] := I$
Objektbezug (O):	$[[A \rightarrow I] \rightarrow A := A$
Interpretantenbezug (J):	$[[[A \rightarrow I] \rightarrow A] \rightarrow I := I(A)$
Qualität (Q)	$[A \rightarrow I]° = [I \rightarrow A] := A(I)$,

dann finden wir 1. Koinzidenz von Q und M, d.h.

$\urcorner, \llcorner \Rightarrow \perp$

im Bereich des Randes der topologischen Darstellung des Zeichen, Objekt-Systems



und 2. Koinzidenz von O und J in der Menge der inneren Punkte des (Z, Ω)-Systems, d.h.

$\urcorner, \ulcorner \Rightarrow \top$,

2. Systemisch gesprochen, gilt also im Randgebiet

$Q \leftrightarrow M \Leftrightarrow (0.a) \leftrightarrow (1.b) \Leftrightarrow [A \rightarrow I] \Leftrightarrow [I \rightarrow A]$

und im Gebiet der inneren Punkte

$O \leftrightarrow J \Leftrightarrow (2.c) \leftrightarrow (3.d) \Leftrightarrow [[[A \rightarrow I] \rightarrow A] \rightarrow I] \Leftrightarrow [I \rightarrow [A \rightarrow [I \rightarrow A]]]$.

Wenn wir diese konverse Abbildung von Semiosen und Retrosemiosen betrachten:

$[[[A \rightarrow I] \rightarrow A] \rightarrow \boxed{I] \Leftrightarrow [I} \rightarrow [A \rightarrow [I \rightarrow A]]]$

$[[A \rightarrow I] \rightarrow A]$,

so sehen wir, wie bereits in Toth (2012a) ausgeführt, daß diese zweite Form partizipativer Austauschrelationen im Gegensatz zur ersten selber vermittelt ist, und zwar fungiert als Vermittlung die Menge der inneren Punkte selber.

Nun ist aber das Verschachtelungsschema der triadischen Peirce-Bense-Zeichenrelation

$ZR^3 = (1.a, (2.b, 3.c))$

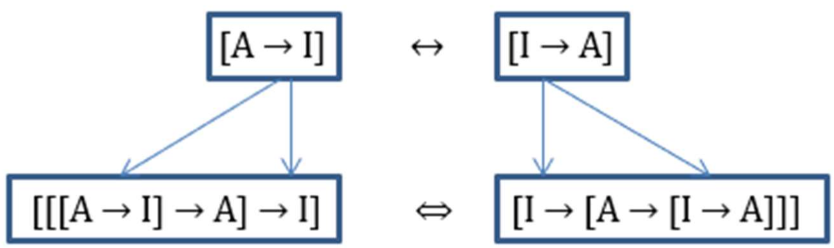
und dasjenige der um die Nullheit erweiterten (Toth 2012b) tetradischen Zeichenrelation, welche also die disponiblen und kategorialen Objekte und Mittel mitumfaßt,

$ZR^4 = (0.a, (1.a, (2.b, 3.c)))$

(die möglichen anderen Positionen von 0 in den Folgen (0, 1, 2, 3), (1, 0, 2, 3), (1, 2, 0, 3) und (1, 2, 3, 0) scheiden wegen der ersten partizipativen Austauschrelation aus). Da also auch in ZR^4 die Zeichenbezüge in einer hierarchischen "Relation über Relationen" so eingebettet sind, daß jeder n-te Bezug in jedem (n+1)-ten Bezug enthalten ist, folgt also, daß dies auch für die Abbildungen zwischen den Bezüge gelten muß, d.h. es muß auch eine semiotische Inklusion zwischen den beiden partizipativen Abbildungen, der unvermittelten von (Q ↔ M) und der über I vermittelten von (O ↔ I) geben. Wegen der Suggestivitätskraft der anhand der Definitionen der Tetralexis gewählten Symbole, d.h. wegen

$\rceil, \lfloor \Rightarrow \perp$ und $\rceil, \lceil \Rightarrow \top$

nennen wir das Verhältnis der topologisch-systemtheoretischen "Resultanten" $\perp$ und $\top$ orthogonal. Explizit beinhaltet Orthogonalität der beiden im (Z, Ω)-System vorhandenen partizipativen Austauschrelationen also die Relation zwischen $[A \rightarrow I] \leftrightarrow [I \rightarrow A]$ und $[[[A \rightarrow I] \rightarrow A] \rightarrow I] \Leftrightarrow [I \rightarrow [A \rightarrow [I \rightarrow A]]]$:



Während also die Relation der beiden Domänen und der beiden Codomänen der Abbildungen in beiden Fällen dasjenige von Semiose und Retrosemiose ist, ist die Relationen ZWISCHEN den beiden Domänen und den beiden Codomänen also rein semiosisch. Dabei werden die Abbildungen der Domänen in beiden Fällen iteriert und doppelt eingebettet.

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Kategoriale Vorthetik

> Jedes der Vier spiegelt in seiner Weise das Wesen der übrigen wieder. Jedes spiegelt sich dabei nach seiner Weise in sein Eigenes innerhalb der Einfalt der Vier zurück.
>
> Heidegger (1997, S. 172)

1. Wie bereits in Toth (2008, S. 36 ff.) gezeigt worden war, setzt die maximale Anzahl der aus den logisch-epistemischen Funktionen Subjekt und Objekt konstruierbaren Paar-Kombinationen, d.h. objektives und subjektives Subjekt sowie subjektives und objektives Objekt, zusätzlich zu den drei Peirceschen eine weitere Kategorie voraus. Identifiziert man, wie z.B. in Toth (2011) den Interpretantenbezug mit dem subjektiven Subjekt, den Objektbezug mit dem objektiven Objekt und den Mittelbezug mit dem subjektiven Objekt, so fehlt also eine der Funktion des objektiven Subjekts korrespondierende Kategorie. Wie zuletzt in Toth (2012a) gezeigt worden waren, rücken mit dieser Bestimmung die fehlende Kategorie x und der Mittelbezug M in ein Konversionsverhältnis, da sich objektives Subjekt zu subjektivem Objekt verhält wie x : M, d.h. wir erhalten sofort: $x = M^{-1}$.

2. Die Einführung der zusätzlichen Kategorie x bedingt natürlich gleichzeitig eine Erweiterung der triadischen zu einer tetradischen Zeichenrelation. Spätestens an diesem Punkt stellt sich also die Frage, wie die neue Kategorie x semiotisch zu interpretieren sei. Wie spätestens seit Walther (1979, S. 58 ff.) bekannt ist, sind ja die Peirceschen "Fundamentalkategorien" ursprünglich modal, insofern dem Mittelbezug die kategoriale Möglichkeit, dem Objektbezug die kategoriale Wirklichkeit und dem Interpretantenbezug die kategoriale Notwendigkeit entspricht. Modal gesehen, lautet die Frage also: Kann es (mit x) eine Kategorie "unterhalb" der Möglichkeit geben? Da diese Frage kaum beantwortbar ist, setzte Bense (1975, S. 44 ff., 65 f.) bei den Peirceschen ordinalen Kategorien Firstness, Secondness, Thirdness an und führte also eine "Zeroness" als der Bereich der "disponiblen" bzw. "vorthetischen" Mittel und Objekte ein. Es handelt sich dabei nach Benses eigenen Bestimmungen um vorzeichenhaften Gebilde, deren Relationszahl $r = 0$ ist, deren Kategorialzahl jedoch $k > 0$ ist. Der Fall $r = k = 0$ ist damit nur für die "absoluten", d.h. nicht

kategorisierten – und damit weder wahrgenommenen noch wahrnehmbaren Objekte erfüllt. Da jedoch für die Ebene der Disponibilität der später innerhalb einer triadischen Zeichenrelation fungierenden Kategorien stets $k > 0$ gilt, weist die der tetradischen Zeichenrelation zugehörige Matrix am "Pol" (0.0) eine Lücke auf

	0	1	2	3
0	—	0.1	0.2	0.3
1	1.0	1.1	1.2	1.3
2	2.0	2.1	2.2	2.3
3	3.0	3.1	3.2	3.3,

d.h. die neue tetradische Zeichenrelation enthält war die in sich eingebettete triadische Peirce-Bensesche Zeichenrelation vollständig, aber die die entsprechende symmetrische 3×3-Matrix enthaltende Obermatrix der tetradischen Zeichenrelation ist unvollständig. Konkret bedeutet das, daß die Variable a in

$$ZR^4 = (0.a, ((1.b), ((2.c), (3.d)))$$

aus einem anderen, nämlich triadischen, Wertevorrat besetzt wird als die Variablen b, c, d, welche einen tetradischen Wertevorrat besitzen. D.h. die tetradische Matrix enthält eine triadisch-trichotomische Submatrix, aber die Differenzmatrix zwischen der tetradischen und der triadischen Matrix ist zwar tetradisch, aber trichotomisch und nicht tetratomisch. Im Gegensatz zur triadischen Submatrix enthält also die tetradische Obermatrix zwar eine Neben-, jedoch keine Hauptdiagonale. Damit gibt es zwar zur triadischen Eigenrealität, nicht aber zur triadischen Kategorienrealität eine tetradische Entsprechung.

3. Eine weitere, rein formal weit weniger dramatisch als inhaltlich, beruht darin, daß die tetradische Matrix wegen ihrer Tri- anstatt Tetratomizität zwar das reine Objekt nicht enthält, dieses aber immerhin als "Zero-Objekt" Teil der Matrix ist. Dasselbe gilt nun aber gerade nicht für das reine Subjekt, denn seine

maximale Approximation ist (3.3), und diese ist Teil der Matrix. Somit ist von der tetradischen Matrix aus betrachtet das Objekt (als "Leerstelle") der Semiotik immanent, das Subjekt jedoch "transzendent", d.h. es steht außerhalb der Matrix und damit außerhalb der Semiotik, da ja nach semiotischer Auffassung nur das gegeben ist, was repräsentierbar ist (Bense 1981, S. 11). Noch prägnanter gesagt: Während die triadische Matrix sowohl Subjekt- als auch Objekttranszendenz besitzt (da sie die nullheitliche Ebene ja nicht enthält), besitzt ihre tetradische Obermatrix zwar Subjekt-, aber nicht Objekttranszendenz! Da wir in Toth (2012b) gezeigt hatten, daß formalsemiotisch kein Unterschied zwischen Benses "disponiblen" Mitteln und seinen "vorthetischen Objekten" besteht, bleibt somit die "vorsemiotische Triade" unvollständig in dem Sinne, daß es in der tetradischen Semiotik zwar disponible oder vorthetische Mittel und Objekte, aber keine disponiblen oder vorthetischen Interpretanten gibt. Diese von Bense (1975) selbst anvisierte Vorthetik oder Präsemiotik ist somit im Gegensatz zur triadischen Semiotik selbst dyadisch, und damit verbirgt sich hinter bzw. "unter" der tetradischen Obermatrix eine dyadische und keine triadische Objektrealität, auch wenn die nullheitlichen Subzeichen der Form (0.a) ja mit $a \in \{1, 2, 3\}$ eine Trichotomie bilden.

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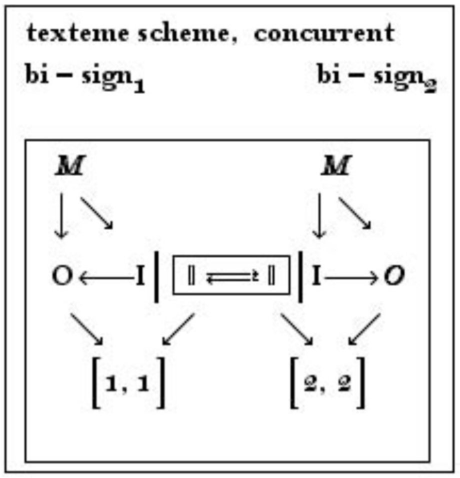
Annäherung an systemische Bi-Zeichen

1. Kaehr (2009a) hatte vorgeschlagen, nicht das Zeichen, sondern ein umfassenderes System, das er Textem nennt, zur Ausgangsbasis der Semiotik zu machen:



Die beiden "Bi-Zeichen" sind wie folgt in ein Textem eingebettet:

texteme :
diamond = (sign + environment)
bi – sign = (diamond + 2 – anchor)
texteme = (composed bi – signs + chiasm).

2. Wie man aus dem Diagramm ersieht, läuft der Zusammenhang der beiden Bi-Zeichen über eine Interpretanten-Umgebung ab, wobei die beiden Interpretanten nicht nur morphismisch, sondern auch, wie Kaehr sich ausdrückt, heteromorphismus auf einander abgebildet werden. Vom Standpunkt der systemischen Semiotik haben wir also folgende vier Abbildungstypen zur Wahl:

morphismisch-semiosisch: $[A_\alpha \rightarrow I_\alpha]$

morphismisch-retrosemiosisch: $[A_\alpha \leftarrow I_\alpha]$

heteromorphismisch-semiosisch: $[A_\alpha \rightarrow I_\beta]$

heteromorphismisch-retrosemiosisch: $[A_\alpha \leftarrow I_\beta]$

(mit $\alpha \neq \beta$)

Wesentlich bei dieser Unterscheidung ist also, daß die Kaerschen Heteromorphismen nicht einfach Retrosemiosen sind (vgl. Kaehr 2009b).

3. Kaehr (2009a) unterscheidet ferner zwischen homogenen und heterogenen Bi-Zeichen-Zusammenhängen. Nehmen wir also Beispiel die beiden systemischen Zusammenhänge, die wir als Beispiele zweier semiotischer Typen von Flächenschluß untersucht hatten (Toth 2012a):

3.1. Homogener semiotischer Flächenschluß

$$ZR_{sys} = [[A \rightarrow I], [[[A \rightarrow I] \rightarrow A], [[[[A \rightarrow I] \rightarrow A] \rightarrow I\boxed{]]] \leftarrow I\ [}A \leftarrow [I \leftarrow A]]]]$$

$$I \equiv I^n$$

3.2. Heterogener semiotischer Flächenschluß

$$ZR_{sys} = [[A \rightarrow I], [[[A \rightarrow I] \rightarrow A], [[[[A \rightarrow I] \rightarrow A] \rightarrow I\boxed{]]] \leftarrow A\ [I} \leftarrow [A \leftarrow I]]]]$$

$$I \equiv A^n$$

In diesen beiden Fällen handelt es sich um strikt monokontexturale "Texteme", d.h. es ist nicht nötig, anstatt Zeichen Bi-Zeichen zu nehmen, und die beiden (semiosischen und retrosemiosischen) morphismischen Abbildungstpyen sind ebenfalls ausreichend. Stellt man sich jedoch die beiden Zusammenhangstypen eingebettet in ein polykontexturales Verbundsystem, dann kann man die Abbildungen, wie oben gezeigt, kontexturalisierungen, d.h. formal mit $\alpha, \beta \in K$ indizieren und somit beide Zusammenhangstypen in der Form von Kaehrschen Textemen notieren. Besonders sei noch darauf hingewiesen, daß wir in Toth (2012b) die These vertreten hatten, daß die systemischen Qualitäten der Form $[A \rightarrow I]° = [A \leftarrow I]$, die ja das "Außen des Innen" in Bezug auf eine Kontextur betreffen, die Funktion der systemischen Perspektivierung ausüben. Sie könnten somit eine semiotische Verankerung übernehmen. Der wichtigste Punkt, auf den wir noch hinzuweisen haben, ist aber, daß eine polykontexturale Form, wie dies auch Kaehr gesehen hat, eine mindestens tetradische Semiotik ist, also z.B. eine solche, wie sie zuletzt in Toth (2012c) skizziert worden war. Eine solche impliziert jedoch nicht nur zwei, sondern mindestens drei Kontexturen. Das bedeutet jedoch für unsere obigen vier Abbildungstypen, daß die beiden heteromorphismischen jeweils 3! = 6 Permutationen in Bezug auf die Verteilung der Kontexturen haben, nämlich (α, β, γ), (α, γ, β), (β, α, γ), (β, γ, α), (γ, α, β) und (γ, β, α).

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Vom "Kristall-Sehen" zum Diamantenmodell

1. In Oskar Panizzas Erzählung "Die Gelbe Kröte" (1896) heißt es: "Wenn wir von einer Summe gleicher Geräusche affizirt und von einer Menge stets sich wiederholender optischer Eindrüke erregt werden, so dauert es einige Zeit, dann werden die äußeren Sinne stumpf, und es hebt sich aus unserem Innern eine Art 'Kristall-Sehen', eine autochtone Macht, eine dritte Bewegung, die wir nicht mehr komandiren können, die sich als 'freier Wille' selbst auf den Schauplaz stelt"[4] (Panizza 1992, S. 84f.). Daß der freie Wille als "dritte Bewegung" zwischen Kognition und Volition steht, wie Gotthard Günther viele Jahrzehnte später unterscheiden sollte (Günther 1971), und zwar indem der freie Wille als *vermittelnde* Instanz die systemische Basisdichotomie von Außen und Innen aufbricht, wurde bereits in Toth (2012a) angedeutet.

2. Wie im folgenden gezeigt werden soll, liest sich Panizzas "Kristall-Sehen" wie eine Vorläuferkonzeption des 2007 von Rudolf Kaehr vorgeschlagenen kategorial-saltatorischen Diamantenmodells, das wie folgt semiotisch dargestellt werden kann (vgl. Toth 2007a):

$$1.3 \longleftarrow 2.3$$

$$1.3 \rightarrow 3.2 \diamond 2.2 \rightarrow 2.3$$

$$1.3 \rightarrow 2.3$$

Die Neuerung Kaehrs innerhalb seines polykontexturalen Diamantenmodells beruht darin, dem kategorietheoretischen Modell mit seinen morphismischen Abbildungen ein saltatorisches Modell mit dessen heteromorphismischen Abbildungen gegenüberzustellen. Aus der Kombination von Akzeptanz und Rejektion, von Morphismen und Heteromorphismen ergibt sich ein Modell, dessen Struktur dem eines Diamanten gleicht. Obzwar es in der Semiotik, wie Kaehr (2010) richtig bemerkt, keine eigentlichen polykontexturalen Hetero-

[4] Panizzas bewußt eigenwillige Orthographie wird beibehalten.

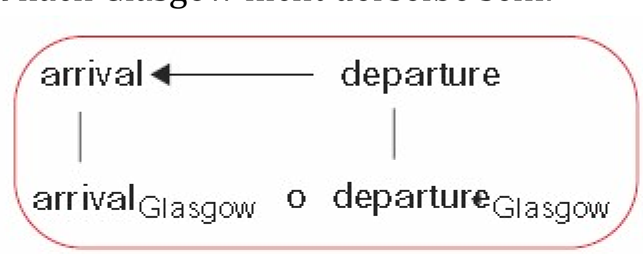
morphismen (sowie damit assoziierte Strukturen, vgl. jedoch Toth 2007b) gibt, bedeutet semiotisch

1. die Umkehrung einer Semiose, d.h. die entsprechende Retrosemiose, wie schon Max Bense bemerkte, nur eine quantitative, aber keine qualitative Inversion. Dies geht schon aus den konversen Subzeichen hervor: Kategorieal läßt sich z.B. das Sinzeichen (1.2) als Abbildung (1 → 2) darstellen, aber deren retrosemiosische Abbildung (1 ← 2) ist kein Sinzeichen, sondern ein Icon, also keine erstheitliche Quantität, sondern eine zweitheitliche Abstraktion (Bense 1979, S. 61). Entsprechendes gilt für alle nicht-genuinen Subzeichen, d.h. für alle nicht-identitiven semiotischen Morphismen.

2. fallen nur im Falle dyadischer Subzeichenrelationen konverse und duale Formen zusammen. Bereits im Falle dreistelliger Zeichengebilde (vgl. Toth 2012b), gibt es zwei Mal vier Abbildungsvarianten:
(a → (b → c), (a → (b ← c)), (a ← (b → c)), (a ← (b ← c))

((a → b) → c), ((a → b) ← c), ((a ← b) → c), ((a ← b) ← c),

und genau diese Situation liegt im Diamentanmodell vor, wenn man sich die a, b, c in den obigen Abbildungstypen als semiotisch dyadische Objekte vorstellt. Panizzas "Kristall-Sehen" ist also impressionistisch gesagt, die Möglichkeit, einen Teil seiner selbst aus sich herauszulösen und zu seinem eigenen Beobachter zu machen. Nichts anderes tun die Heteromorphismen, denn, wie Kaehr (2077, S. 18) sehr schön am Beispiel einer Reise in England gezeigt hat, muß der Weg von und nach Glasgow nicht derselbe sein:



zooming into the composition

genauso wenig, wie z.B. die "gleiche" Landschaft bei Tageslicht und in der Dunkelheit der Nacht nicht die "selbe" ist (vgl. dieses Stilmittel Panizzas ap. Bauer 1984, S. 74). Das schönste Beispiel für das diamantentheoretische

"Kristall-Sehen" finden wir jedoch wie zu erwarten bei Panizza. In einer seiner besten Erzählungen, dem "Corsetten-Fritz", heißt es vom Ich-Erzähler, der, wie sein Vater, Priester geworden ist: "Unwillkürlich schaute ich hinunter auf die Kirchenbänke, und: da saß ich, als Junge, mit gläsernem, starrem Blick: und gleichzeitig hörte ich die breite, wiederhallende Predigerstimme meines Vaters" (1992, S. 78). Solche Aufhebungen der Individualität setzen voraus, daß in einem logischen System der Identitätssatz außer Kraft gesetzt ist, und als Folge davon wird der Satz vom ausgeschlossenen Dritten aufgelöst. Somit kann eine Person mehrere Identitäten besitzen, und eine Form davon ist z.B. das Sich-selbst-Begegnen, ein Motiv übrigens, das sich durch Panizzas erzählerisches Werk zieht und das er in seinem philosophischen Hauptwerk "Der Illusionismus" (1895) aus metapyhsischer Sicht vor dem Hintergrund des deutschen transzendentalen Idealismus sowie des solipsistischen Idealismus Stirners detailliert abgehandelt hatte. Eine Logik jedoch, in der der Drittensatz außer Kraft gesetzt ist, muß eine mindestens dreiwertige Logik sein, d.h. eine Logik, welche im Gegensatz zur zweiwertigen aristotelischen Logik Platz für ein vermittelndes Glied hat – das berühmte Panizzasche "Dritte", beispielsweise eben der sich verselbständigende freie Wille.

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Zum erkenntnistheoretischen Status des Zeichens

1. Bekanntlich besitzen Quanten-Objekte die duale Eigenschaft, sowohl als Teilchen (Objekte) als auch als Wellen (Funktionen) auftreten zu können. In gewisser Weise vergleichbar damit ist Benses Feststellung, daß das Zeichen als Funktion "die Disjunktion zwischen Welt und Bewußtsein" überbrückt (1975, S. 16). Damit haben auch Zeichen die duale Eigenschaft, sowohl als Objekte als auch als Abbildungen aufzutreten.

2. In meiner "Theory of the Night" (Toth 2011) hatte ich die vier aus der Kombination von Subjekt (S) und Objekt (O) möglichen epistemischen Funktionen (sS, oS, oO, sO) zur erkenntnistheoretischen Bestimmung des Zeichens herangezogen. Kaehr (2011), der sich intensiv mit der Theorie der Nacht auseinandergesetzt hatte, gibt folgendes Korrespondenzschema

subjective subject (sS) ≅ Thirdness (interpretant relation, I)
objective object (oO) ≅ Secondness (Object relation, O)
subjective object (sO) ≅ Firstness (medium relation, M)
objective subject (oS) ≅ Zeroness (quality, Q)

3. Damit erhebt sich allerdings die Frage, ob es wirklich sinnvoll sei, die Peirce-Benseschen sog. gebrochenen Kategorien, mit denen bekanntlich die Zeichenklassen sowie die Realitätsthematiken modal bestimmt werden, auf die vier epistemischen Funktion abzubilden. Die Kritik an meiner eigenen Konzeption einer epistemischen Semiotik umfaßt folgende Punkte:

3.1. Wenn das Zeichen zwischen Welt und Bewußtsein und damit zwischen Objekt und Subjekt vermittelt, dann kann es sinnvollerweise nur als subjektives Objekt, und das heißt als durch ein Subjekt gesetzte (thetisch eingeführte) Objektskopie bestimmt werden. Die beiden homogenen epistemischen Funktion fallen damit automatisch dem zeichensetzenden und zeichenverwendenden Subjekt sowie dem durch das Zeichen bezeichneten Objekt zu. Damit haben wir

Zeichen = sO

Subjekt = sS

Objekt = oO.

3.2. Somit erhebt sich die Frage nach dem semiotischen Status der verbleibenden Funktion des objektiven Subjekts. Dieses ist definitionsgemäß ein Subjekt, das für ein anderes Subjekt ein Objekt ist, und also keinesfalls die epistemische Funktion der Qualität. Ferner besteht somit auch kein duales Austauschverhältnis der Form (sO × oS) zwischen dem Mittelbezug und der Qualität oder Substanz des Zeichens. Zwar wäre es möglich zu sagen, daß ein Subjekt B, das für ein Subjekt A ein objektives Subjekt ist, von sich selbst aus gesehen in diesem Moment natürlich ein subjektives Objekt ist, aber dieser Dualismus betrifft die Ontik und nicht die Semiotik, d.h. es handelt sich um die Personalunion zweier dualer Funktionen zwischen zwei Subjekten und weder um die Relation eines Zeichens zu seinem bezeichneten Objekt noch um diejenige eines Zeichens zu seinem setzenden Subjekt.

3.3. Wenn das Zeichen zwischen einem bezeichneten Objekt und einem setzenden Subjekt vermittelt, dann benötigen wir zur vollständigen Bestimmung des Zeichens nicht nur eine Theorie des Zeichens (Semiotik), sondern auch eine Theorie des Objektes (Ontik) sowie eine Theorie des Subjektes. Während, von meinen eigenen, bislang noch sehr rudimentären, Arbeiten zu einer Ontik im Sinne einer Theorie bezeichneter Objekte abgesehen, die Heideggersche Fundamentalontologie immer noch die beste Theorie des Objektes darstellt, besitzen wir auch heute noch nicht einmal ansatzweise eine Theorie des Subjektes im Sinne einer Theorie der zeichensetzenden und zeichenverwendenden Subjekte. Im Grunde genommen sind wir nach beinahe einem halben Jahrhundert noch nicht über Benses äußerst lakonische Bestimmung hinausgekommen: "Zeichen ist alles, was zum Zeichen erklärt wird und nur was zum Zeichen erklärt wird. Jedes beliebige Etwas kann (im Prinzip) zum Zeichen erklärt werden. Was zum Zeichen erklärt wird, ist selbst kein Objekt mehr, sondern Zuordnung (zu etwas, was Objekt sein kann); gewissermaßen Metaobjekt" (1967, S. 9).

3.4. Ein ungleich komplexeres und schwerer wiegendes Problem erhebt sich nun allerdings aus dem Umstand, daß das Zeichen keine dem objektiven

Subjekt entsprechende epistemische Funktion zu besitzen scheint. Der Grund hierfür liegt ohne Zweifel darin, daß das Zeichen sowohl von seinem bezeichneten Objekt als auch von seinem setzenden Subjekt durch je eine Kontexturgrenze (im Sinne von Gotthard Günthers polykontexturaler Logik und Ontologie) getrennt ist. Dies bedeutet also, daß ein Zeichen, das ein Objekt bezeichnet, niemals mit seinem bezeichneten Objekt zusammenfallen, d.h. identisch werden kann. Und es darf eine solche Koinzidenz auch nie eintreten, denn träte sie ein, so wären Zeichen und Objekt nicht mehr unterscheidbar, und damit wäre mindestens das Zeichen überflüssig. Ferner darf das Zeichen aber auch nicht mit seinem Subjekt zusammenfallen, denn es vertritt ja im Sinne eines subjektiven Objektes das setzende Subjekt gerade hinblicklich eines bezeichneten Objektes. Dennoch folgt natürlich bereits aus Benses Bestimmung der Zeichenfunktion als Vermittlung zwischen Welt und Bewußtsein, daß das Zeichen zwischen der Kontextur des Objektes und der Kontextur des Subjektes vermittelt. Die aus dieser Feststellung resultierende Frage, ob denn somit das Zeichen entweder der Objekt-, der Subjekt-, beiden Kontexturen angehöre oder selbst eine eigene Kontextur bestimme, kann man also ebenfalls sogleich dahingehend beantworten, daß das Zeichen, indem es Objekt und Subjekt in Beziehung zueinander setzt, auch zwischen den Kontexturen des Objektes und des Subjektes vermittelt. Obwohl das Zeichen also a priori nicht poly-kontextural ist, ist es dennoch kontexturenvermittelnd. Daraus folgt übrigens auch, daß es prinzipiell unmöglich ist, entweder die Semiotik auf die Logik oder die Logik auf die Semiotik abzubilden, wenigstens solange man darunter die zweiwertige aristotelischen Logik meint, denn diese ist, wie G. Günther gezeigt hatte, strikt monokontextural. Will man also die Semiotik auf eine Logik abbilden, daß muß man ein Verbundsystem mindestens zweier Logiken heranziehen, wobei diese beiden Logiken miteinander vermittelt werden müssen, d.h. man benötigt die in der klassischen Logik völlig unbekannten Güntherschen Transoperatoren, um von einer Kontextur in die andere zu gelangen. Damit ergibt sich für das Zeichen die ganz absonderliche Folgerung, daß es zwar hinsichtlich seines Objektcharakters monokontextural, hinsichtlich seines Funktionscharakters aber polykontextural ist. Während also der von de Broglie formulierte Welle-Teilchen-Dualismus der Materie hinsichtlich seiner Kontexturalität monovalent ist, ist der von Bense formulierte Objekt-Funktions-Dualismus des Zeichens hinsichtlich seiner Kontexturalität bivalent. Erst in der Domäne des Geistes stellt sich die Frage nach der Kontexturalität, d.h. nach der Verortung der Logik, denn diese stellt sich erst nach dem Erscheinen der Subjekte ein. In der Domäne der Materie spielt die Frage der Verortung scheinbar paradoxerweise keine Rolle.

3.5. Zeichen müssen daher doppelt kontexturiert werden, denn sie vermitteln ja zwischen bezeichneten Objekten und setzenden Subjekten. Wenn ich ein Taschentuch verknote, um mich daran zu erinnern, daß ich morgen meine Tochter vom Kindergarten abholen muß, dann gehören 1. das Taschentuch als Zeichen und das von ihm bezeichnete Ereignis nicht der gleichen Kontextur an, und 2. gehöre ich selbst selbstverständlich weder der Kontextur des Taschentuchs noch derjenigen des Ereignisses an, d.h. Zeichen, Objekt und Subjekt befinden sich in drei verschiedenen Kontexturen. Im Falle der in der semiotischen Literatur mindestens ebenso berühmten Haarlocke meiner Geliebten gehören zwar Zeichen und Objekt der gleichen Kontextur an, aber ich selbst befinde mich immer noch in einer von beiden geschiedenen Kontextur. Doch damit sind wir bereits an der äußerste Grenze angelangt, da sich Objekt und Subjekt nie in der gleichen Kontextur befinden können, denn würden sie es tun, wären sie nicht mehr voneinander unterscheidbar, und es hätte dann überhaupt keinen Sinn mehr, von Subjekt und Objekt zu sprechen. Doch genau diesem Umstande verdankt das Zeichen seine Entstehung und seine Legitimation: als subjektives Objekt umschifft es den prinzipiell unmöglichen Zusammenfall von Subjekt und Objekt im Sinne eines behelfsmäßigen Substitutes, und zwar sowohl für das von ihm bezeichnete Objekt, als dessen Kopie das Zeichen auftritt, als auch für das setzende Subjekt, dem es seine Existenz verdankt.

4.1. Aus den obigen Feststellungen resultiert, daß innerhalb der Peirce-Benseschen Zeichenrelation

$ZR = R(M, O, I)$

natürlich nur der Mittelbezug M die epistemische Funktion eines subjektiven Objektes ausübt, denn der Objektbezug O ist per definitionem die Relation des

Mittelbezugs als Repräsentamen zum vom Zeichen bezeichneten Objekt (Ω), d.h.

$$O = R(M, \Omega),$$

und der Interpretantenbezug I ist die Relation von O zum das Zeichen setzenden und verwendenden Subjekt (Σ), d.h.

$$I = R(R(M, \Omega), \Sigma).$$

Doch damit ist M nichts anderes als das Zeichen selbst, das innerhalb von ZR in doppelte Beziehung zu seinem Objekt und Subjekt gesetzt wird:

$$ZR = R(Z, R(M, \Omega), R(R(M, \Omega), \Sigma)),$$

denn nur M kann ja die definitorische Zeichenfunktion der Vermittlung zwischen Welt und Bewußtsein bzw. Objekt und Subjekt ausüben. Man könnte daher die zehn Benseschen Zeichenklassen hinsichtlich ihres Anteils an Vermittlung, d.h. des Vorkommens der Kategorie M, wie folgt anordnen

(I.M, O.M, M.M)	$M = 4/6$	(I.M, O.O, M.O)	$M = 2/6$
		(I.M, O.O, M.I)	$M = 2/6$
(I.M, O.M, M.O)	$M = 3/6$	(I.M, O.I, M.I)	$M = 2/6$
(I.M, O.M, M.I)	$M = 3/6$		

(I.O, O.O, M.O)	$M = 1/6$
(I.O, O.O, M.I)	$M = 1/6$
(I.O, O.I, M.I)	$M = 1/6$
(I.I, O.I, M.I)	$M = 1/6$

Zeichenklassen mit dem gleichen M-Wert sind damit vermittlungsmäßig gleich, d.h. bei ihnen unterscheidet sich nur die Repräsentationsstärke des Zeichens relativ zu seinem bezeichneten Objekt und seinem setzenden Subjekt. Ordnet man die Zeichenklassen weiterhin nach der Repräsentationsstärke des Zeichens relativ zu seinem Objekt, ergibt sich folgende Ordnung

(I.M, O.M, M.M)	M = 4/6	O = 1/6	S = 1/6
(I.M, O.M, M.O)	M = 3/6	O = 2/6	S = 1/6
(I.M, O.M, M.I)	M = 3/6	O = 1/6	S = 2/6
(I.M, O.O, M.O)	M = 2/6	O = 3/6	S = 1/6
(I.M, O.O, M.I)	M = 2/6	O = 2/6	S = 2/6
(I.M, O.I, M.I)	M = 2/6	O = 1/6	S = 3/6
(I.O, O.O, M.O)	M = 1/6	O = 4/6	S = 1/6
(I.O, O.O, M.I)	M = 1/6	O = 3/6	S = 2/6
(I.O, O.I, M.I)	M = 1/6	O = 2/6	S = 3/6
(I.I, O.I, M.I)	M = 1/6	O = 1/6	S = 4/6

Es gibt also nur zwei Zeichenklassen (und nicht etwa drei!), bei welchen Objekt und Subjekt gleich stark repräsentiert sind, eine einzige Zeichenklasse, bei denen dies für Zeichen, Objekt und Subjekt gilt, auffälligerweise nur eine einzige Zeichenklasse, bei der die Stärke der Vermittlung derjenigen der Repräsentanz des Subjektes entspricht, und ebenfalls nur eine einzige Zeichenklasse, bei der die Stärke der Vermittlung derjenigen der Repräsentanz des Objektes korrespondiert.

4.2. Diese neuerliche Erkenntnis führt uns zu einer Neudarstellung des Peirce-Benseschen Systems der Zeichenklassen auf der Basis der Repräsentanzstärken von Zeichen, Objekt und Subjekt, und dazu benutzen wir die Zähler der obigen Bruchzahldarstellung der Repräsentanzwerte.

(I.M, O.M, M.M)	:=	(M^4, O^1, S^1)
(I.M, O.M, M.O)	:=	(M^3, O^2, S^1)
(I.M, O.M, M.I)	:=	(M^3, O^1, S^2)

(I.M, O.O, M.O) := (M^2, O^3, S^1)

(I.M, O.O, M.I) := (M^2, O^2, S^2)

(I.M, O.I, M.I) := (M^2, O^1, S^3)

(I.O, O.O, M.O) := (M^1, O^4, S^1)

(I.O, O.O, M.I) := (M^1, O^3, S^2)

(I.O, O.I, M.I) := (M^1, O^2, S^3)

(I.I, O.I, M.I) := (M^1, O^1, S^4).

Wie man leicht erkennt, sind die Repräsentationsmöglichkeiten des Zeichens als eines subjektiven Objektes (M^i, O^j, S^k) mit $\Sigma i,j,k = 6$, d.h. in den drei möglichen Formen (M^1, M^2, M^3, M^4), auch tatsächlich bereits ausgeschöpft, d.h. das Peirce-Bensesche System ist repräsentationstheoretisch betrachtet vollständig und sollte also (im Gegensatz auch zu meinen eigenen früheren Annahmen) weder relational (d.h. durch Erhöhung der n-adischen Haupt- und/oder der n-tomischen Stellenwerte) noch durch Aufhebung der Inklusionsbeschränkung $a \leqq b \leqq c$ für a, b, c $\in$ {.1, .2, .3} in der Zeichenklassenform (3.a, 2.b, 1.c)) erweitert werden. Konkret gesagt, bedeutet dies also, daß weder Gebilde der Form (3.a, 2.b, 1.c 0.d) noch Zeichenklassen wie z.B. (3.2 2.1 1.3) oder (3.1 2.3 1.1) Repräsentationsschemata sind, d.h. es handelt sich hier nicht nur um irreguläre, sondern um inexistente Pseudo-Zeichenklassen, wie man übrigens leicht selber nachprüft.

4.3. Die Frage der Kontexturierung von Zeichenklassen wurde bereits weiter oben gestreift (vgl. Kap. 3.4). Die bereits von Kaehr (2009) vorgeschlagenen Kontexturierungen sind durchwegs Subjektskontexturierungen, da in der polykontexturalen Logik das Subjekt und nicht das Objekt eine Kontextur bestimmt, denn diese Logik ist eine Logik für mehr als ein Subjekt und nicht für mehr als ein Objekt, d.h. es ändert sich mit dem Wechsel von der aristotelischen zur Günther-Logik für das Objekt überhaupt nichts, und darüber können auch die gebrochenen epistemischen Funktionen des subjektiven Objekts und des objektiven Subjekts, auf die Günther immer wieder hingewiesen hatte, nichts ändern. Wenn man also nicht wieder einer pansemiotischen Zeichentheorie wie derjenigen von Peirce huldigen möchte, die zwar qua Metaobjektivation das Objekt einerseits voraussetzt, es dann aber nach Abschluß des Metaobjektivationsprozesses sogleich wieder vergißt und als angeblichen Realitätsbezug nur mehr die ad hoc erzeugten sog. Realitätsthematiken zuläßt, dann muß man explizite Objektkontexturierungen in Ergänzung zu den Subjektkontexturierungen einführen, da das Zeichen, wie bereits gesagt, ja nicht nicht nur die epistemischen Funktionen von Objekt und Subjekt, sondern natürlich auch deren Kontexturen vermittelt.

4.4. Damit stellt sich abschließend die bereits angeschnittene Problematik der von Bense (1975, S. 100 ff.; 1976, S. 53 ff.) eingeführten Realitätsthematiken. Sie entsprechen formal den Konversionen der Zeichenklassen, d.h. sie haben die allgemeine Form

$Rth := Zkl^{-1} = (3.a, 2.b, 1.c)^{-1} = (c.1, b.2, a.3)$.

Damit kehrt sich das Verhältnis der sog. Subzeichen um, d.h. es wird z.B. ein Legizeichen (1.3) zum Rhema (3.1), d.h. es soll angeblich der Realitätstbezug z.B. eines konventionell vermittelnden Zeichens qua "Dualisation" ($\times(1.3) = (3.1)$) ein offener und logisch nicht beurteilbarer Zeichenzusammenhang im Sinne der Thematik der Realität sein. Dagegen besitzen die beiden anderen möglichen Mittelbezuge, d.h. das Qual- (1.1) und das Sinzeichen (1.2), überhaupt keine Zeichenzusammenhänge als Realitätskorrelat, sondern nur sich selber ($\times(1.1) = (1.1)$) sowie die iconische Relation (2.1) des Zeichens zu seinem bezeichneten Objekten ($\times(1.2) = (2.1)$). Es dürfte klar sein, daß diese beispielhaft erwähnten sowie alle übrigen neun möglichen Fälle jeglicher sinnvollen Interpretation entbehren. Vor allem aber ändert die "Dualisation" einer Zeichenklasse in ihre Realitätsthematik rein gar nichts an der repräsentationstheoretischen Struktur der Zeichenklassen (vgl. Kap. 4.2). Z.B. haben wir

$\times(I.M, O.M, M.O) = (O.M, M.O, I.M) = (M^3, O^2, S^1)$.

Realitätsthematiken sind also vom repräsentationstheoretischen Standpunkt aus, der das Zeichen als subjektives Objekt in Relation zu seinem bezeichneten objektiven Objekt und zu seinem setzenden subjektiven Subjekt betrachtet, vollkommen überflüssig.

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Elementare Anforderungen an eine Logik der Ontik

1. Wie ich seit Toth (2012) gezeigt habe, ist der von Peirce und Bense begründeten Semiotik als Theorie der Zeichen eine Ontik als Theorie der Objekte gegenüberzustellen. Diese Notwendigkeit ergibt sich aus der Tatsache, daß die Dichotomie D = (Objekt, Zeichen) isomorph ist zur logischen Basisdichotomie L = (0, 1). Würde ein Objekt, wie es Peirce, Bense und ihre Nachfolger behaupten, durch Wahrnehmung automatisch zum Zeichen („Alles, was wir wahrnehmen, nehmen wir durch Zeichen wahr"), gäbe es in dieser Welt nur Zeichen, denn selbst gesetzt, es gäbe trotzdem noch Objekte, wie sollten diese ohne Wahrnehnung erkennbar sein? Die Peirce-Bense-Semiotik ist, wie übrigens die meisten Semiotiken, pansemiotisch. Diese Tatsache bedeutet also vermöge Isomorphie dasselbe, als würde man eine Logik entweder durch L = 0 oder durch L = 1 definieren, also einen vollständigen Unsinn.

2. Objekte setzen Subjekte voraus, so, wie Subjekte Objekte voraussetzen, denn Objekte können vermöge Wahrnehmung nur für Subjekte Objekte sein, und umgekehrt setzt die Wahrnehmung eines Objektes das wahrnehmende Bewußtsein eines Subjektes voraus. Daraus folgt, daß wir statt von

L = (0, 1)

auszugehen haben von

0 = f(1)

1 = f(0),

d.h.

L* = (f(1), f(0)).

Es gibt somit weder subjektive Subjekte noch objektive Objekte, sondern nur objektive Subjekte und subjektive Objekte. Die subjektiven Objekte sind die von einem Subjekt wahrgenommenen Objekte, und die objektiven Subjekte sind die von einem Objekt „affizierten" Subjekte. Das bedeutet aber, daß L* nicht nur zwei, sondern vier mögliche Relationen aufweist

L* = ((0, (0)), (0, (1)), (1, (0)), (1, (1))).

Einfacher ausgedrückt, wird die klassische Definition der 2-wertigen Logik

$L = (0, 1)$

durch die nicht-klassische Definition

$L^* = (E, (0, 1))$,

darin E den Einbettungsoperator bedeutet, ersetzt. Man bedenke, daß L* immer noch 2-wertig ist, denn die erkenntnistheoretisch relativen Werte von L* unterscheiden sich von den erkenntnistheoretischen absoluten Werten von L lediglich dadurch, daß sie in funktionale Abhängigkeit voneinander gesetzt werden. Man könnte sogar soweit gehen, zu sagen, daß die vier Werte von L* zwischen den zwei Werten von L vermitteln

$0 = 0(0) \rightarrow 0(1) \rightarrow 1(0) \rightarrow 1(1) = 1.$

Die Vermittlungsstufen 0(1) und 1(0) verstoßen jedoch nicht gegen das Grundgesetzt des tertium non datur, denn mit E wird ja kein weiterer logischer Wert in L* eingeführt.

3. L* ist genau die Logik, welche man benötigt, um als Grundlage der Dichotomie

D = (Objekt, Zeichen)

bzw.

D = (Ontik, Semiotik)

zu fungieren, denn wie man leicht einsieht, ist

subjektives Objekt = Objekt → Ontik

objektives Subjekt = Zeichen → Semiotik,

d.h. die beiden Relationen

O(0) und 0(1)

sind die logischen Basisrelationen der Ontik, und die beiden Relationen

1(0) und 1(1)

sind die logischen Basisrelationen der Semiotik.

Wie Rudolf Kaehr in einer grundlegenden Arbeit gezeigt hatte, ist die von mir zuerst skizzierte „quadralektische“ Logik L* kompatibel mit der polykontexturalen Diamantentheorie (vgl. Toth 2011), d.h. einem mehrwertigen logischen System, das auf der Dissemination theoretisch unendlich vieler 2-wertiger „Monokontexturen“ beruht.

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Hierarchische Vermittlung von logischer Zweiwertigkeit

1. Im folgenden präsentieren wir ein kleines und harmlos ausschauendes, aber nichts desto weniger „gefährliches“ Spiel. Wie zuletzt in Toth (2017) gezeigt, ist es möglich, eine „quadralektische“ (vgl. Kaehr 2011) Logik zu konstruieren vermittels eines Einbettungsoperators, d.h. eine Logik der Basisdefinition

$L^* = (E, (0, 1))$,

ohne gegen die Grundgesetze des Denkens, in Sonderheit ohne gegen das Gesetz des tertium non datur zu verstoßen. Es ist dann

$L^* = ((0, (0)), (0, (1)), (1, (0)), (1, (1)))$

mit

$0 = 0(0) \rightarrow 0(1) \rightarrow 1(0) \rightarrow 1(1) = 1.$

2. Sei nun

$L = (A, B)$,

dann bekommen wir als Anwendung eines „Vermittlungsoperators“ V

$V(L) = V(A, B) = (A, C, B)$.

Diese erststufige Anwendung von V ist also bijektiv. Die Bijektion ist jedoch bereits bei der zweitstufigen Anwendung von V aufgehoben, denn wir bekommen

$V(A, C, B) = ((A, D, C), (C, D, B))$.

Als drittstufige Anwendung von V bekommen wir

$V(A, D, C) = ((A, E, D), (D, E, C), (C, E, D), (D, E, B))$, usw.

Die Progression der V-Hierarchie von zweiwertigen Strukturen folgt also natürlich dem Schema

$2^0 = 1 \rightarrow 2^1 = 2 \rightarrow 2^2 = 4 \rightarrow 2^3 = 8 \rightarrow 2^4 = 16 \rightarrow \ldots$.

2. Gehen wir nun von der Logik aus, gilt selbstverständlich

$L = (0, 1)$ oder $L = (1, 0)$,

und das bedeutet, daß für die zusätzlich eingeführten Werte C, D, E, ... natürlich ebenfalls gelten kann

$C, D, E \in (0, 1)$.

Dieses Ergebnis ist nun in der Tat erstaunlich, denn dadurch präsentiert sich unsere obige Hierarchie wie folgt

$L = (0, 1)$ oder $(1, 0)$

$V(L) = V(0, 1)$ oder $V(1, 0) = (0, 0, 1)$ oder $(0, 1, 1)$ oder $(1, 0, 1)$ oder $(1, 1, 1)$,

d.h. bereits auf der 1. Stufe der Vermittlung ergibt sich das von uns zuletzt in Toth (2017) zugrunde gelegte „quadralektische" Schema relativer statt absoluter logischer Werte.

Verzichtet man also auf die Einführung anderer logischer Werte als 0 und 1 und wahrt die logische Zweiwertigkeit, ohne gegen das Gesetz des tertium non datur zu verstoßen, wächst die Progression der V-Hierarchie von zweiwertigen Strukturen doppelt so schnell an, d.h durch

$2^0 = 2 \rightarrow 2^1 = 4 \rightarrow 2^2 = 8 \rightarrow 2^3 = 16 \rightarrow 2^4 = 32 \rightarrow \dots$.

Damit ist eine echte Fortsetzung des elementaren Schemas einer Logik als Grundlage für die Ontik und die Semiotik

$L^* = (E, (0, 1))$,

mit

$L^* = ((0, (0)), (0, (1)), (1, (0)), (1, (1)))$

erreicht, da ja jeder der jeweils 3 Werte einer Vermittlungstruktur eingebettet oder nicht eingebettet erscheinen kann, vgl. etwa

$L^* = (0, 0, (1)), (0, (0), 1), ((0), 0, 0); (o, (0, 0), ...; ((0, 0, 0))$

und da wegen iterierter Anwendung von E natürlich auch eingebettete Vermitlungsstrukturen von Formen wie z.B.

$L^* = (0, (0), ((0))), (0, ((0)), (((0))))$, usw.

denkbar sind. Man wird der Hilfe elektronischer Rechenanlagen bedürfen, um die astronomisch hohe Zahl von möglichen Einbettungsgraden vermittelter Strukturen von L^* zu berechnen. Alle Einzelwerte dieser Strukturen sind und bleiben jedoch natürlich logisch zweiwertig, da die zusätzlichen Werte die beiden Werte der aristotelischen Logik sind und da E nicht gegen den logischen Drittensatz verstößt!

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Die Semiotik als 2-wertiges logisches Vermittlungssystem

1. In Toth (2017a) waren wir von der von uns schon früher eingeführten modifizierten Logik mit der Definition

$L^* = (E, (0, 1))$,

d.h.

$L^* = ((0, (0)), (0, (1)), (1, (0)), (1, (1)))$

sowie

$0 = 0(0) \rightarrow 0(1) \rightarrow 1(0) \rightarrow 1(1) = 1$

Ausgegangen. D.h., L* unterscheidet sich von der klassischen aristotelischen Logik mit der Definition

$L = (0, 1)$

lediglich durch zwei Vermittlungswerte, die durch den Einbettungsoperator E erzeugt werden und widerspricht somit nicht der 2-Wertigkeit dieser Logik.

2. In Toth (2017b) waren wir noch einen Schritt weiter gegangen und von einem progressiv wachsenden Schema von Vermittlungsstrukturen der elementaren Form

$V(A, B) = (A, C, B)$

ausgegangen. Das Besondere hierbei ist die Möglichkeit, daß, anders als es die mehrwertigen Logiken getan hatten, der Wert C nicht nur eine 3, sondern auch 0 oder 1 sein kann, d.h. daß für Vermittlungswerte W(V) allgemein gilt

$W(V) \in L$.

Damit bekommt man also bereits für die erststufige Vermittlungsebene

$V(A, C, B) = ((A, D, C), (C, D, B))$

statt zwei nun vier mögliche Vermittlungsstrukturen

$V(L) = V(0, 1)$ oder $V(1, 0) = ((0, 0, 1)$ oder $(0, 1, 1)$ oder $(1, 0, 1)$ oder $(1, 1, 1))$.

3. Nun ist, wie bereits aus dem Namen des Mediums, das Peirce einführte und das in der Terminologie Benses „Mittelbezug" heißt, M die Kategorie der Vermittlung, d.h. wir können statt

$Z = (M, O, I)$

oder besser

$Z = (O, M, I)$

auch schreiben

$M = V(O, I)$.

Tatsächlich resultieren ja die nun seit fast hundert Jahren anhaltenden Bemühungen, die triadische Semiotik mit der binären Logik (oder umgekehrt) zu begründen, nicht nur aus der verschiedenen Anzahl der Kategorien bzw. Werte, sondern aus dem epistemisch unklaren Status von M. M wird ja von Walther (1979, S. 58) ausdrücklich als 1-stellige Relation eingeführt und somit vom materialen Zeichenträger (vgl. Bense/Walther 1973, S. 137) differenziert. Da ganz offenbar die semiotische Kategorie des Objektbezugs dem logischen Objekt und die semiotische Kategorie des Interpretantenbezugs dem logischen Subjekt entspricht, spricht nichts dagegen, daß wir unser obigen Verfahren der freien Wertwahl bei logischen Vermittlungsstrukturen auch auf die Semiotik anwenden. Wir haben dann natürlich wieder die beiden Möglichkeiten

$M = O$

Oder

$M = I$.

Verfährt man auf diese Weise, ergeben sich zunächst zwei verschiedene Matrizen

$1 \longrightarrow 0$				$1 = 1$		
0.0	0.2	0.3		1.1	1.2	1.3
2.0	2.2	2.3		2.1	2.2	2.3
3.0	3.2	3.3		3.1	3.2	3.3,

wobei die rechte Matrix mit derjenigen identisch ist, die Bense (1975, S. 37) eingeführt hatte.

Nun können die beiden semiotischen Kategorien, wenn wir an der kanonischen Ordnung von Z = (M, O, I) festhalten, wiederum zweifach substituiert werden:

$2 \longrightarrow 0$ $\qquad\qquad$ $2 \longrightarrow 1$

$3 \longrightarrow 1$ $\qquad\qquad$ $3 \longrightarrow 0$.

Dadurch ergeben sich also, ausgehend von den obigen beiden Matrizen, zwei weitere, sowohl semiotisch als auch logisch zweiwertige Matrizen:

0.0	0.0	0.1		1.1	1.1	1.0
0.0	0.0	0.1		1.1	1.1	1.0
1.0	1.0	1.1		0.1	0.1	0.0 .

Wie man leicht erkennt, repräsentieren aber diese beiden Matrizen genau die von Kaehr (2011) als „quadralektisch“ bezeichnete und von mir eingeführte Vermittlungslogik der Form

$L^* = ((0, (0)), (0, (1)), (1, (0)), (1, (1)))$,

wobei noch zu beweisen wäre, ob die beiden Matrizen wirklich isomorph sind, d.h. letztlich, ob

$L^* = \quad ((0, (0)), (0, (1)), (1, (0)), (1, (1))) =$

$L^{*-1} = \ ((1, (1)), (1, (0)), (0, (1)), (0, (0)))$

gilt. Die Isomorphie von $L = (0, 1) = L^1 = (1, 0)$ wurde immerhin, auf informale Weise, von Günther bewiesen: "Beide Werte einer solchen Logik aber sind metaphysisch äquivalent. Das heißt, man kann sie beliebig miteinander vertauschen. Sie verhalten sich zueinander in einer totalen logischen Disjunktion, wie rechts und links. Es gibt keinen theoretischen Grund, welche Seite rechts und welche Seite links von der Zugspitze ist. Die Benennung beruht auf einer willkürlichen Entscheidung, und wenn man seinen Standpunkt wechselt, sind die rechte und die linke Seite miteinander vertauscht" (Günther 2000, S. 230 f.).

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Die drei Grundtypen polykontexturaler Diamanten

1. Aus kategorietheoretischer Sicht ist die Besonderheit der von Kaehr in die qualitative Mathematik eingeführten Diamanten (vgl. Kaehr 2007, 2009 und zahlreiche weitere Arbeiten) das Auftreten eines bislang unbekannten Typus von Abbildung: des Heteromorphismus, der im folgenden, Kaehrs Notation folgend, in der allgemeinen Form

$(O^{\sim}_{\omega} \leftarrow O^{\sim}_{\alpha})$

notiert wird. Man beachte in Sonderheit, daß für n-kontexturale Systeme mit n > 2 gilt, daß

$(A \rightarrow B) \neq (A \rightarrow B)^{-1}$.

Für die Semiotik bedeutet dies also, daß die Konversen der als Morphismen deutbaren Abbildungen, d.h. die sog. Retrosemiosen, in der monokontexturalen Peirce-Bense-Semiotik keineswegs Heteromorphismen sind.

2. Im folgenden sei gezeigt, daß es in Ergänzung zu den zwei von Kaehr (2007 usw.) komponierten polykontexturalen Diamanten noch einen dritten gibt, die wir relativ zu ihren jeweiligen Heteromorpismen als $O^{\sim}$-, $M^{\sim}$ und $I^{\sim}$-Diamanten kategorisieren können.

2.1. Polykontexturaler Diamant I

$$\begin{array}{ccc}
 & (O^{\sim}_{\omega} \leftarrow O^{\sim}_{\alpha}) & \\
\text{dif} \diagup & & \diagdown \text{dif} \\
((M_{\alpha} \rightarrow O_{\omega}) & \diamond & (O_{\alpha} \rightarrow I_{\omega})) \\
\diagdown \text{id} & & \diagup \text{id} \\
(M_{\alpha} & \longrightarrow & I_{\omega})
\end{array}$$

2.2. Polykontexturaler Diamant II

$$\begin{tikzcd}
 & (M^{\sim}_{\omega} & & M^{\sim}_{\alpha}) \arrow[ll] & \\
((O_{\alpha} \arrow[r] \arrow[d, dash, "\text{id}"] & M_{\omega}) \arrow[u, dash, "\text{dif}"] & \diamond & (M_{\alpha} \arrow[r] \arrow[u, dash, "\text{dif}"'] & I_{\omega})) \arrow[d, dash, "\text{id}"] \\
(O_{\alpha} \arrow[rrrr] & & & & I_{\omega})
\end{tikzcd}$$

2.3. Polykontexturaler Diamant III

$$\begin{tikzcd}
 & (I^{\sim}_{\omega} & & I^{\sim}_{\alpha}) \arrow[ll] & \\
((O_{\alpha} \arrow[r] \arrow[d, dash, "\text{id}"] & I_{\omega}) \arrow[u, dash, "\text{dif}"] & \diamond & (M_{\alpha} \arrow[r] \arrow[u, dash, "\text{dif}"'] & I_{\omega})) \arrow[d, dash, "\text{id}"] \\
(O_{\alpha} \arrow[rrrr] & & & & I_{\omega})
\end{tikzcd}$$

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Der Zusammenhang zwischen systemtheoretischer und funktionaler Semiotik

1. In Walther (1979, S. 138 ff.) findet sich eine funktionale Einteilung der Zeichen in Formation (M), Information (M→O) und Kommunikation (M→O→I), die m.W. bisher nur in meiner semiotischen Dissertation (Toth 1992) Anwendung gefunden hat. Die Besonderheit dieser Klassifikation beruht darin, dass hier nicht den drei Korrelaten des Zeichens – etwa wie bei der bekannten Morrisschen Zuordnung von Syntaktik, Semantik und Pragmatik zu M, O und I -, sondern den drei Funktionen, d.h. der 1-stelligen M-Funktion, der 2-stelligen (M→O)-Funktion und der 3-stelligen (M→O→I)-Funktion des Zeichens Modelle zur Interpretation zugeordnet werden.

2. Um den Zusammenhang zur systemtheoretischen Semiotik (vgl. Toth 2011) herzustellen, ist es jedoch nötig, von der erweiterten tetradischen Zeichenrelation

4ZR = (.0., .1., .2., .3.),

welche die Peircesche Zeichenrelation und das eingebettete kategoriale Objekt als Qualität enthält. Man kann dann den vier Fundamentalkategorien wie folgt die entsprechenden systemtheoretischen Funktoren zuordnen:

.0. → OI → ⌞

.1. → IO → ⌟

.2. → OO → ⌜

.3. → II → ⌝

Damit ergeben sich die folgenden Zusammenhänge zwischen der funktionalen und der systemtheoretischen Konzeption der Semiotik:

(.0. → OI → ⌞) → Materie (i.S. der realen Objektwelt)

(.1. → IO → ⌟) → Formation

((.1. → .2.) → IO → OO → ⌟ → ⌜) → Information

((.1. → .2. → .3.) → IO → OO → II → ⌞ → ⌟ → ⌜)) → Kommunikation

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Notizen zur Quadralektik des Zeichens

1. In mehreren Arbeiten (z.B. in Toth 2009a, b) hatte ich versucht, die Dichotomie von Zeichen und Objekt unter dem Verhältnis des Eigenen zum Anderen darzustellen. Die besondere Problematik, die sich hierbei stellt und die etwa sprachlich in Wendungen wie

a) Ich bin noch hier, aber die anderen sind schon weg,

noch stärker aber in Fügungen wie

b) Was willst du noch hier? Geh doch zu den anderen Idioten!

zum Ausdruck kommt, ist die, dass hier das jeweilige Andere am Eigenen und damit logischerweise auch das jeweilige Eigene am Anderen partizipiert. Zwischen dem Eigenen und dem Anderen besteht also in anderen Worten kein Abbruch von zwei Kontexturen, sondern eine Brücke bzw. ein Gebiet, in dem sich Eigenes und Anderes treffen, d.h., wie ich andernorts extensiv dargestellt habe: eine mereotopologische Relation, die also ein riesiges Intervall zwischen blosser tangentialer Berührung von Eigenem und Anderem in einem Punkt bis zum „Überlappen" des Anderen über das Eigene (bzw. das „Unterlappen des Eigenen unter das Andere) erstrecken kann.

2. Darüber hinaus hat die Betrachtung des Zeichens unter dem Aspekt von Eigenem und Anderem die Frage aufgeworfen, woher denn die Transzendenz stamme, denn vom Zeichen aus ist zwar das Objekt, und vom Objekt aus ist zwar das Zeichen transzendent, aber wohin gehört die Partizipation, die mereotopologische Verbindung? Die Frage lautet dann: Woher kommt denn die Transzendenz? Ist sie dem Zeichen präexistent oder wird sie erst durch das Zeichen geschaffen? In einer Welt ohne Brücke zwischen Eigenem und Anderem führt diese Frage zu einem unendlichen Regress: Ist die Transzendenz, wie z.B. Heidegger meinte, dem Objekt eigen, dann ermöglicht die Transzendenz das Zeichen, aber die Frage bleibt, woher das Objekt seinen eigenen Überstieg hernimmt. Ist die Transzendenz hingegen, wie dies gemeinhin angenommen wird, dem Zeichen eigen, dann stellt sich hinwiederum die Frage, woher sie das nimmt und damit selbst ermöglicht.

3. Ein ganz neues und ebenso revolutionäres wie geniales Modell verdanken wir seit neuestem Rudolf Kaehr (Kaehr 2011): Die sog. Quadralektik, eine polykontexturale Erweiterungen (oder besser: Neubestimmung) der Spencer Brownschen „Laws of Form“, seinen Namen dem „Vierfachen Anfangen“ verdankend:

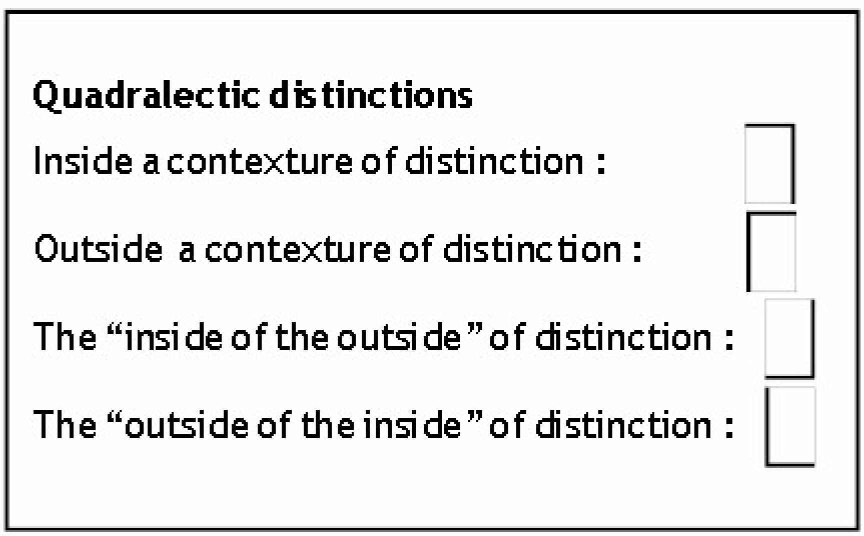
Quadralectics

The quadralectic (tetralemmatic, diamond) notation is enabling operations on the parts of the diamond complexions consisting of *Inside*, *Outside* and *inside*, *outside*, i.e. $[[A\,|\,a]\,|\,[a\,|\,A]]$, short: $[a\ |\,A\,|\,a]$.

Those operations applied to the quadralectic complexion have to preserve the rules of retrograde recursivity.

$[[A\,|\,a]\,|\,[a\,|\,A]]$:
[Inside | Outside] | [outside| inside]:
[*Inside* of inside | *Outside* of inside] | [*outside* of Outside | *inside* of Outside].

Damit unterscheidet Kaehr 4 quadralektische Unterschiede:



Wie man leicht sieht, ist damit auch ein engstens damit zusammenhängendes Problem gelöst, nämlich das folgende: Geht man von Spencer Browns „Laws of Form“ auf, dann muss der Unterschied mit dem Zeichen zusammenfallen. Das Zeichen IST dann der Unterschied, da es kein Drittes gibt. Daraus folgt aber, dass der leere Raum, in den der Unterschied „eingeschrieben“wird, der Raum des Objektes sein muss, da ja in einem zweiwertigen System nur Zeichen und

Objekte vorkommen. Der Ausgangsraum der Laws of Form ist damit klarerweise die Ontologie, und es ist das Zeichen (und sein semiotischer Raum), der ihm als Transzendenz gegenübersteht. Damit muss sich aber, sobald die „Marke" (wie Spencer Brown sagt) gesetzt ist, der ganze Calculus im semiotischen Raum abspielen. Semiotik und Logik fallen damit zusammen, und der ontologische Raum wird im Grunde – sehr ähnlich übrigens wie bei Peirce – nur noch als Ausrede dazu gebraucht, wie Zeichen überhaupt entstehen: sie werden nämlich aus Objekten gemacht, sind als Zeichen eingeführte „Meta-Objekte", wie Bense (1967, S. 9) ausdrücklich sagt. Hier kann man allerdings o.B.d.A. den Spiess umkehren und aller sog. Evidenz zum Trotz z.B. behaupten: Das Setzen des Unterschiedes führt das Objekt ein, und das Zeichen ist demnach ein Etwas, das erst zum Objekt erklärt werden muss. Transzendenz gehört somit in den semiotischen Raum und ermöglicht erst die Kreation von Objekten. Gott selbst schafft ja die Objekte dieser Welt durch den Logos, d.h. durch das Zeichen.

So unsinnig diese Umkehrung klingen mag, eine wissenschaftlich vertretbare Semiotik, die mehr als eine Mythologie ist, die Hilfskonstruktionen wie das „vorgegebene" Objekt, die magische „thetische Introduktion" und die durch sie bewirkte mystisch-mysteriöse Verwandlung des Objektes in ein Zeichen durch den plötzlich als deus/diabolus ex machina erscheinenden „Interpretanten" bedarf, bedarf beider Richtungen: der Semiose vom Objekt zum Zeichen und der Kenose vom Zeichen zum Objekt. Eine revolutionäre Idee Günthers war es, die Objekte aufzulösen und durch Morphogramme bzw. kenomische Matrizen (Kaehr) zu ersetzen. Jeden Fall liegt hier der grosse Schwachpunkt der Spencer Brownschen Laws of Form, die sich damit klar als monokontextural erweisen und zwar etwas abstrakter als die aristotelische Logik formuliert sind, aber im Grunde sonst nichts Neues bringen: Das Eigene ist das, was vom Anderen abrupt unterschieden ist, es gibt keine Partizipation, zwischen Immanenz und Transzendenz führt, wie Felix Hausdorff in seiner an Nietzsche orientierten Studie (1976) es überdeutlich gesagt hatte: kein Brücke hinüber oder herüber. Beschäftigungen mit dem jeweils Anderen sind daher unwissenschaftlich und bilden daher, wie Günther so schön sagte, von unserem zweiwertigen Denken ausgechlossenene Denkrest-Asyle.

4. Gehen wir zuerst also vom Objekt aus, dann bekommen wir mit dem quadralektischen Schema:

4.1.

Objekt | Zeichen bzw. Zeichen | Objekt

und damit korrespondierend:

Zeichen Objekt bzw. Objekt Zeichen

Die quadralektische Fassung der Laws of Form ermöglicht also sowohl Semiose wie Kenose. Sowohl das Zeichen wie das Objekt können das Eigene und das jeweilig Andere sein, denn sie stehen nun in einer Austausch- und nicht mehr in einer Ausschlussrelation.

Ferner führen die sich aus zweiwertigen Systemen ergebenden Standpunkt-Paradoxienin quadralektischer Fassung zu den sog. semiotischen Objekten (vgl. Walther 1979, S. 122 f.), den von Bühler (1985) so genannten Hybriden zwischen Zeichen und Objekt, zwischen denen in diesen Fällen die viel diskutierte "symphysische“ Relation besteht:

Das Innere des Äusseren Das Äussere des Inneren

$\underbrace{\text{Zeichen} \mid \text{Objekt}}_{\text{Zeichenobjekt}}$ bzw. $\underbrace{\mid \text{Objekt} \quad \text{Zeichen}}_{\text{Objektzeichen}}$

Ein Zeichenobjekt ist genauso wenig eine Addition eines Zeichens und eines Objektes wie ein Objektzeichen eine Addition eines Objektes und eines Zeichens wäre, denn erstens würde dies der bekannten Addition von Äpfeln und Birnen entsprechen, und zweitens müsste man dann begründen können, warum hier offenbar $1 + 2 \neq 2 + 1$ gilt. Vielmehr ist ein Zeichenobjekte eine „symphysische", d.h., einmal vollzogen, nicht mehr in ihre Bestandteile abtrennbare Verbindung von Zeichen und Objekt, z.B. bei einem Wegweiser, wo der Zeichenanteil (Orts- und Richtungsangaben) allein genauso sinnlos ist wie der Objektanteil (der Ständer bzw. Träger). Noch deutlicher wird dies beim Objektzeichen, z.B. einer Prothese: Sie ist insofern Objekt, als sie ein reales Bein physisch ersetzt, und insofern Zeichen, als sie dem ursprünglichen (d.h. zu ersetzenden) phyischen Objekt iconisch, d.h. zeichenhaft nachgebildet ist. Solche „hybriden" semiotischen Objekte dürfte es nach klassischer Semiotik eigentlich nicht geben, und doch begegnen sie einem auf Schritt und Tritt. Wie ich kürzlich gezeigt habe, gibt es sogar eine neben den Kardinal- und den Ordinalzahlen vergessene Zahlensorte, die ein semiotisches Objekt darstellt, die Nummer: Während nämlich bei den gewöhnlichen Zahlen diese immer eindeutig einem Objekt beim Zählvorgang zugeordnet werden muss (da sonst das Zählen nicht stattfindet bzw. der ganze Vorgang sinnlos) ist, ist die Zuordnung von Nummern viel freier: Das Zuordnungs-Intervall reicht von den Hausnummern, welche wie Ordinalzahlen den Häusern zugeordnet werden, zu den Nummer von Bussen, welche nicht diese, sondern die von ihnen befahrenen Strecken numerieren, so dass es sein kann, dass eine ordinale Reihenfolge von Bussen z.B. 2-25-1-17-3 ist, ohne dass die Ordnung der Nummer hier gegen die Ordnung der Zahl verstösst.

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Quadralektische Distinktionen zur systemtheoretischen Notation von Zeichenprozessen

1. In Toth (2011) hatte ich die Korrespondenzen zwischen der formativen Basisunterscheidung zwischen Innen und Aussen und den entsprechenden kenogrammatischen, logisch-epistemologischen, fundamentalkategorialen sowie numerisch-semiotischen Begriffen wie folgt dargestellt:

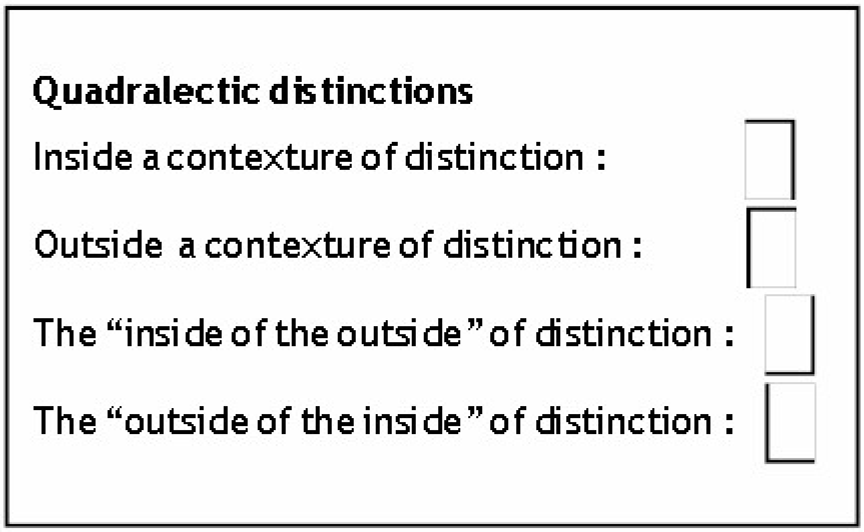
$$\text{Innen} \lceil \text{Aussen}$$
$$\downarrow$$
$$[\bigcirc \lceil \square] \lceil [\square \lceil \bigcirc]$$
$$\downarrow$$
$$[sS \lceil oS] \lceil [oO \lceil sO]$$
$$\downarrow$$
$$[I \lceil Q] \lceil [O \lceil M]$$
$$\downarrow$$
$$[(3.a \lceil 0.d] \lceil [2.b \lceil 1.c]$$
$$\downarrow \qquad\qquad \downarrow$$

Subjekt (System) Objekt (Umgebung)

wobei für die Zeichenklassen a, b, c, d ∈ {0, 1, 2, 3} gilt. Wie man sieht, läuft diese systemtheoretische Unterscheidung – die natürlich bereits in der Spencer Brownschen Dichotomie von „Leere“ vs. „Distinktion“ angelegt ist, auf ein Zeichenmodell heraus, das aus einer Dyaden von Dyaden besteht und in dessen Struktur sich das Verhältnis von Repräsentation von Subjekt- und Objektpol, das bei Peirce über ein ganzes, aus Zeichen- und Realitätsthematik bestehendes Dualsystem distribuiert ist, innerhalb einer einzigen Repräsentationsklasse, bestehend aus 4 anstatt 3 Fundamentalkategorien und ebenso viele Werten, d.h. einem balancierten System, spiegelt. Anders gesagt: Wie jede Peircesche Zeichenklasse ihre duale Realitätsthematik enthält, enthält jede dyadische Zeichenrelation nicht nur ihr eigenes System, sondern auch ihre eigene

Umgebung, allerdings in einer und nicht in zwei dual miteinander verbundenen Repräsentationsrelationen.

2. Nun hat Rudolf Kaehr eine Art von universaler Notation für das, was er „quadralektische Distinktionen“ nennt, eingeführt (Kaehr 2011, S. 12):



Wegen den oben tabellierten Korrespondenzen bekommen wir somit

oS ↔ Q (.0.) ↔ oI ↔ └

sO ↔ M (.1.) ↔ iO ↔ ┘

oO ↔ O (.2.) ↔ oO ↔ ┌

sS ↔ I (.3.) ↔ iI ↔ ┐

Weil in dieser Notation also der Unterschied zwischen triadischen und trichotomischen Peirce-Zahlen aufgehoben ist, kann sie zur formalen Notation von Zeichenklassen und anderen Zeichenrelationen verwendet werden:

(┐.┘ ┌.┘ ┘.┘)

(┐.┘ ┌.┘ ┘.┌)

(┐.┘ ┌.┘ ┘.┐)

(┐.┘ ┌.┌ ┘.┌)

(⏋.⌟ ⌈.⌈ ⌟.⏋)

(⏋.⌟ ⌈.⏋ ⌟.⏋)

(⏋.⌈ ⌈.⌈ ⌟.⌈)

(⏋.⌈ ⌈.⌈ ⌟.⏋)

(⏋.⌈ ⌈.⏋ ⌟.⏋)

(⏋.⏋ ⌈.⏋ ⌟.⏋)

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Elemente einer quadralektischen semiotischen Systemtheorie

1. Wir gehen wiederum aus von den in der folgenden Tabelle aus Toth (2011a) auf Grund von Kaehr (2011) und eigenen Vorarbeiten zusammengestellten Korrespondenzen:

oS	↔	Q (.0.)	↔	oI	↔	$\llcorner$
sO	↔	M (.1.)	↔	iO	↔	$\lrcorner$
oO	↔	O (.2.)	↔	oO	↔	$\ulcorner$
sS	↔	I (.3.)	↔	iI	↔	$\urcorner$

Ferner setzen wir mit Toth (2011b) voraus, dass gerichtete semiotische Mengen der Form

$\rightarrow := \emptyset \rightarrow O$

$\leftarrow: = I \leftarrow \emptyset$

defektiv sind, da sie systemtheoretisch nur System und Umgebung, aber nicht ihre dyadischen Kombinationen in Übereinstimmung mit der logisch-epistemologischen Tetras objektives und subjektives Subjekt sowie subjektives und objektives Objekt berücksichtigen (sS, oS; sO, oO). Wir haben daher definiert:

$\rightleftarrows := \emptyset \rightleftarrows OI$

$\leftrightarrows := \emptyset \leftrightarrows IO$,

vgl. hochdeutsch hinaus/herein; heraus/hinein.

2. Diese Voraussetzungen genügen zur systemtheoretischen Darstellung einer quadralektischen Semiotik, um den Kaehrschen Begriff zu verwenden.

2.1. Tetradisch-tetravalente Kategorien

$(\urcorner_{\rightarrow a}\ \ulcorner_{\rightarrow b}\ \lrcorner_{\rightarrow c}\ \llcorner_{\rightarrow d})$ $\quad$ $(\urcorner_{\rightarrow a}\ \ulcorner_{\leftarrow b}\ \lrcorner_{\leftarrow c}\ \llcorner_{\leftarrow d})$

$(\urcorner_{\rightarrow a}\ \ulcorner_{\rightarrow b}\ \lrcorner_{\leftarrow c}\ \llcorner_{\rightarrow d})$ $\quad$ $(\urcorner_{\leftarrow a}\ \ulcorner_{\leftarrow b}\ \lrcorner_{\leftarrow c}\ \llcorner_{\leftarrow d})$

$(\urcorner_{\rightarrow a}\ \ulcorner_{\leftarrow b}\ \lrcorner_{\rightarrow c}\ \llcorner_{\rightarrow d})$ $\quad$ $(\urcorner_{\leftarrow a}\ \ulcorner_{\rightarrow b}\ \lrcorner_{\rightarrow c}\ \llcorner_{\rightarrow d})$

$(\urcorner_{\leftarrow a}\ \ulcorner 2_{\rightarrow b}\ \lrcorner_{\rightarrow c}\ \llcorner_{\rightarrow d})$ $\qquad$ $(\urcorner_{\leftarrow a}\ \ulcorner_{\rightarrow b}\ \lrcorner_{\rightarrow c}\ \llcorner_{\leftarrow d})$

$(\urcorner_{\rightarrow a}\ \ulcorner_{\rightarrow b}\ \lrcorner_{\leftarrow c}\ \llcorner_{\leftarrow d})$ $\qquad$ $(\urcorner_{\leftarrow a}\ \ulcorner_{\rightarrow b}\ \lrcorner_{\leftarrow c}\ \llcorner_{\leftarrow d})$

$(\urcorner \rightleftarrows_a\ \ulcorner \rightleftarrows_b\ \lrcorner \rightleftarrows_c\ \llcorner \rightleftarrows_d)$ $\qquad$ $(\urcorner \rightleftarrows_a\ \ulcorner \leftrightarrows_b\ \lrcorner \leftrightarrows_c\ \llcorner \leftrightarrows_d)$

$(\urcorner \rightleftarrows_a\ \ulcorner \rightleftarrows_b\ \lrcorner \leftrightarrows_c\ \llcorner \rightleftarrows_d)$ $\qquad$ $(\urcorner \leftrightarrows_a\ \ulcorner \leftrightarrows_b\ \lrcorner \leftrightarrows_c\ \llcorner \leftrightarrows_d)$

$(\urcorner \rightleftarrows_a\ \ulcorner_{\leftarrow b}\ \lrcorner \rightleftarrows_c\ \llcorner \rightleftarrows_d)$ $\qquad$ $(\urcorner \leftrightarrows_a\ \ulcorner \rightleftarrows_b\ \lrcorner \rightleftarrows_c\ \llcorner \rightleftarrows_d)$

$(\urcorner \leftrightarrows_a\ \ulcorner \rightleftarrows_b\ \lrcorner \rightleftarrows_c\ \llcorner \rightleftarrows_d)$ $\qquad$ $(\urcorner \leftrightarrows_a\ \ulcorner \rightleftarrows_b\ \lrcorner \rightleftarrows_c\ \llcorner \leftrightarrows_d)$

$(\urcorner \rightleftarrows_a\ \ulcorner \rightleftarrows_b\ \lrcorner \leftrightarrows_c\ 0 \leftrightarrows_d)$ $\qquad$ $(\urcorner \leftrightarrows_a\ \ulcorner \rightleftarrows_b\ \lrcorner \leftrightarrows_c\ \llcorner \leftrightarrows_d)$

2.2. Quadralektische Closure-Gesetze[5]

$$
\left.\begin{array}{ll}
\varnothing^{\rightarrow} = c(\varnothing^{\rightarrow}) & c(c(x^{\rightarrow})) \subseteq c(x^{\rightarrow}) \\
\varnothing^{\rightarrow} \neq c(\varnothing^{\leftarrow}) & c(c(x^{\rightarrow})) \subseteq c(x^{\leftarrow}) \\
\varnothing^{\leftarrow} \neq c(\varnothing^{\rightarrow}) & c(c(x^{\leftarrow})) \subseteq c(x^{\rightarrow}) \\
\varnothing^{\leftarrow} = c(\varnothing^{\leftarrow}) & c(c(x^{\leftarrow})) \subseteq c(x^{\leftarrow}) \\
\varnothing\rightleftarrows = c(\varnothing\rightleftarrows) & c(c(x\rightleftarrows)) \subseteq c(x\rightleftarrows) \\
\varnothing\rightleftarrows \neq c(\varnothing\leftrightarrows) & c(c(x\rightleftarrows)) \subseteq c(x\leftrightarrows) \\
\varnothing\leftrightarrows \neq c(\varnothing\rightleftarrows) & c(c(x\leftrightarrows)) \subseteq c(x\rightleftarrows) \\
\varnothing\leftrightarrows = c(\varnothing\leftrightarrows) & c(c(x\leftrightarrows)) \subseteq c(x\leftrightarrows)
\end{array}\right\}\ x \in \{\llcorner, \lrcorner, \ulcorner, \urcorner\}
$$

[5] Von hier an überschneidet sich der Text mit demjenigen von Toth (2011b), ausser eben, dass man für die x bzw. x und y $\{\llcorner, \lrcorner, \ulcorner, \urcorner\}$ einsetzen muss.

$$\left.\begin{array}{ll}
x^{\rightleftarrows} \subseteq c(x^{\rightleftarrows}) & c(x^{\rightleftarrows}) \cup c(y^{\rightleftarrows}) = c(x^{\rightleftarrows} \cup y^{\rightleftarrows}) \\
x^{\rightleftarrows} \not\subset c(x^{\leftrightarrows}) & c(x^{\rightleftarrows}) \cup c(y^{\leftrightarrows}) = c(x^{\rightleftarrows} \cup y^{\leftrightarrows}) \\
x^{\leftarrow} \not\subset c(x^{\rightleftarrows}) & c(x^{\leftrightarrows}) \cup c(y^{\rightleftarrows}) = c(x^{\leftrightarrows} \cup y^{\rightleftarrows}) \\
x^{\leftarrow} \subseteq c(x^{\leftarrow}) & c(x^{\leftrightarrows}) \cup c(y^{\leftrightarrows}) = c(x^{\leftrightarrows} \cup y^{\leftrightarrows}) \\
x^{\rightleftarrows} \subseteq c(x^{\rightleftarrows}) & c(x^{\rightleftarrows}) \cup c(y^{\rightleftarrows}) = c(x^{\rightleftarrows} \cup y^{\rightleftarrows}) \\
x^{\rightleftarrows} \not\subset c(x^{\leftrightarrows}) & c(x^{\rightleftarrows}) \cup c(y^{\leftrightarrows}) = c(x^{\rightleftarrows} \cup y^{\leftrightarrows}) \\
x^{\leftrightarrows} \not\subset c(x^{\rightleftarrows}) & c(x^{\leftrightarrows}) \cup c(y^{\rightleftarrows}) = c(x^{\leftrightarrows} \cup y^{\rightleftarrows}) \\
x^{\leftrightarrows} \subseteq c(x^{\leftrightarrows}) & c(x^{\leftrightarrows}) \cup c(y^{\leftrightarrows}) = c(x^{\leftrightarrows} \cup y^{\leftrightarrows})
\end{array}\right\} x,y \in \{ \llcorner, \lrcorner, \ulcorner, \urcorner \}$$

2.3. Äquivalenzen des Zusammenhangs

(auch für die folgenden Gesetze gilt: $x,y \in \{ \llcorner, \lrcorner, \ulcorner, \urcorner \}$)

$C_1 (x, y) \Leftrightarrow x^{\rightarrow} \cap y^{\rightarrow} \neq \varnothing$ / $x^{\rightarrow} \cap y^{\leftarrow} = \varnothing$ / $x^{\leftarrow} \cap y^{\rightarrow} = \varnothing$ / $x^{\leftarrow} \cap y^{\leftarrow} \neq \varnothing$

$C_2 (x, y) \Leftrightarrow x^{\rightarrow} \cap c(y^{\rightarrow}) \neq \varnothing$ / $x^{\rightarrow} \cap c(y^{\leftarrow}) \neq \varnothing$ / $x^{\leftarrow} \cap c(y^{\rightarrow}) \neq \varnothing$ / $x^{\leftarrow} \cap c(y^{\leftarrow}) \neq \varnothing$

od. $c(x^{\rightarrow}) \cap y^{\rightarrow} \neq \varnothing$ / $c(x^{\rightarrow}) \cap y^{\leftarrow} \neq \varnothing$ / $c(x^{\leftarrow}) \cap y^{\rightarrow} \neq \varnothing$ / $c(x^{\leftarrow}) \cap y^{\leftarrow} \neq \varnothing$

$C_3 (x, y) \Leftrightarrow c(x^{\rightarrow}) \cap c(y^{\rightarrow}) \neq \varnothing$ / $c(x^{\rightarrow}) \cap c(y^{\leftarrow}) \neq \varnothing$ / $c(x^{\leftarrow}) \cap c(y^{\rightarrow}) \neq \varnothing$ / $c(x^{\leftarrow}) \cap c(y^{\leftarrow}) \neq \varnothing$

$C_1 (x, y) \Leftrightarrow x^{\rightleftarrows} \cap y^{\rightleftarrows} \neq \varnothing$ / $x^{\rightleftarrows} \cap y^{\leftrightarrows} = \varnothing$ / $x^{\leftrightarrows} \cap y^{\rightleftarrows} = \varnothing$ / $x^{\leftrightarrows} \cap y^{\leftrightarrows} \neq \varnothing$

$C_2 (x, y) \Leftrightarrow x^{\rightleftarrows} \cap c(y^{\rightleftarrows}) \neq \varnothing$ / $x^{\rightleftarrows} \cap c(y^{\leftrightarrows}) \neq \varnothing$ / $x^{\leftrightarrows} \cap c(y^{\rightleftarrows}) \neq \varnothing$ / $x^{\leftrightarrows} \cap c(y^{\leftrightarrows}) \neq \varnothing$

od. $c(x^{\rightleftarrows}) \cap y^{\rightleftarrows} \neq \varnothing$ / $c(x^{\rightleftarrows}) \cap y^{\leftrightarrows} \neq \varnothing$ / $c(x^{\leftrightarrows}) \cap y^{\rightleftarrows} \neq \varnothing$ / $c(x^{\leftrightarrows}) \cap y^{\leftrightarrows} \neq \varnothing$

$C_3 (x, y) \Leftrightarrow c(x^{\rightleftarrows}) \cap c(y^{\rightleftarrows}) \neq \varnothing$ / $c(x^{\rightleftarrows}) \cap c(y^{\leftrightarrows}) \neq \varnothing$ / $c(x^{\leftrightarrows}) \cap c(y^{\rightarrow}) \neq \varnothing$ / $c(x^{\leftrightarrows}) \cap c(y^{\leftrightarrows}) \neq \varnothing$,

usw.

2.4. Mereotopologische Basis-Definitionen

(auch für die folgenden Gesetze gilt: $x, y \in \{ \llcorner, \lrcorner, \ulcorner, \urcorner \}$)

5.1.	$O(x^{\rightarrow}, y^{\rightarrow})$	$:=$	$\exists z(P(z^{\rightarrow}, x^{\rightarrow}) \wedge P(z^{\rightarrow}, y^{\rightarrow}))$	
	$O(x^{\leftarrow}, y^{\leftarrow})$	$:=$	$\exists z(P(z^{\leftarrow}, x^{\leftarrow}) \wedge P(z^{\leftarrow}, y^{\leftarrow}))$	Überlappung
5.2.	$A(x, y \quad)$	$:=$	$C(x^{\rightarrow}, y^{\rightarrow}) \wedge \neg O(x^{\rightarrow}, y^{\rightarrow})$	
	$A(x^{\leftarrow}, y^{\leftarrow})$	$:=$	$C(x^{\leftarrow}, y^{\leftarrow}) \wedge \neg O(x^{\leftarrow}, y^{\leftarrow})$	Angrenzung
5.3.	$E(x, y)$	$:=$	$P(x^{\rightarrow}, y^{\rightarrow}) \wedge P(y^{\rightarrow}, x^{\rightarrow})$	
	$E(x, y)$	$:=$	$P(x^{\leftarrow}, y^{\leftarrow}) \wedge P(y^{\leftarrow}, x^{\leftarrow})$	Gleichheit
5.4.	$PP(x, y)$	$:=$	$P(x^{\rightarrow}, y^{\rightarrow}) \wedge \neg P(y^{\rightarrow}, x^{\rightarrow})$	
			$P(x^{\leftarrow}, y^{\leftarrow}) \wedge \neg P(y^{\leftarrow}, x^{\leftarrow})$	echter Teil
5.5.	$TP(x, y)$	$:=$	$P(x^{\rightarrow}, y^{\rightarrow}) \wedge \exists z^{\rightarrow}(A(z^{\rightarrow}, x^{\rightarrow}) \wedge A(z^{\rightarrow}, y^{\rightarrow}))$	
			$P(x^{\leftarrow}, y^{\leftarrow}) \wedge \exists z^{\leftarrow}(A(z^{\leftarrow}, x^{\leftarrow}) \wedge A(z^{\leftarrow}, y^{\leftarrow}))$	tangentialer Teil
5.6.	$O(x^{\rightleftarrows}, y^{\rightleftarrows})$	$:=$	$\exists z(P(z^{\rightleftarrows}, x^{\rightleftarrows}) \wedge P(z^{\rightleftarrows}, y^{\rightleftarrows}))$	
	$O(x^{\leftrightarrows}, y^{\leftrightarrows})$	$:=$	$\exists z(P(z^{\leftrightarrows}, x^{\leftrightarrows}) \wedge P(z^{\leftrightarrows}, y^{\leftrightarrows}))$	Überlappung
5.7.	$A(x, y)$	$:=$	$C(x^{\rightleftarrows}, y^{\rightleftarrows}) \wedge \neg O(x^{\rightleftarrows}, y^{\rightleftarrows})$	
			$A(x^{\leftrightarrows}, y^{\leftrightarrows}) := C(x^{\leftrightarrows}, y^{\leftrightarrows}) \wedge \neg O(x^{\leftrightarrows}, y^{\leftrightarrows})$	Angrenzung
5.8.	$E(x, y)$	$:=$	$P(x^{\rightleftarrows}, y^{\rightleftarrows}) \wedge P(y^{\rightleftarrows}, x^{\rightleftarrows})$	
	$E(x, y)$	$:=$	$P(x^{\leftrightarrows}, y^{\leftrightarrows}) \wedge P(y^{\leftrightarrows}, x^{\leftrightarrows})$	Gleichheit
5.9.	$PP(x, y)$	$:=$	$P(x^{\rightleftarrows}, y^{\rightleftarrows}) \wedge \neg P(y^{\rightleftarrows}, x^{\rightleftarrows})$	
			$P(x^{\leftrightarrows}, y^{\leftrightarrows}) \wedge \neg P(y^{\leftrightarrows}, x^{\leftrightarrows})$	echter Teil

5.10. $TP(x, y) \quad := \quad P(x\rightleftarrows, y\rightleftarrows) \wedge \exists z\rightleftarrows (A(z\rightleftarrows, x\rightleftarrows) \wedge A(z\rightleftarrows, y\rightleftarrows))$

$P(x\leftrightarrows, y\leftrightarrows) \wedge \exists z\leftrightarrows (A(z\leftrightarrows, x\leftrightarrows) \wedge A(z\leftrightarrows, y\leftrightarrows))$

tangentialer Teil

Literatur

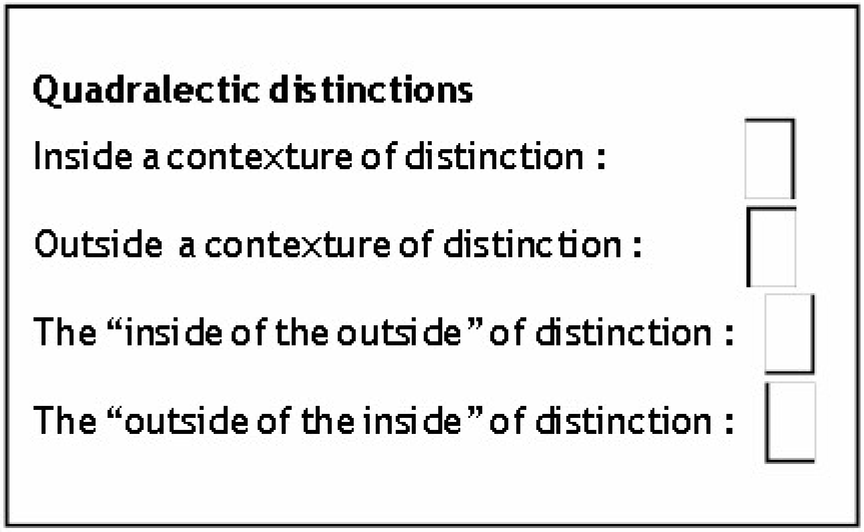
Kaehr, Rudolf, Diamond Calculus of Formation of Forms. http://www.thinkartlab.com/pkl/media/Diamond%20Calculus/Diamond%20Calculus.pdf (2011)

Toth, Alfred, Quadralektische Distinktionen zur systemtheoretischen Notation von Zeichenprozessen. In: Electronic Journal of Mathematical Semiotics (2011a)

Toth, Alfred, Gerichtete quadralektische Mengen. In: Electronic Journal of Mathematical Semiotics (2011b)

Quadralektische Distinktionen, Ligaturen und Gestalten

1. Man kann die von Rudolf Kaehr in einer kürzlich veröffentlichten Arbeit zusammengestellten „quadralektischen Distinktionen“ (Kaehr 2011, S. 12)



in der folgenden Tabelle mit ihren logisch-epistemologischen, fundamental-kategorialen und semiotischen Entsprechungen zusammenbringen:

oS	↔	Q (.0.)	↔	oI	↔	└
sO	↔	M (.1.)	↔	iO	↔	┘
oO	↔	O (.2.)	↔	oO	↔	┌
sS	↔	I (.3.)	↔	iI	↔	┐

2. Eine Besonderheit dieser Korrespondenzen liegt darin, dass sie die Differenz zwischen semiotischem Haupt- und Stellenwert bzw. zwischen triadischer und trichotomischer Peirce-Zahl hintergehen. Diese Tatsache erlaubt uns, die semiotische Matrix Benses rein systemtheoretisch zu notieren:

	⌊	⌋	⌈	⌉
⌊	⌊⌊	⌊⌋	⌊⌈	⌊⌉
⌋	⌋⌊	⌋⌋	⌋⌈	⌋⌉
⌈	⌈⌊	⌈⌋	⌈⌈	⌈⌉
⌉	⌉⌊	⌉⌋	⌉⌈	⌉⌉

Eine tetradisch-tetravalente Zeichenklasse hat daher die allgemeine Form:

Zkl_4^4 = (⌊a ⌋ b ⌈c ⌉ d) mit a...d ∈ { ⌊, ⌋ , ⌈, ⌉ }.

Für die Dualisation gilt:

(× ⌊) = (×.0.) = ⌋ = (.1.), d.h. ⌊ × ⌋

(× ⌈) = (×.2.) = ⌉ = (.3.), d.h. ⌈ × ⌉

Demgegenüber bilden

(.0.)/(.2.) = ⌊ ⌈

(.0.)/(.3.) = ⌊⌉

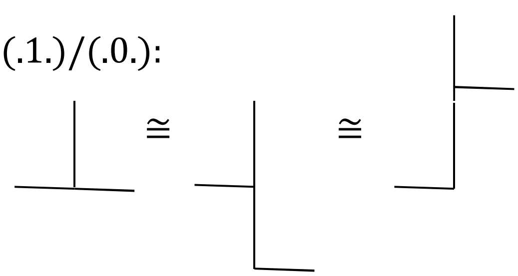
(.1.)/(.2.) = ⌋ ⌈

(.1.)/(.3.) = ⌋ ⌉

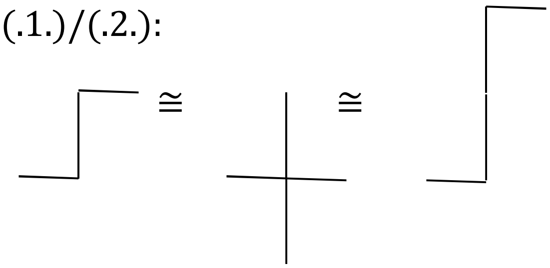
keine Gestaltpaare.

Allerdings kann man alle 4 Zeichengestalten zu Mengen isomorpher „Ligaturen“ zusammenfassen, vgl. z.B.

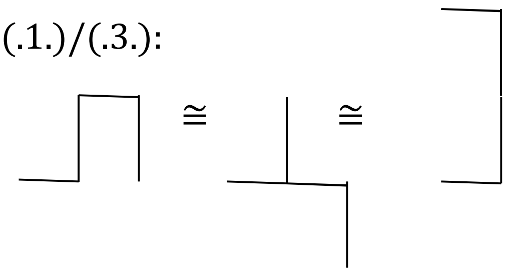
(.1.)/(.0.):



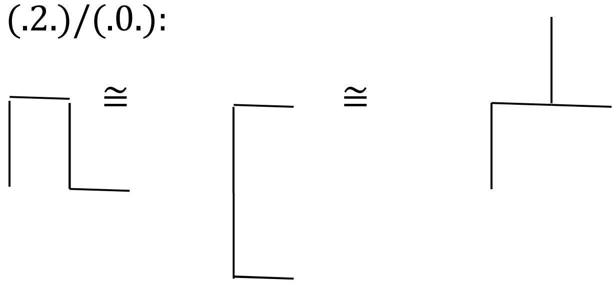
(.1.)/(.2.):



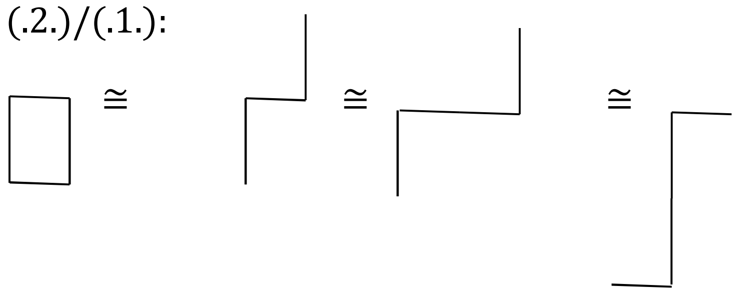
(.1.)/(.3.):



(.2.)/(.0.):

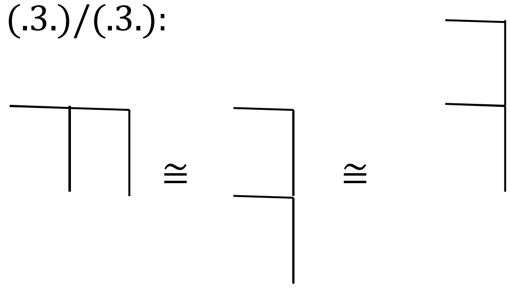


(.2.)/(.1.):



...

(.3.)/(.3.):



(!!)

Wird also neben der Dichotomie Innen/Aussen auch diejenige zwischen Oben/Unten einbezogen, dann verliert sich natürlich der isomorphe Status der meisten der oben präsentierten Zeichengestalten. Der Zeichentext wird dann zur Partitur.

Literatur

Kaehr, Rudolf, Diamond Calculus of Formation of Forms. http://www.thinkartlab.com/pkl/media/Diamond%20Calculus/Diamond%20Calculus.pdf (2011)

Zeichenzusammenhänge im 4-partiten systemtheoretischen Zeichenmodell

1. Wie in Toth (2012a, b, c) gezeigt wurde, läßt sich über der systemtheoretisch-intrinsischen triadischen Zeichenrelation

$ZR_{int} := [\omega, [\omega, 1], [[\omega, 1], 2]]$

ein 4-fältiges Zeichenmodell der Form

	V	H
A	AV	AH
I	IV	IH

konstruieren, wodurch bei allen 10 semiotischen Haupt-Dualsystemen zwischen Vorder- und Hintergrundsperspektivierung unterschieden werden kann:

$V_1 = ((((\omega, 1), 2), \omega)\ ((\omega, 1), \omega)\ (\omega, \omega)) \times$
$H_1 = ((\omega, \omega)\ (\omega, (\omega, 1))\ (\omega, ((\omega, 1), 2)))$

$V_2 = ((((\omega, 1), 2), \omega)\ ((\omega, 1), \omega)\ (\omega, (\omega, 1))) \times$
$H_2 = (((\omega, 1), \omega)\ (\omega, (\omega, 1))\ (\omega, ((\omega, 1), 2)))$

$V_3 = ((((\omega, 1), 2), \omega)\ ((\omega, 1), \omega)\ (\omega, ((\omega, 1), 2))) \times$
$H_3 = ((((\omega, 1), 2), \omega)\ (\omega, (\omega, 1))\ (\omega, ((\omega, 1), 2)))$

$V_4 = ((((\omega, 1), 2), \omega)\ ((\omega, 1), (\omega, 1))\ (\omega, (\omega, 1))) \times$
$H_4 = (((\omega, 1), \omega)\ ((\omega, 1), (\omega, 1))\ (\omega, ((\omega, 1), 2)))$

$V_5 = ((((\omega, 1), 2), \omega)\ ((\omega, 1), (\omega, 1))\ (\omega, ((\omega, 1), 2))) \times$
$H_5 = ((((\omega, 1), 2), \omega)\ ((\omega, 1), (\omega, 1))\ (\omega, ((\omega, 1), 2)))$

$V_6 = ((((\omega, 1), 2), \omega)\ ((\omega, 1), ((\omega, 1), 2))\ (\omega, ((\omega, 1), 2))) \times$
$H_6 = ((((\omega, 1), 2), \omega)\ (((\omega, 1), 2), (\omega, 1))\ (\omega, ((\omega, 1), 2)))$

$V_7 = ((((\omega, 1), 2), (\omega, 1))\ ((\omega, 1), (\omega, 1))\ (\omega, (\omega, 1))) \times$
$H_7 = (((\omega, 1), \omega)\ ((\omega, 1), (\omega, 1))\ ((\omega, 1), ((\omega, 1), 2)))$

$V_8 = ((((\omega, 1), 2), (\omega, 1))\ ((\omega, 1), (\omega, 1))\ (\omega, ((\omega, 1), 2))) \times$
$H_8 = ((((\omega, 1), 2), \omega)\ ((\omega, 1), (\omega, 1))\ ((\omega, 1), ((\omega, 1), 2)))$

$V_9 = ((((\omega, 1), 2), (\omega, 1))\ ((\omega, 1), ((\omega, 1), 2))\ (\omega, ((\omega, 1), 2))) \times$
$H_9 = ((((\omega, 1), 2), \omega)\ (((\omega, 1), 2), (\omega, 1))\ ((\omega, 1), ((\omega, 1), 2)))$

$V_{10} = ((((\omega, 1), 2), (((\omega, 1), 2))\ ((\omega, 1), ((\omega, 1), 2))\ (\omega, ((\omega, 1), 2))) \times$
$H_{10} = ((((\omega, 1), 2), \omega)\ (((\omega, 1), 2), (\omega, 1))\ (((\omega, 1), 2), (((\omega, 1), 2))),$

2.1. Aus der obigen Tabelle der systemtheoretischen semiotischen Dualsysteme können wir direkt eine erste Art von Zeichenzusammenhängen ablesen: die Chreoden, die als Schnittmenge zwischen dem Vorder- und Hintergrund jedes Dualsystems definiert sind:

$\chi(V_1, H_1) = (\omega, \omega)$

$\chi(V_2, H_2) = (((\omega, 1), \omega)\ (\omega, (\omega, 1)))$

$\chi(V_3, H_3) = ((((\omega, 1), 2), \omega), (\omega, ((\omega, 1), 2)))$

$\chi(V_4, H_4) = (((\omega, 1), (\omega, 1)))$

$\chi(V_5, H_5) = ((((\omega, 1), 2), \omega)\ ((\omega, 1), (\omega, 1))\ (\omega, ((\omega, 1), 2)))$

$\chi(V_6, H_6) = ((((\omega, 1), 2), \omega), (\omega, ((\omega, 1), 2)))$

$\chi(V_7, H_7) = (((\omega, 1), (\omega, 1))) = X(V_4, H_4)$

$\chi(V_8, H_8) = (((\omega, 1), (\omega, 1))) = X(V_7, H_7) = X(V_4, H_4)$

$\chi(V_9, H_9) = ((((\omega, 1), 2), (\omega, 1)), ((\omega, 1), ((\omega, 1), 2)))$

$\chi(V_{10}, H_{10}) = (((\omega, 1), 2), (((\omega, 1), 2))$

2.2. Es stellt sich nun die Frage, wie der Zusammenhang zwischen dem Außen und dem Innen in jedem der 10 Dualsysteme aussieht. Da diese Art des Zusammenhangs extern natürlich über die entsprechenden interne Zusammenhänge abläuft, genügt deren Untersuchung. Von den Basisabbildungen der dyadischen Partialrelationen einer triadischen Relation thematisieren der Mittel- und der Interpretantenbezug das Innen und der Objektbezug erwartungsgemäß das Außen des jeweiligen Systems:

M := (A → I) (objektives Subjekt)

J → (((A → I) → A)) → I)) (subjektives Subjekt)

O → ((A → I) → A)) (objektives Objekt),

d.h. eine systemtheoretische Semiotik wird über die logisch-epistemischen Basisrelationen motiviert. Allerdings fehlt in einer triadischen Semiotik eine Abbildung für das subjektive Objekt; hierzu bedarf es einer mindestens tetradischen Semiotik. Für die systemtheoretische Zeichenrelation bedeuten die obigen Entsprechungen also, daß systemische Zeichenzusammenhänge sensu proprio durch die Korrespondenzen

oS: M ↔ (A → I) ↔ ω

sS: I ↔ (((A → I) → A)) → I)) ↔ [ω, 1]

oO: O ↔ ((A → I) → A)) ↔ [[ω, 1], 2]]

Wie man leicht erkennt, tritt also die basale Abbildung ω bereits in einer triadischen Relation in dreifacher Verschachtelungsstufe auf, d.h. als ω, [ω] und [[ω]]. Mit anderen Worten: Um den Zusammenhang der Partialrelationen zu formalisieren, muß man auch die in Toth (1997, S. 21 ff.) definierten semiotisch-kategorietheoretischen (einfachen und komponierten sowie inversen) Morphismen redefinieren: α: = (M → O), β := (O → I). Bei der Tieferlegung der Semiotik auf die Systemtheorie werden somit die innerhalb der Peirce-Bense-Semiotik noch qualitativ definierten und unterschiedenen Morphismen zu einer einzigen rein quantitativen Abbildung, bei der allerdings verschiedene Einbettungsstufen zu unterscheiden sind. Wir wollen definieren:

$\omega := A \rightarrow I$

$\gamma_1 := \omega \rightarrow [\omega]$

$\gamma_2 := \omega \rightarrow [[\omega]]$,

allgemein gilt also: $\gamma_n := \omega \rightarrow [\omega]_n$.

Zur Definition einer systemtheoretischen Semiotik benötigt man somit nur drei Dinge:

1. Die Parameter [± Innen] und [± Vordergrund],

2. Die Abbildung $\omega := A \rightarrow I$,

3. Den Einbettungsoperator $\gamma_n := \omega \rightarrow [\omega]_n$.

Literatur

Toth, Alfred, Entwurf einer semiotisch-relationalen Grammatik. Tübingen 1997

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Zu einer Modelltheorie der systemischen Semiotik

1. In Toth (2012) war festgestellt worden, daß die von mir eingeführte systemische Semiotik sich als Tripel

$\Sigma = [P, \omega, \gamma_n]$

einer Menge von Parametern P, einer Abbildung ω und eines Einbettungsoperators definieren läßt, wobei

$P := [[\pm \text{Innen}], [\pm \text{Vordergrund}]]$,

$\omega := A \rightarrow I$,

$\gamma_n := \omega \rightarrow [\omega]_n$

gilt. Als Basisrelation der triadischen systemischen Semiotik sind die Zeichenrelation

$ZR_{int} := [\omega, [\omega, 1], [[\omega, 1], 2]]$

und das Perspektivierungsschema (V = Vordergrund, H = Hintergrund)

	V	H
A	AV	AH
I	IV	IH

definiert.

2. Es stellt sich hiermit die Frage, ob sich die bislang definierten Begriffe auch dazu eignen, um zu definieren, ob ein Etwas ein Zeichen ist oder nicht. Man erinnert sich an frühere Arbeiten von mir (z.B. Toth 2009, 2010) oder aus dem Bereich der verbalen Semiotik z.B. an Hugo Balls Frage, warum ein Baum nicht „Pluplusch" heißen können - wenn es geregnet habe, aber „Pluplubasch". Vom Standpunkt der Peirce-Bense-Semiotik würde sich eine Stellungnahme zu dieser Frage auf die (sicherlich korrekte) Feststellung beschränken, weder Pluplusch noch Pluplubasch seien sprachliche Zeichen im Sinne des konventionellen Mittelbezugs (1.3). Vom modelltheoretischen Standpunkt aus bedeutet die Frage aber, daß es einer Bewertung oder Interpretation bedarf,

um zu entscheiden, ob ein bestimmtes Zeichen Z wie Pluplusch oder Pluplubasch Element des Mittelrepertoires {M} einer bestimmten Sprache L ist oder nicht. Formal ausgedrückt: Die systemische Abbildung

$M := (A \rightarrow I)$

ist zu ersetzen durch

$\{M\} := \{(A \rightarrow I)_1, (A \rightarrow I)_2, (A \rightarrow I)_3, \ldots, (A \rightarrow I)_n)$.

Dies gilt nun natürlich nur für eine bestimmte Sprache L, also z.B. für das Deutsche. Soll aber geprüft werden, ob Pluplusch und Pluplubasch *irgendeiner* anderen (natürlichen oder künstlichen) Sprache angehört, müssen wir von einer Menge {L} ausgehen, von denen jede natürlich ein Repertoire {M} enthält. Damit benötigen wir nun aber (vereinfacht ausgedrückt)

$\{\{M\}\} := \{\{(A \rightarrow I)_1, (A \rightarrow I)_2, (A \rightarrow I)_3, \ldots, (A \rightarrow I)_n)\}\}$,

wobei man noch höherstufige Abbildungen annehmen kann, da es Sprachen gibt, die z.B. innerhalb ihres Lexikon noch sog. Register unterscheiden, wie z.B. das Javanische, wo es etymologisch unverbundene Wörter zur Bezeichnung ein und desselben Objekts gibt, je nachdem, welchem sozialen Status der Gesprächspartner angehört.

3. Was den Objekt- und den Interpretantenkonnex anbetrifft, so können wir hier etwas summarischer argumentieren, da die Probleme hier eher als beim Mittelbezug als bekannt vorausgesetzt werden können. Z.B. führt die Einführung logischer möglicher Welten in die systemische Semiotik dazu, daß man von

$O \rightarrow \quad ((A \rightarrow I) \rightarrow A))$

zu

$\{O\} \rightarrow \{((A \rightarrow I) \rightarrow A))_1, ((A \rightarrow I) \rightarrow A))_2, ((A \rightarrow I) \rightarrow A))_3, \ldots, ((A \rightarrow I) \rightarrow A))_n\}$

übergehen muß, wobei sich höhere Ableitungsstufen wohl erübrigen. Z.B. könnte rein theoretisch Lewis Carroll's „The White Knight‘s Song” in einer anderen als der uns vertrauten Ontologie sinnvoll sein. Speziell von hier aus ergeben sich natürlich Verbindungen zur Polykontexturalitätstheorie.

Ersetzen wir aus Parallelitätsgründen

$J \rightarrow \quad (((A \rightarrow I) \rightarrow A)) \rightarrow I))$

durch

$\{J\} \rightarrow ((((A \rightarrow I) \rightarrow A)) \rightarrow I))_1, (((A \rightarrow I) \rightarrow A)) \rightarrow I))_2, (((A \rightarrow I) \rightarrow A)) \rightarrow I))_3, \ldots, (((A \rightarrow I) \rightarrow A)) \rightarrow I))_n),$

so können wir hiermit das in der Peirce-Bense-Semiotik ebenfalls unbehandelbare Problem der Idio-, Sozio- und Dialekte behandeln, das keineswegs auf sprachliche Zeichensysteme beschränkt ist, wenn man etwa an die landestypisch verschiedenen Verkehrszeichen, Gestik, Mimik usw. denkt (letzteres steht z.B. explizit, jedoch ohne semiotische Referenz, in jedem Lehrbuch für angehende Hotelangestellte).

Damit wird also das elementare systemtheoretische Zeichenmodell

$ZR_{int} := [\omega, [\omega, 1], [[\omega, 1], 2]]$

durch

$ZR_{int} := [\{\omega\}, \{[\omega, 1]\}_n, \{[[\omega, 1], 2]\}]$

ersetzt, und das erstere stellt somit einen 1-stufigen Spezialfall des zweiten, n-stufigen übergeordneten Modells dar.

Literatur

Toth, Alfred, Zu einer semiotischen Modelltheorie. In: Electronic Journal for Mathematical Semiotics, 2009

Toth, Alfred, Multivariante Semiotik und Modelltheorie. In: Electronic Journal for Mathematical Semiotics, 2010

Toth, Alfred, Zeichenzusammenhänge im 4-partiten systemtheoretischen Zeichenmodell. In: Electronic Journal for Mathematical Semiotics, 2012

Relationale Einbettungszahlen

1. Die über den dyadischen Partialrelationen

$\omega := (A \to I)$

$[\omega, 1] := ((A \to I) \to A)$

$[[\omega, 1], 1] := (((A \to I) \to A) \to I)$

definierbare triadische systemtheoretische Zeichenklasse

$ZR_{sys} = [\omega, [\omega, 1], [[\omega, 1], 1]]$

(Toth 2012) weist im Grunde nur die eine Abbildung ω, allerdings in drei verschiedenen Einbettungsstufen, auf, die man durch [ω], [[ω]], [[[ω]]] kennzeichnen könnte. Damit kann man allerdings die theoretisch unendlich vielen Einbettungen durch einen einzigen indizierten Einbettungsoperator definieren. Da ferner ω nur ein Spezialfall für eine theoretisch beliebige Abbildung zwischen den beiden Gliedern einer beliebigen Dichotomie ist, wollen wir nun definieren. Sei

$D := [a, b]$

eine beliebige Dichotomie und

$1 := a(b) = b \to a$

eine beliebige Abbildung der Glieder von D. Ferner bedeute „1", daß diese Abbildung eine „Oberflächenabbildung" sei, d.h. daß die Einbettungsstufe 0 vorliege:

$1 = [1_0] := 1_0$.

Damit können wir das obige systemtheoretische semiotische Minimalsystem wie folgt notieren

$\omega = 1$

$[\omega, 1] = 1_{-1}$

$[[\omega, 1], 1] = 1_{-2}$,

d.h. wir können hiermit nicht nur die Semiotik auf die Systemtheorie (wie in meinen letzten Arbeiten gezeigt) zurückführen, sondern die letztere durch Paar

$RE = <1, {}_{n}]>$,

bestehend aus einer Abbildung 1 und einem n-stufigen Einbettungsoperator ${}_{n}]$ definieren und nennen dieses Paar RE eine RELATIONALE EINBETTUNGSZAHL.

2. Was haben wir mit dieser weiteren Abstraktion erreicht? War die Rückführung der Peirce-Benseschen Zeichenrelation ZR = (M, O, I) auf die systemtheoretische Zeichenrelation ZR = (1, (1, 2), ((1, 2), 3)) mit „Verlängerung" für n-adische Relationen ZR^n = (1, (1, 2), ((1, 2), 3), (((1, 2), 3)), 4), ...) und der Ersetzung der qualitativ definierten Partialrelationen bzw. semiotischen Funktionen durch allgemeinere systemtheoretische Abbildungen die Verabschiedung des substantiellen Rests der ansonsten relationalen Zeichenrelation ZR = (M, O, I), so werden durch die Einführung der relationalen Einbettungszahlen nun auch noch die letzten statischen Momente der Relation ZR^n durch Morphismen ersetzt und somit die systemtheoretische Basis der Zeichenrelation ZR^n selbst so weit wie nur möglich verallgemeinert.

Damit haben wir also ein TRIPARTITES SEMIOTISCHES SYSTEM vor uns: Wir geben nachstehend für jede der 10 Peirce-Benseschen Zeichenklassen zunächst die traditionelle Notation in Form der semiotischen Kategorien, dann die system-theoretischen Entsprechungen und hernach ihre Transformationen in Teilsysteme relationaler Einbettungszahlen. (Da die Realitätsthematiken ja dual zu ihren Zeichenklassen sind, erübrigt sich hier ihre gesonderte Darstellung.)

1. Zkl = (3.1 2.1 1.1) $\rightarrow S_1 = ((((\omega, 1), 2), \omega)\ ((\omega, 1), \omega)\ (\omega, \omega)) \rightarrow$
   $RE = [[1_{-3}, 1], [1_{-2}, 1], [1, 1]]$.

2. Zkl = (3.1 2.1 1.2) $\rightarrow S_2 = ((((\omega, 1), 2), \omega)\ ((\omega, 1), \omega)\ (\omega, (\omega, 1)))$
   $RE = [[1_{-3}, 1], [1_{-2}, 1], [1, 2]]$.

3. Zkl = (3.1 2.1 1.3) → S_3 = ((((ω, 1), 2), ω) ((ω, 1), ω) (ω, ((ω, 1), 2))) → RE = [[1_{-3}, 1], [1_{-2}, 1], [1, 3]].

4. Zkl = (3.1 2.2 1.2) → S_4 = ((((ω, 1), 2), ω) ((ω, 1), (ω, 1)) (ω, (ω, 1))) → RE = [[1_{-3}, 1], [1_{-2}, 2], [1, 2]].

5. Zkl = (3.1 2.2 1.3) → S_5 = ((((ω, 1), 2), ω) ((ω, 1), (ω, 1)) (ω, ((ω, 1), 2))) →
RE = [[1_{-3}, 1], [1_{-2}, 2], [1, 3]].

6. Zkl = (3.1 2.3 1.3) → S_6 = ((((ω, 1), 2), ω) ((ω, 1), ((ω, 1), 2)) (ω, ((ω, 1), 2))) → RE = [[1_{-3}, 1], [1_{-2}, 3], [1, 3]].

7. Zkl = (3.2 2.2 1.2) → S_7 = ((((ω, 1), 2), (ω, 1)) ((ω, 1), (ω, 1)) (ω, (ω, 1))) →
RE = [[1_{-3}, 2], [1_{-2}, 2], [1, 2]].

8. Zkl = (3.2 2.2 1.3) → S_8 = ((((ω, 1), 2), (ω, 1)) ((ω, 1), (ω, 1)) (ω, ((ω, 1), 2))) → RE = [[1_{-3}, 2], [1_{-2}, 2], [1, 3]].

9. Zkl = (3.2 2.3 1.3) → S_9 = ((((ω, 1), 2), (ω, 1)) ((ω, 1), ((ω, 1), 2)) (ω, ((ω, 1), 2))) → RE = [[1_{-3}, 2], [1_{-2}, 3], [1, 3]].

10. Zkl = (3.3 2.3 1.3) → S_{10} = ((((ω, 1), 2), (((ω, 1), 2)) ((ω, 1), ((ω, 1), 2)) (ω, ((ω, 1), 2))) → RE = [[1_{-3}, 3], [1_{-2}, 3], [1, 3]].

Die Konstanz der Struktur [[1_{-3}, -], [1_{-2}, -], [1, -]] erweckt hier den Eindruck der Redundanz der RE. Das ändert sich jedoch schnell, wenn man die Permutation der Partialrelationen zuläßt, wie dies z.B. bereits Bense bei seiner Definition der Realitätsthematiken, Kommunikations- und Kreationsschemata getan hatte. Dann erhalten wir also z.B. für das obige Teilsystem 10 die folgenden 6 Möglichkeiten:

[[1_{-3}, 3], [1_{-2}, 3], [1, 3]], [[1_{-3}, 3], [1, 3], [1_{-2}, 3]], [[1_{-2}, 3], [1_{-3}, 3], [1, 3]], [[1_{-2}, 3], [1, 3], [1_2, 3]], [[1, 3], [1_{-3}, 3], [1_{-2}, 3]], [[1, 3], [1_{-2}, 3], [1_3, 3]].

Ferner sollte man sich bewußt sein, daß die Anwendung systemtheoretischer Relationen gerade in komplexen Zeichenklassen nicht in der Form diskreter semiotischer Repräsentationssysteme geschieht, so daß solche in mannigfacher Kombination auftreteten, d.h. die ursprüngliche triadisch-retrosemiosische Struktur der Peirce-Benseschen Zeichenklassen kann durch Einfügung einer beliebigen Anzahl beliebiger Partialrelationen unterbrochen, überbrückt und noch anders modifiziert werden. Ferner kommen nach der Definition von ZR^n ja nicht nur triadische, sondern auch höherstufige Relationen vor. Zusammen mit den Permutationsmöglichkeiten ergeben sich damit hochgradisch komplexe semiotische Zeichenstrukturen, Zeichensysteme und Zeichenprozesse, bei denen die scheinbare Konstanz der RE, wie sie für den Grenzfall der triadischen Repräsentationssysteme erscheint, die einzige Möglichkeit der Gliedern in Typen (via triadische und trichotomische Werte, d.h. Abbildungen) und Stufen (via Einbettungen) darstellt.

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Relationale Einbettungszahlen und Ordnungsrelationen

1. Die Ordnungsrelationen der Peirce-Bense-Semiotik sind von mir schon in Toth (1996) behandelt worden, vgl. auch Toth (2006/08, S. 64 ff.). Nach Toth (2012a) sind relationale Einbettungszahlen (REZ) definiert durch ein Paar

$RE = \langle m, {}_{n}] \rangle$,

aus $m \in$ Peanozahlen und einem n-stufigen Einbettungsoperator, und eine ihrer auffälligsten Eigenschaften, wie bereits in Toth (2012b) bemerkt, besteht darin, daß bei ihnen im Gegensatz zu den Benseschen Relationszahlen (Bense 1981, S. 26 ff.) Dualia und Konversionen nicht zusammenfallen, vgl.

$\times(1.3) = (1.3)^0 = (3.1)$,

hingegen

$\times[1, 2] \neq [1_{-1}, 1]$

$\times[1, 3] \neq [1_{-2}, 1]$

$\times[2, 3] \neq [1_{-2}, 2]$.

Allgemein gilt also

$[1_{-a}, b] = [a(+1), b]$

und daher

$\times[1_{-a}, b] = [b, a(+1)]$,

d.h. jede REZ ist eine Menge von zwei REZ

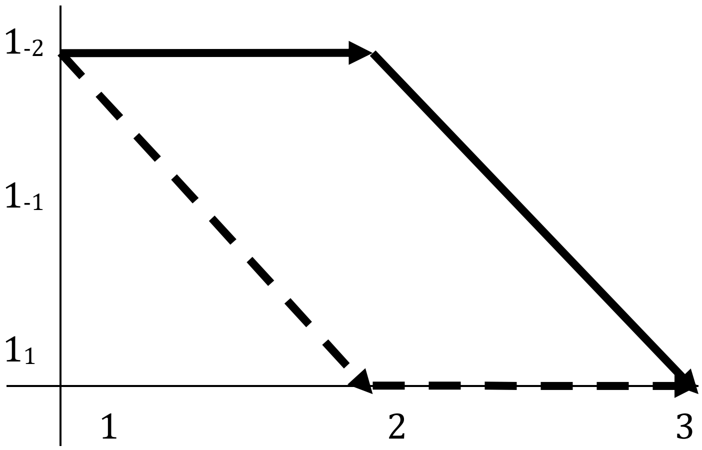
$[a, b] = \{[a_{-(a-1)}, b], [a, b]\}$.

2. Nehmen wir als erstes Beispiel die Peirce-Bensesche Zeichenklasse (3.1 2.1 1.3) und deren duale Realitätsthematik (3.1 1.2 1.3). In der entsprechenden REZ-Relation ausgedrückt:

$R_{REZ} = [[[1_{-2}, 1], [1_{-1}, 2]], [1, 3]]$

$R_{REZ}{}^{\circ} = [[1_{-2}, 1], [[1_{1}, 2], [1_{1}, 3]]]$

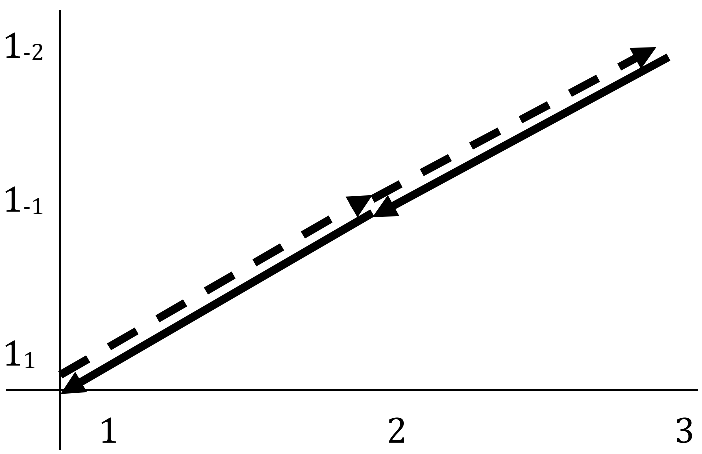
Ihr graphische Darstellung sieht also wie folgt aus (R_{REZ} ausgezogen):



Als zweites Beispiel stellen wir die Peircesche Kategorienklasse (3.3 2.2 1.1) und deren duale Kategorienthematik (1.1 2.2 3.3) dar. Als REZ-Relationen:

$R_{REZ} = [[[1_{-2}, 3], [1_{-1}, 2]], [1, 1]]$

$R_{REZ}{}^{\circ} = [[1, 1], [[1_{-1}, 2], [1_{-2}, 3]]]$



Wie man bereits anhand dieser beiden Beispiele sieht, gibt es also in REZ-Ordnungsrelationen weder reflexive, noch symmetrische noch transitive Relationen, was das Verhältnis der beiden zueinander dualen Strukturen jeder REZ-Repräsentationsrelation anbelangt. Dieses Ergebnis mag auf den ersten Blick sehr überraschen, da REZ ja, ebenso wie Benses Relationszahlen, Relationen über Peanozahlen sind. Im Gegensatz zu den Relationszahlen sind jedoch die relationalen Einbettungszahlen, was ihre Peanozahl-Basis betrifft, hierarchisch gestuft, so daß z.B. keine Loops in den Graphen von Ordnungsrelationen auftreten können.

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Relationale Einbettungszahlen und komplexe Zahlen

1. In einem gewisse Sinne könnte man die in Toth (2012a) eingeführten relationalen Einbettungszahlen (REZ) als komplexe Zahlen bezeichnen, da sie, wie die komplexen Zahlen, flächige Zahlen darstellen (vgl. Toth 2012b). Eine REZ ist allgemein definiert als

$RE = \langle m, {}_{n}]\rangle$,

wobei gilt

$[a, b] = \{[a_{-(a-1)}, b], [a, b]\}$.

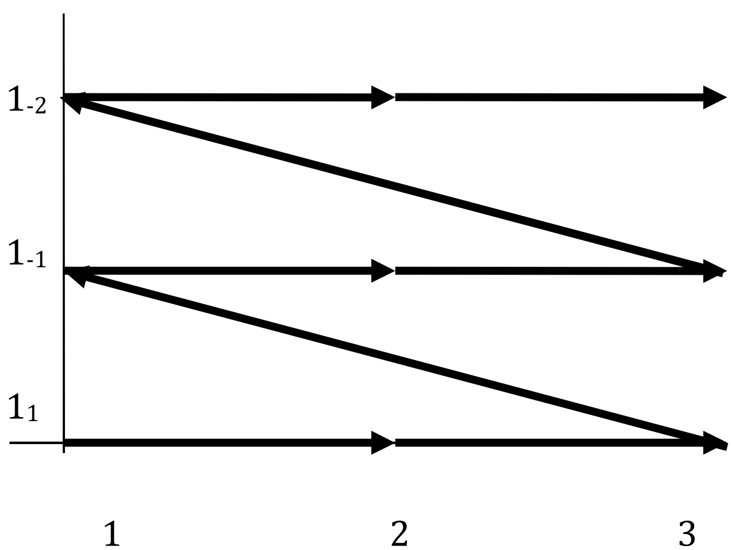
Setzen wir m = n = 3, dann können wir eine Peirce-Bensesche Zeichenklasse in der REZ-Form

$R_{REZ}{}^{3,3} = [[1, a], [[1_{-1}, b], [1_{-2}, c]]]$,

wobei die dyadischen Abbildungen der a, b, c $\in$ m wie folgt definiert sind

$[1, 1] := id_1$	$[1, 2] := \alpha$	$[1_{-1}, 3] := \beta$	$[1, 3] := \beta\alpha$
$[1_{-1}, 2] := id_2$	$[1_{-1}, 1] := \alpha^0$	$[3, 1_{-1}] := \beta^0$	$[1_{-2}, 1] := \alpha^0\beta^0$
$[1_{-2}, 3] := id_3$.			

2. Erinnern wir uns an die graphische Repräsentation der Zahlenfläche der REZ, wie sie in Toth (2012c) gegeben worden war



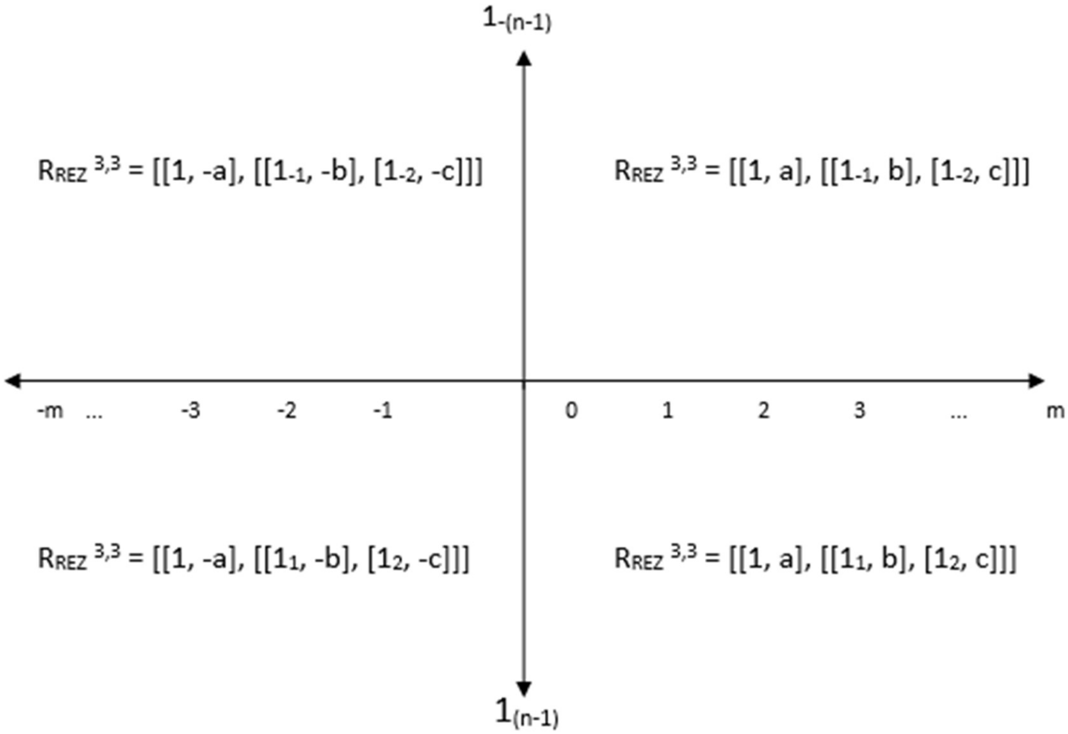
Was nun die Menge der Peanozahlen $m \in \mathbf{N}$ in RE = $<m, {}_n]>$ betrifft, so hatten wir bereits in Toth (2001) gezeigt, daß man in Peirce-Benseschen dyadischen Partialrelationen der Form (a.b) anstatt positiver auch negative Zahlen verwenden kann und damit das Peirce-Bensesche semiotische System dadurch auf die Gaußsche Zahlenebene abbilden kann, daß man die drei weiteren Formen von Subzeichen (-a.b), (a.-b) und (-a.-b) einführt. Wir sind somit berechtigt, auch für m negative Werte einzusetzen. Da die Vorstellung negativer Einbettungen einige Kopfschmerzen bereitet, wenn wir also Einbettungsoperatoren der Form ${}_n]$ nun auch für negatives n einführen, sei auf Toth (2012d) verwiesen, wo wir gezeigt hatten, daß speziell das REZ-System über einen „negativen" Droste-Effekt verfügt. Das bedeutet, impressionistisch ausgedrückt, daß hier – im Gegensatz zum Peirce-Bense-System, wo ein positiver Droste-Effekt herrscht, wo also die Relationen durch Einsetzen immer „länger" werden – die Relationen im REZ-System immer „kürzer" werden, bis sie an einem Punkt wegen $1_{-2} + 1_{-1} + 1 = 0$ in 0, d.h. mit der Peirce-Bense-Semiotik, koinzidieren. Geht man also unter 0 hinunter, so hat man auf semiotischer REZ-Ebene ein ähnliches Phänomen vor sich wie im Bereiche der Zahlentheorie bei derNon-Standard-Analysis (vgl. Ebbinghaus 1992, S. 255 ff).

Nach diesen Überlegungen sind wir also berechtigt, komplexe REZ-Relationen der Form

$$R_{REZ}{}^{m,n} = [[1, \pm 1], [[1_{\pm 1}, \pm 2], [1_{-2}, \pm 3]] \ldots [1_{\pm(n-1)}, \pm m]]] \ldots {}_n]$$

(1 ist natürlich Abkürzung für "$1_{\pm 0}$".)

und der oben wiedergegebene Quadrant aus einem REZ-Koordinatensystem präsentiert sich innerhalb einer REZ-Zahleneben natürlich in der Form



(Wink mit dem Zaunpfahl: negative Einbettungsrelationen sind im positiven Teil des Quadrantensystems, und umgekehrt!)

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Konversion und "interne Konversion" relationaler Einbettungszahlen

1. Wie in Toth (2012) gezeigt, weisen die durch

RE := $<1_{m, n}]>$

definierten sowie unter der Beschränkung von m, n ∈ {1, ..., n} auf m = n = 3 konstruierten und in der folgenden triadisch-trichotomische REZ-Matrix angeordneten

[1, 1]	[1, 2]	[1, 3]
$[1_{-1}, 1]$	$[1_1, 2]$	$[1_{-1}, 3]$
$[1_{-2}, 1]$	$[1_{-2}, 2]$	$[1_{-2}, 3]$

relationalen Einbettungszahlen (REZ) folgende Basis-Ambiguität auf

$[a, b] = \{[a_{-(a-1)}, b], [a, b]\}$.

Diese führt dazu, daß einer der beiden semiotischen Basismorphismen der Peirce-Bense-Semiotik in Bezug auf seine REZ-Darstellung mit seinem inversen Morphismus koinzidiert

$[1, 1] := id_1$	$[1, 2] := \alpha$	$\boxed{[1_{-1}, 3] := \beta}$	$[1, 3] := \beta\alpha$
$[1_{-1}, 2] := id_2$	$[1_{-1}, 1] := \alpha^o$	$\boxed{[1_{-1}, 3] := \beta^o}$	$[1_{-2}, 1] := \alpha^o\beta^o$
$[1_{-2}, 3] := id_3$.			

2. Nehmen wir nun z.B. die REZ

$[1_{-2}, 3]$,

so kann man nach der obigen Festsetzung nicht nur eine, sondern zwei Konversen bilden

$[1_{-2}, 1]° = \{[1, 1_{-2}], [1, 3]\}$

Nun hat [1, 3] aber selber eine Konverse

$[1, 3]° = [3, 1]$,

sodaß also jeder REZ 4 Strukturen der allgemeinen Form

$[a, b], [b, a], [a_{-(a-1)}, b], [b, a_{-(a-1)},]$

zukommen. Da [a, b] jedoch die Peanostruktur von Benses "Primzeichen"-Darstellung (vgl. Bense 1981, S. 17 ff.) ist, ist [b, a] auch nichts anderes als ihre (Peano-)Konverse. Wir wollen somit die beiden REZ-Strukturen relativ zu den beiden Peano-Strukturen als "interne Konversen" bezeichnen (da die REZ wegen ihres Einbettungsoperators, der sie zu flächigen Zahlen macht, quasi die interne Struktur der semiotischen Peanozahlen offenlegen). Die durch zwei REZ-Konversen verursachte Ambiguitäten im triadisch-trichotomischen 9er-System der Semiotik kann man z.B. durch die folgende Darstellung illustrieren

$[1, 1]° = [1, 1]$ //

$[1, 2]° = [2, 1]$ / $[1_{-1}, 1]$

$[1, 3]° = [3, 1]$ / $[1_{-2}, 1]$

$[1_{-1}, 1]° = [1, 1_{-1}]$

$[1_{-1}, 2]° = [2, 1_{-1}]$ //

$[1_{-1}, 3]° = [3, 1_{-1}]$

$[1_{-2}, 1]° = [1, 1_{-2}]$

$[1_{-1}, 3]° = [3, 1_{-1}]$

$[1_{-2}, 3]° = [3, 1_{-2}]$

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Grundlegung einer logischen Semiotik

1. Im folgenden seien die wichtigsten Probleme der Peirce-Bense-Semiotik zusammengefaßt.

1.1. Sie ist eine Pansemiotik, d.h. "ein nicht-transzendentales, ein nicht-apriorisches und nicht-platonisches Organon" (Gfesser 1990, S. 133). Dennoch wird ein sowohl der Semiose als auch dem Zeichen vorgegebenes und damit ontisches Objekt vorausgesetzt (Bense 1967, S. 9).

1.2. In der Bestimmung der thetischen Introduktion als Metaobjektivation (Bense 1967, S. 9) wird ein Objekt durch die Semiose auf ein Zeichen abgebildetes, das jedoch erst durch diese Abbildung entsteht. Ferner wird das für diesen Prozeß notwendige Subjekt zwar vorausgesetzt, aber nicht prozessual operationalisiert.

1.3. Entgegen einer verbreiteten Ansicht ist wegen 1.1. und 1.2. weder ein ontisches noch ein kategoriales Objekt (vgl. Bense 1975, S. 65 f.) in die Zeichenrelation eingebunden, sondern diese enthält lediglich die *Relation* des Zeichens zum externen Objekt, nämlich das sog. interne Objekt (vgl. Bense 1986, S. 15). Entsprechend ist zwischen dem Mittelbezug als Relation des Zeichens auf seinen Zeichenträger und diesem selbst, d.h. dem Mittel, sowie dem Interpretantenbezug und einem zu supponierenden Interpretanten zu unterscheiden: Das Peircesche Zeichen kann als "Relation über Relationen" (Bense 1979, S. 53, 67) weder ontisches Mittel, Objekt noch Subjekt enthalten, vielmehr müßte zum Zwecke ihrer Einbettung in die Zeichenrelation eine zusätzliche Kategorie der "Nullheit" eingeführt werden (Bense 1975, S. 39 ff., 64 ff.).

1.4. Die trichotomische Unterteilung der drei Triaden ist inhaltlich gesehen uneinheitlich. Z.B. ist nicht einleuchtend, weshalb im Mittelbezug die Essenz in der Subkategorisierung (Qualität – Quantität – Essenz) (Bense 1979, S. 61) an Stelle der Relation erscheint. Die Relation erscheint allerdings als zweitheitliche Zweitheit im Objektbezug in der Subkategorisierung (Abstraktion – Relation – Komprehension), die jedoch überhaupt keine ist, da die drei Unterteilungen inhaltlich keine Trichotomie bilden (wie dies etwa bei [Qualität –

Quantität – Relation] der Fall wäre). Das bedeutet also, daß die von Bense die Trichotomisierung von Triaden erzeugende generative Operation inhaltlich nicht nachvollziehbar ist.

1.5. Während der iconische und der symbolische Objektbezug des Zeichens sich mengentheoretisch im Sinne nicht-leerer sowie leerer Durchschnitte der Merkmalsmengen von Objekt und Zeichen formalisieren lassen, ist dies beim indexikalischen Objektbezug nicht möglich. Ferner decken dessen inhaltliche Bestimmung als "kausale", "nexale", "kontiguitäre" oder Teilmengenrelation zwischen Objekt und Zeichen seine Verwendungen nicht ab. Andererseits kann mereotopologisch zwischen mindestens drei indexikalischen Hauptrelationen unterschieden werden (vgl. Toth 2010), die semiotisch innerhalb der einfachen triadischen Relation mit dyadischen Partialrelationen nicht thematisierbar sind. Deshalb wurde in Toth (2012a) argumentiert, Indices als gerichtete Objektrelationen zu definieren.

1.6. Der Interpretantenbezug amalgamiert mehrere semiotisch differente Funktionen, v.a. die Konnexbildung von Zeichen einerseits (für die Bense [1971, S. 33 ff.] jedoch die Operationen der Adjunktion, Superisation und Iteration, die Interpretantenfeldern erzeugen, eingeführt hatte und von denen aus somit die Funktion der Konnexbildung von Interpretanten redundant ist) und die Superposition einer "zweiten Bedeutung" über dem Objektbezug (vgl. Ditterich 1990, S. 37), d.h. dessen Kontextuierung. Ferner hatte bereits Peirce zwischen zahlreichen logisch geschiedenen Interpretanten unterschieden (vgl. Walther 1979, S. 73 ff. u. 90 ff.), deren Unterscheidung durch die semiotische Repräsentation jedoch wiederum aufgehoben wird.

2.1. Vonnöten ist also, kurz gesagt, eine erstens sowohl formal als auch inhaltlich einheitliche und damit nachvollziehbare und erst dann operationalisierbare Semiotik, und zweitens eine Semiotik, die mit der zweiwertigen aristotelischen Logik, auf der ja bekanntlich alle (übrigen) Wissenschaften gegründet sind, kompatibel ist. Da die Konnexbildungen von Zeichen sich bereits durch die drei Operationen der Adjunktion, Superisation und Iteration erzeugen lassen (vgl. 1.6) und da die durch sie konstruierten Interpretantenfelder (vgl. Bense/Walther 1973, S. 45) sich problemlos als Kontextuierungen der

Objektbezüge der Zeichen interpretieren lassen, gehen wir also statt von einer triadischen von einer dyadischen Zeichenrelation der Form

$ZR^{2,3} = \langle a, b \rangle$

aus (vgl. meine Darstellung der logischen Menne-Semiotik [Toth 2012b]), wobei a Symbol für das Bezeichnende im Sinne des Saussureschen Signifikanten bzw. des Peirceschen Mittelbezugs und b Symbol für das Bezeichnete im Sinne eines realen, d.h. ontischen Objektes ist. (Innerhalb von $ZR^{2,3}$ muß dieses freilich als kategoriales Objekt, d.h. als 0-stellige Relation repräsentiert werden.)

2.2. Wir definieren nun folgende semiotischen Werte mit $x, y, z \in \mathbb{N}$

Bezeichnendes	Bezeichnetes
$\langle 1, x \rangle :=$ Ereignis	$\langle x, 1 \rangle :=$ Art
$\langle 2, y \rangle :=$ Gestalt	$\langle y, 2 \rangle :=$ Gattung
$\langle 3, z \rangle :=$ Funktion	$\langle z, 3 \rangle :=$ Familie

Bezeichnenden-Seite: Unter Ereignis verstehen wir das konkrete, realisierte, manifeste Zeichen und unter Gestalt die Isomorphieklasse aller konkreten, realisierten, manifesten Zeichen. Die Funktion ist der operationale Status isomorpher Zeichen, also z.B. die grammatische Differenzierung von ansonsten gleichen Wörtern (vgl. Menne 1992, S. 43 f.).

Bezeichneten-Seite: Wie man leicht bemerkt, korrespondiert die zunehmende Abstraktion von der Trichotomie (Art – Gattung- Familie) genau derjenigen von (Ereignis – Gestalt – Funktion), d.h. ordo essendi und ordo cognoscendi sind korrespondent konzipiert. Menne unterteilt die Bezeichnetenseite seines logischen Zeichenbegriffs durch die Trichotomie (Dinge – Begriffe – Sachverhalte), die wiederum derjenigen von (Art – Gattung – Familie) korrespondiert. D.h. die Art bzw. das Ding ist semiotisch gesprochen das individuelle und isolierte Objekt, während dessen Gattung bzw. Begriff die ihm zugehörige Objektfamilie und die Familie bzw. der Sachverhalt im Sinne eines Gefüges von Begriffen (Menne 1992, S. 45) eine Familie von Objektfamilien ist. Somit stellt die Bezeichnetenseite des Zeichens eine mengentheoretische Abstraktionsfolge der Form (x, {x}, {{x}}) dar, die nach Voraussetzung somit ebenfalls die Abstraktionsfolge der Bezeichnendenseite des Zeichens darstellt. Das dyadische Zeichen ist also eine binäre logische Relation, deren Wertrelationen isomorph sind und das ein (minimales) System mit Umgebung darstellt.

2.3. Zur Transformation zwischen den einzelnen trichotomischen Stufen in den Triaden wie in den Trichotomien genügt somit ein einziger Abstraktionsoperator α, der wegen der beiden Seiten des dyadischen Zeichens gemeinsamen mengentheoretischen Struktur bzw. Ordnung (x, {x}, {{x}}) als Einbettungsoperator definiert werden kann. Operiert α über Triadenwerten, so lassen wir ihn unbezeichnet; operiert er über Trichotomienwerten, so kennzeichnen wir ihn durch α'. Damit haben wir

$\alpha(\langle 1, x\rangle) = (\langle 1, y\rangle)$ $\qquad \alpha^{-1}(\langle 1, y\rangle) = (\langle 1, x\rangle)$

$\alpha(\langle 1, y\rangle) = (\langle 1, z\rangle)$ $\qquad \alpha^{-1}(\langle 1, z\rangle) = (\langle 1, y\rangle)$

$\alpha^2(\langle 1, x\rangle) = (\langle 1, z\rangle)$ $\qquad (\alpha^{-1})^2(\langle 1, z\rangle) = (\langle 1, x\rangle)$

$\alpha'(\langle 1, x\rangle^{-1}) = (\langle 1, y\rangle^{-1})$ $\qquad \alpha'^{-1}(\langle 1, y\rangle^{-1}) = (\langle 1, x\rangle^{-1})$

$\alpha' (\langle 1, y\rangle^{-1}) = (\langle 1, z\rangle^{-1})$ $\qquad \alpha'^{-1}(\langle 1, z\rangle^{-1}) = (\langle 1, y\rangle^{-1})$

$\alpha'^2(\langle 1, x\rangle^{-1}) = (\langle 1, z\rangle^{-1})$ $\qquad (\alpha'^{-1})^2(\langle 1, z\rangle^{-1}) = (\langle 1, x\rangle^{-1})$

α und α' sind also nur dann zyklisch, wenn die x, y, z Elemente einer endlichen oder begrenzten Menge sind, also z.B. hier im gewählten triadisch-trichotomischen Fall. Da man jedoch theoretisch die Folge (x, {x}, {{x}}, {{{x}}}, ...) beliebig vermehren, d.h. die Einbettungen von x iterieren kann, gibt es weder formal noch inhaltlich einen zwingenden Grund, die Folge bei den Triaden abzubrechen (zur "trinitären" Triadizität von Peirce vgl. Günther [1978, S. xi ff.]).

2.4. Wie bereits gesagt, kann man somit innerhalb der Ordnungsstruktur

$ZR^{2,3} = \langle a, b\rangle$ mit $a, b \in \mathbb{N}$

die a's z.B. im Sinne des Peirceschen Mittelbezugs auffassen. Wegen der Definition der a's gilt dies jedoch nur oberflächlich, denn <1, 1>, <1, 2>, <1, 3> entsprechen gemäß unseren Definitionen keineswegs der Peirceschen Mitteltrichotomie von Quali-, Sin- und Legizeichen. Vielmehr ist <1, 1> im Sinne von <1, a> mit a = 1 ein realisiertes Objekt (Ding), <1, a> mit a = 2 die Abstraktion aller durch <1, 1> realisierten Dinge, und <1, a> mit a = 3 deren Funktion. Z.B. ist ein phonetisch realisierter Laut <1, 1>, sein zugehöriges Phonem <1, 2> und sein Fungieren innerhalb von Silben (Morphemen) oder Wörtern (Lexemen) <1, 3>. Da in <1, a> jedoch a ∈ ℕ ist, hindert uns natürlich nichts daran (entgegen den entsprechenden Verhältnissen in der Peirceschen Semiotik; vgl. Walther 1979, S. 100), den Laut auch in Überworteinheiten, also z.B. in Satzteilen, Sätzen, Diskursen, Texten (z.B. mit "phonostilistischen" Funktionen) zu betrachten.[6] Wegen der Isomorphie von ordo cognoscendi und essendi bzw. Bezeichnendem und Bezeichnetem sind also die konvertierten geordneten Paare der allgemeinen Form <a, 1> mit ∈ ℕ natürlich keine Zeichen (wie es die konversen Dyaden der Peirce-Bense-Semiotik sind), sondern die ontischen Gegenstücke der semiotischen Zeichen, d.h. es ist z.B. <1, 1> die Identität zwischen einem Phonem und seinem "Lautsubstrat", aber <2, 1> ist die Nicht-Identität eines Phonems mit dem letzteren, denn das Phonem bezieht sich gemäß Definition nicht auf ein Objekt, d.h. einen konkreten, realisierten Laut (wie das Phon), sondern auf eine Isomorphieklasse von Lauten, d.h. auf einen Begriff, nämlich auf eine lautliche Abstraktion (und genauso ist das Phonem ja in der theoretischen Linguistik definiert). Entsprechend ist <3, 1> die Nicht-Identität der Phonotaktik mit dem Lautsubstrat, da die Kombination von Phonemen, aufgefaßt als Funktion, einen Sachverhalt und also weder den Laut, d.h. das Objekt selber, noch ein einzelnes Phonem, d.h. den Begriff des Lautes, darstellt. Der Sachverhalt als ontisches Gegenstück der Phonotaktik ist somit wortwörtlich als der "Verhalt" der als "Sachen" aufgefaßten und von den Lauten als Dingen unterschiedenen Phoneme aufzufassen.

[6] Im Gegensatz zur Stratifikationsgrammatik ist also auch die Anzahl der "Strata", d.h. der grammatischen Ebenen, wegen ∈ ℕ theoretisch unbegrenzt.

Es dürfte nach dieser illustrativen Explikation somit keinerlei Zweifel mehr unterliegen, daß die Bezeichnetenseite von $ZR^{2,3}$ keinesfalls mit dem Peirceschen Objektbezug zusammenfällt, da dieser das interne oder semiotische Objekt (vgl. Bense/Walther 1973, S. 70 f.), jene aber das externe oder ontische Objekt betrifft. Zwischen dem Peirceschen Zeichen und $ZR^{2,3}$ gibt es somit einzig und allein eine oberflächliche (und darüber hinaus triviale) Verwandtschaft zwischen der Bezeichnendenseite und den Signfikantenseiten der Legion von Zeichenmodellen von der Antike bis zu de Saussure (und nach ihm), aber es gibt keine Verwandtschaft zwischen der Bezeichnetenseite und der Signfikatenseite, denn in $ZR^{2,3}$ wird logisch streng zwischen Ding, Begriff und Sachverhalt bzw. mengentheoretisch zwischen Elementen und ihren Mengenabbildungen unterschieden.

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Typen dyadischer Semiosen

1. Wir gehen aus von dem Zeichenmodell der in Toth (2012) eingeführten logischen Semiotik

$ZR^{2,n} = \langle a, b \rangle$

und der folgenden semiotisch-ontischen Subkategorisierung für den Fall n = 3

semiotische Subkategorisierung	ontische Subkategorisierung	mengentheoret. Einbettungsstufe
Ereignis (E)	Art (A)	x
Gestalt (Ge)	Gattung (Ga)	{x}
Funktion (Fu)	Familie (Fa)	{{x}}
...	...	...

Diese Semiotik ist also binär, da sie

1. die Isomorphie der logischen Position und Negation auf die Zeichen überträgt, d.h. jede mit semiotischen Werten belegte Struktur $ZR^{2,n}$ ist sowohl logisch als auch semiotisch interpretierbar. Diese Semiotik erlaubt es somit erstmals, neben der Repräsentationsfunktion auch die Wahrheitsfunktion von Zeichen zu bestimmen bzw. sie erlaubt erstmals eine semiotische Interpretation der Kalküle der zweiwertigen aristotelischen Logik.

2. die Binarität von Position und Negation auf die Subkategorisierung der Zeichen überträgt, d.h. Semiotik und Ontik sind isomorph definiert. Dies bedingt die Ersetzung des absoluten, d.h. objektiven Objektes durch ein subjektives Objekt und die Ersetzung des absoluten, d.h. subjektiven Subjektes durch ein objektives Subjekt. Somit spiegelt sich die Ontik in der Semiotik sowie die Semiotik in der Ontik, und die logische Semiotik ist also sowohl eine Zeichen- als auch eine Objekttheorie.

Ferner ist diese Semiotik n-adisch, da die doppelte (d.h. sowohl semiotische als auch ontische) Subklassifizierung natürlich nicht auf der Einbettungsstufe {{x}} stehen bleiben muß, genauso wenig wie die Phänomenologie sich mit einer Triade von Art, Gattung und Familie beschränken muß, vgl. dazu Menne (1992, S. 94 ff.). Da sich Zeichen und Objekt gegenseitig spiegeln, entstehen durch die Einbettungsoperation unendliche Folgen, die man mit der bekannten konkreten Situation zweier leicht schräg gegenüber liegender Spiegel (z.B. in einem Coiffeur-Salon) vergleichen kann. Wesentlich ist also, daß der durch diese Spiegelungen verursachte Prozeß $n \rightarrow \infty$ die Binarität von Zeichen und Objekt keinesfalls stört, denn wir haben für die ersten Stufen z.B.

$ZR^{2,1} = \langle a, b\rangle$

$ZR^{2,2} = \langle\langle a, b\rangle, c\rangle \,/\, \langle a, \langle b, c\rangle\rangle$

$ZR^{2,3} = \langle\langle\langle a, b\rangle, c\rangle, d\rangle \,/\, \langle d, \langle\langle a, b\rangle, c\rangle\rangle$,

so daß man auch hieran sieht, daß sich jedes n-tupeln als binäre Relation darstellen läßt (vgl. Schwabhäuser 1954).

2. Da also Zeichen und Objekt in der logischen Semiotik sowohl semiotisch als auch logisch interpretierbar sind, sind sie auch nicht nur durch Wahrheitswertfunktionen, sondern auch durch Zeichenfunktionen, d.h. Semiosen beschreibabr, d.h. die statische Komplementarität von Zeichen und Objekt, wie sie in der verdoppelten Subkategorisierung zum Ausdruck kommt, wird ihrerseits auf die dynamische Komplementarität von Wahrheitswertfunktionen und Semiosen übertragen. Wir können die folgenden Typen dyadischer Semiosen unterscheiden.

Für n = 1:

1.a $S_{ZR2,1} = \langle x, y\rangle$ 1.b $S_{ZR2,1} = \langle y, x\rangle$

Für n = 2:

1.a $S_{ZR2,2} = \langle x, \langle y, z\rangle\rangle$ 1.b $S_{ZR2,2} = \langle\langle x, y\rangle, z\rangle$

2.a $S_{ZR2,2} = \langle x, \langle z, y\rangle\rangle$ 2.b $S_{ZR2,2} = \langle\langle x, z\rangle, y\rangle$

3.a $S_{ZR2,2} = \langle y, \langle x, z\rangle\rangle$ 3.b $S_{ZR2,2} = \langle\langle y, x\rangle, z\rangle$

4.a $S_{ZR2,2} = \langle y, \langle z, x\rangle\rangle$ 4.b $S_{ZR2,2} = \langle\langle y, z\rangle, x\rangle$

5.a $S_{ZR2,2} = <z, <x, y>>$ 5.b $S_{ZR2,2} = <<z, x>, y>$

6.a $S_{ZR2,2} = <z, <y, x>>$ 6.b $S_{ZR2,2} = <<z, y>, x>$

Für n = 3:

1.a $S_{ZR2,3} = <x, <y, <z, w>>>$ 1.b $S_{ZR2,2} = <<<x, y>, z>, w>$

2.a $S_{ZR2,3} = <x, <y, <w, z>>>$ 2.b $S_{ZR2,2} = <<<x, y>, w>, z>$

3.a $S_{ZR2,3} = <x, <w, <y, z>>>$ 3.b $S_{ZR2,2} = <<<x, w>, y>, z>$

4.a $S_{ZR2,3} = <x, <w, <z, y>>>$ 4.b $S_{ZR2,2} = <<<x, w>, z>, y>$

5.a $S_{ZR2,3} = <x, <z, <w, y>>>$ 5.b $S_{ZR2,2} = <<<x, z>, w>, y>$

6.a $S_{ZR2,3} = <x, <z, <y, w>>>$ 6.b $S_{ZR2,2} = <<<x, z>, y>, w>$

7.a $S_{ZR2,3} = <y, <x, <z, w>>>$ 7.b $S_{ZR2,2} = <<<y, x>, z>, w>$

8.a $S_{ZR2,3} = <y, <x, <w, z>>>$ 8.b $S_{ZR2,2} = <<<y, x>, w>, z>$

9.a $S_{ZR2,3} = <y, <w, <x, z>>>$ 9.b $S_{ZR2,2} = <<<y, w>, x>, z>$

10.a $S_{ZR2,3} = <y, <w, <z, x>>>$ 10.b $S_{ZR2,2} = <<<y, w>, z>, x>$

11.a $S_{ZR2,3} = <y, <z, <w, x>>>$ 11.b $S_{ZR2,2} = <<<y, z>, w>, x>$

12.a $S_{ZR2,3} = <y, <z, <x, w>>>$ 12.b $S_{ZR2,2} = <<<y, z>, x>, w>$

13.a $S_{ZR2,3} = <z, <y, <x, w>>>$ 13.b $S_{ZR2,2} = <<<z, y>, x>, w>$

14.a $S_{ZR2,3} = <z, <y, <w, x>>>$ 14.b $S_{ZR2,2} = <<<z, y>, w>, x>$

15.a $S_{ZR2,3} = <z, <w, <y, x>>>$ 15.b $S_{ZR2,2} = <<<z, w>, y>, x>$

16.a $S_{ZR2,3} = <t, <w, <x, y>>>$ 16.b $S_{ZR2,2} = <<<z, w>, x>, y>$

17.a $S_{ZR2,3} = <z, <x, <w, y>>>$ 17.b $S_{ZR2,2} = <<<z, x>, w>, y>$

18.a $S_{ZR2,3} = <z, <x, <y, w>>>$ 18.b $S_{ZR2,2} = <<<z, x>, y>, w>$

19.a $S_{ZR2,3} = <w, <y, <z, x>>>$ 19.b $S_{ZR2,2} = <<<w, y>, z>, x>$

20.a $S_{ZR2,3} = <w, <y, <x, z>>>$ 20.b $S_{ZR2,2} = <<<w, y>, x>, z>$

21.a $S_{ZR2,3} = <w, <x, <y, z>>>$ 21.b $S_{ZR2,2} = <<<w, x>, y>, z>$

22.a $S_{ZR2,3} = <w, <x, <z, y>>>$ 22.b $S_{ZR2,2} = <<<w, x>, z>, y>$

23.a $S_{ZR2,3} = <w, <z, <x, y>>>$ 23.b $S_{ZR2,2} = <<<w, z>, x>, y>$

24.a $S_{ZR2,3} = <w, <z, <y, x>>>$ 24.b $S_{ZR2,2} = <<<w, z>, y>, x>$

Es gibt nur schon bei Beschränkung auf maximal triadische Subkategorisierung bereits 37 Typen dyadischer Zeichen-Semiosen. Wegen der Isomorphie von Zeichen und Objekt kommen dazu natürlich noch 37 Typen dyadischer Objekt-Semiosen.

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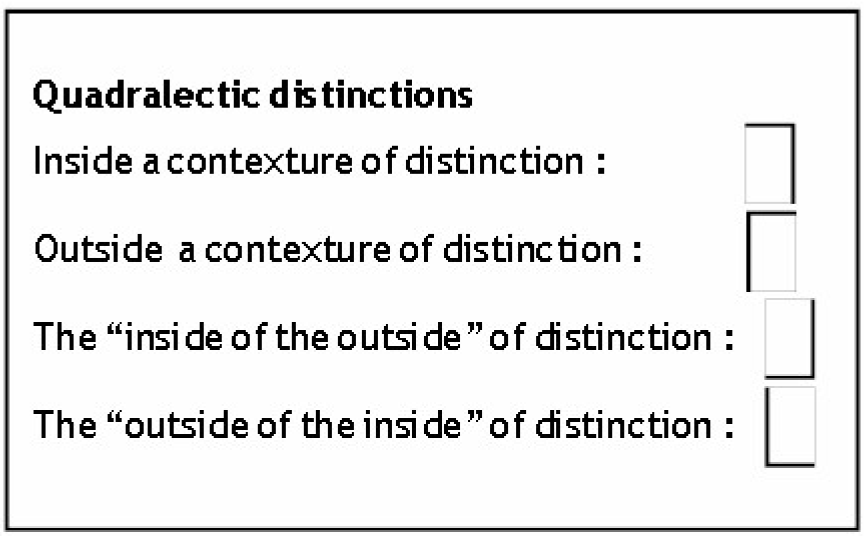
Bivalenz und Tetravalenz

1. Wie zuletzt in Toth (2012a, b) gezeigt, weisen die logischen Semiotiken von Albert Menne (Menne 1992, S. 39 ff.) und von Georg Klaus (1965, 1973) als zentrale Gemeinsamkeit auf, daß sie auf einem Axiom der Isomorphie von Zeichen und Objekt bzw. von Signifikanten- und Signifikatsseite des Zeichens basieren, das eine direkte Konsequenz der zweiwertigen Logik darstellt. In einer solchen Semiotik fallen Abstraktionsklassenbildung und Superisation zusammen (vgl. Toth 2012c), d.h. der Weg vom konkreten zum abstrakten Zeichen und weiter zu einer theoretisch unendlichen Hierarchie von Superzeichen geschieht durch "kulminierte" iterative Mengenbildung. Wegen des Isomorphieaxioms können sowohl die Menne- als auch die Klaus-Semiotik als verdoppeltes von Neumann-Universums darsgestellt werden (vgl. Toth 2012d), deren Strukturschema wie folgt aussieht

$$
\begin{array}{lcl}
x & \cong & y \\
\{x\} & \cong & \{y\} \\
\{\{x\}\} & \cong & \{\{y\}\} \\
\{\{\{x\}\}\} & \cong & \{\{\{y\}\}\} \\
\{\{\{\{x\}\}\}\} & \cong & \{\{\{\{y\}\}\}\} \\
\{\{\{\{\{x\}\}\}\}\} & \cong & \{\{\{\{\{y\}\}\}\}\} \\
\{\{\{\{\{\{x\}\}\}\}\}\} & \cong & \{\{\{\{\{\{y\}\}\}\}\}\} \\
\vdots & & \vdots
\end{array}
$$

Das weder von Menne noch von Klaus je auch nur erwähnte, geschweige denn besprochene Problem besteht jedoch darin, daß in der Semiotik nach Saussure Signifikant und Signifikat bekanntlich so zusammenhängen wie Recto- und Versoseite eines Blattes Papier. Falls dies korrekt, muß wegen des Isomorphieaxioms ein solcher Zusammenhang auch zwischen Zeichen und Objekt in der Logik existieren. Wegen des Tertium non Datur-Axioms definiert eine zweiwertige Kontextur einen ontologischen Ort, der in dieser Zweiwertigkeit absolut und vollständig determiniert ist, d.h. Zeichen und Objekt hängen tatsächlich bis auf Isomorphie so zusammen, wie es Signifikant und Signifikat tun.

2. Die isomorphe "Parallelisierung" von Objekt und Zeichen sowie von Signifikat und Signifikant wird somit durch die Ontologie gestützt, deren zweiwertige Interpretation besagt, daß ein Etwas entweder existiert oder nicht existiert, d.h. daß es nur Sein oder Nichts gibt. Allerdings wird die Parallelisierung nicht durch die Epistemologie gestützt, denn in der zweiwertigen Logik und ihrer korrespondierenden Ontologie gibt es keine Möglichkeit, neben den "reinen" Kategorien von Subjekt und Objekt die "gemischten" oder besser: vermittelnden Kategorien des subjektives Objekts und des objektiven Subjekts zu designieren bzw. zu thematisieren. wie Kaehr (2011)gezeigt hat, somit somit bivalente Semiotiken und Logiken auch systemtheoretisch defizient:



In Toth (2011a) hatte ich deshalb eine tetravalente Semiotik folgendermaßen systemtheoretisch definiert:

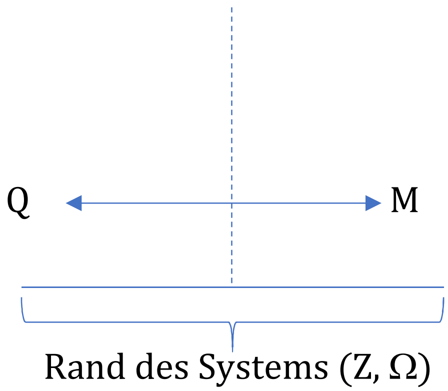
Mittelbezug (M): $[A \rightarrow I] := I$
Objektbezug (O): $[[A \rightarrow I] \rightarrow A := A$
Interpretantenbezug (J): $[[[A \rightarrow I] \rightarrow A] \rightarrow I := I(A)$
Qualität (Q) $[A \rightarrow I]° = [I \rightarrow A] := A(I)$.

Wie man aus der Konversionsbeziehung zwischen der ersten und der letzten Definition erkennt, gilt also

Mittelbezug (M): $[A \rightarrow I] := I$

Qualität (Q) $[A \rightarrow I]° = [I \rightarrow A] := A(I)$,

d.h. es ist M° = Q und Q° = M. Das bedeutet aber, daß der tetravalenten Semiotik ein Systemmodell zugrunde liegt, das man wie folgt schematisieren könnte:



Der Rand des Systems partizipiert somit sowohl am "semiotischen Raum" als auch am "ontischen Raum" (vgl. dazu Bense 1975, S. 65 f.), d.h. Q und M stehen in einer PARTIZIPATIVEN AUSTAUSCHRELATION, und der Übergang vom semiotischen zum ontischen Raum erfolgt durch einen chiastischen Austausch der Systemkategorien A und I:

3.heit $[[[A \rightarrow I] \rightarrow A] \rightarrow I]$

2.heit $[[A \rightarrow I] \rightarrow A]$

1.heit $[A \rightarrow I]$

0.heit $[I \rightarrow A]$,

d.h. dieser fällt unter die von Günther (1971) entdeckte Proemialrelation und führt somit unter die Ebene der (zweiwertigen) Logik.

3. Wie man leicht einsieht, ergibt sich auf dieser sowohl unter der Logik als auch unterhalb der auf dieser basierenden Semiotik liegenden Ebene nicht nur ein System von zwei, sondern von (16-4 =) 12 erkenntnistheoretischen Vermittlungsrelationen

	└	┘	┌	┐
└	└ └	└┘	└ ┌	└┐
┘	┘ └	┘ ┘	┘ ┌	┘ ┐
┌	┌ └	┌┘	┌ ┌	┌┐
┐	┐ └	┐ ┘	┐ ┌	┐ ┐.

Definieren wir wie in Toth (2011b)

$\omega := (A \rightarrow I)$

$[\omega, 1] := ((A \rightarrow I) \rightarrow A)$

$[[\omega, 1], 1] := (((A \rightarrow I) \rightarrow A) \rightarrow I)$,

so haben wir die ja wegen der der tetravalenten Semiotik zugrunde liegenden Systemdefinition vorhandene Parallelisierung von Zeichen und Objekt insofern "gerettet", als wir nur von einer einzigen Abbildung ω, allerdings in drei verschiedenen Einbettungsstufen, ausgehen, die man genauso gut durch [ω], [[ω]], [[[ω]]], also wie in den kumulativen Mengenhierarchien von Menne und von Klaus, bezeichnen könnte. Versuchen wir nun also, noch abstrakter zu sein und den Systembegriff selbst als Spezialfall einer beliebigen Dichotomie D zu definieren. Sei

$D := [a, b]$

eine beliebige Dichotomie und

$1 := a(b) = b \rightarrow a$

eine beliebige Abbildung der Glieder von D. Ferner bedeute „1“, daß diese Abbildung eine „Oberflächenabbildung“ sei, d.h. daß die Einbettungsstufe 0 vorliege:

$1 = [1_0] := 1_0$.

Damit können wir das obige systemtheoretische semiotische Minimalsystem wie folgt notieren

$\omega = 1$

$[\omega, 1] = 1_{-1}$

$[[\omega, 1], 1] = 1_{-2}$,

d.h. wir können hiermit nicht nur die Semiotik auf die Systemtheorie zurückführen, sondern die letztere durch das Paar

$RE = \langle 1, {}_{n}] \rangle$,

bestehend aus einer Abbildung 1 und einem n-stufigen Einbettungsoperator ${}_{n}]$ definieren und nennen dieses Paar RE eine relationale Einbettungszahl. Wir erhalten dann z.B. für die Bensesche Zeichenklasse des vollständigen Mittelbezugs, d.h. für das Klaussche Zeichenexemplar und für das Mennesche Lalem:

Zkl	=	$\{3.1, 2.1\ 1.1\} =$
S_1	=	$((((\omega, 1), 2), \omega)\ ((\omega, 1), \omega)\ (\omega, \omega)) =$
$*S_1$	=	$\{\{\{\{\omega\}\}\}, \{\{\omega\}\}, \{\omega\}\})\}$
RE		$[[1_{-3}, 1], [1_{-2}, 1], [1, 1]]$.

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Zur Logik des Gleichheitszeichens

> La libération de l'espérence est la libération totale.
>
> Unica Zürn (Der Mann im Jasmin. Frankfurt am Main 1977, S. 56)

1. Die Gleichheitsrelation

$a = b$

bedeutet, daß die zu f: a → b konverse Relation f^{-1}: b → a existiert, d.h. daß

$b = a$

ist. Daraus folgt – wie bereits Kronthaler (1986) in anderem Zusammenhang festgestellt hatte –, daß. a nichts enthalten kann, was b nicht enthält – denn es fehlt ein drittes, vermittelndes Glied (Tertium non datur), dies aber wiederum bedeutet, daß die Gleichheitsrelation eine Reflexion ist. Nochmals anders gesagt: Das Gleichheitszeichen markiert lediglich die Differenz zwischen a und b, d.h. sie gibt an, daß es sich bei der Gleichheit um eine Relation zwischen zwei Objekten handelt, während es sich bei der Identität um eine Relation an einem Objekt handelt. (M.W. hat diese Tatsache niemand so deutlich erkannt oder mindestens formuliert wie Menne [1991, S. 99].) Identität ist daher immer Selbst-Identität, d.h. die definitorische logische Eigenschaft der ontologischen Selbstgegebenheit von Objekten, die damit der Selbst-Reflexitität der Subjekte gegenübersteht. Somit bedeutet die Identiätsrelation

$a \equiv b$

genau dasselbe wie die Identitätsrelationen

$a \equiv a$

$b \equiv b$.

2. Ganz anders aber verhält es sich bei den von Bense (1981, S. 17 ff.) eingeführten "Primzeichen", die er ausdrücklich als "Zeichenzahlen" verstanden haben wollte und die er daher mittels der Peano-Axiome eingeführt hatte

(vgl. Bense 1975, S. 167 ff.; vgl. ebenfalls Bense 1983, S. 192 ff.), denn semiotische Subrelationen sind als kartesische Produkte definiert, die in Form von geordneten Paaren

$S = <a.b>$

notiert werden, und somit ist

$<a.b> \neq <b.a>$,

denn die triadische Zeichenzahl

$P_{td} = (a.)$

hat einen höheren Einbettungsgrad als die trichotomische Zeichenzahl

$P_{tt} = (.b)$.

Somit kann man die folgenden Gleichungen aufstellen

$S = <a.b> = [a[b]]$

$S = <b.a> = [b[a]]$.

S bekommt damit aber die Form selbsteinbettender Systeme, die seit Toth (2012) durch

$S^* = [S, U]$

$U^* = [U, S]$

definiert sind, und diese Definition wiederum ist ontisch isomorph zur semiotischen Definition der Zeichenrelation, die Bense (1979, S. 53) gegeben hatte

$Z = R(M \rightarrow ((M \rightarrow O) \rightarrow (M \rightarrow O \rightarrow I)))$

und die Bense explizit als "Relation über Relationen" (ibd.) bezeichnet hatte.

3. Damit setzen Zeichenzahlen allerdings ein Tertium datur in Form eines Einbettungsoperator E voraus, der, auf die Menge der Zeichenzahlen

$P = (1, 2, 3)$

angewandt, entscheidet, ob ein Element aus P Teilmenge von P_{td} oder von P_{tt} ist. Dieser Schluß hat nun die erstaunliche weitere Konsequenz, daß die immer wieder behauptete "Selbstdualität" der sogenannten "genuinen" Subrelationen (1.1), (2.2) und (3.3) überhaupt nicht existiert, denn vermöge S haben wir ja

(1.1) = [1[1]]

(2.2) = [2[2]]

(3.3) = [3[3]],

und die zugehörigen Dualitätsrelationen sind

$\times[1[1]] \neq [[1]1]$

$\times[2[2]] \neq [[2]2]$

$\times[3[3]] \neq [[3]3]$,

d.h. es gelten für $S = \langle a.b \rangle$ nicht nur die trivialen Ungleichungen für $a \neq b$, d.h. für Verschiedenheit, sondern auch für $a = a$ bzw. $b = b$, d.h. für Gleichheit. In Sonderheit folgt daraus, daß es keine semiotische Identität und damit auch keine Selbstidentität der Dualität von Repräsentationsschemata, d.h. keine von Bense (1981, S. 155; 1992) so genannte Eigenrealität, gibt.

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Logische Einbettungen

1. Den mythologischen Schöpfungsprozeß des Alten Testamentes, den Günther (1980 , S. 14 ff.) im Rahmen der polykontexturalen Logik detailliert dargestellt hatte, gipfelt im bemerkenswerten Satz: "Das Du ist abgeleitetes und vermitteltes Subjektsein" (1980, S. 27).

2.1. Wenn wir davon ausgehen, daß am Anfang der Schöpfung der Schöpfergott als Subjekt steht, dann ist dieses logisch gesehen natürlich ein Ich-Subjekt, und seine Geschöpfe bilden relativ zu ihm die Menge der Du-Subjekte. Der günthersche Satz bedeutet nun aber, daß eine simple, der Basisrelation der aristotelischen Logik isomorphe dichotomische Relation

$S = [S_{ich}, S_{du}]$

ebenso falsch wie sinnlos wäre. Falsch ist S deshalb, weil Ich- und Du-Subjekt in S nicht vermittelt sind, sinnlos ist S, weil daraus folgt, das Schöpfer und Geschöpf einander ebenbürtig sind, was angesichts der Allmacht des Schöpfergottes klarerweise falsch ist.

2.2. In Toth (2014) war daher vorgeschlagen worden, anstatt einen künstlichen dritten Wert in die 2-wertige Logik einzuschmuggeln, das Tertium-Gesetz durch Anwendung eines Einbettungsoperator E zu eliminieren. Damit bekommen wir

$E(S) = [S_{ich}, [S_{du}]]$

oder

$E(S) = [[S_{ich}], S_{du}]$.

Beide Fälle stellen also Vermittlung durch Ersetzung der heterarchischen Austauschrelation in S durch die hierarchischen Austauschrelationen in E(S) dar. Der erste Fall ist derjenige der Bibel. Der zweite Fall würde bedeuten, daß Gott den Menschen an seine Stelle setzt – und selbst zum Menschen wird.

2.3. Noch interessanter als diese Ergebnisse ist auf theoretischem Boden, daß beide Formen von E(S) isomorph sind anderen hierarchischen Einbettungen, die man durch Selbsteinbettungen definieren kann, wie z.B. diejenige zwischen

Subjekt und Objekt. Denn obwohl die logische Position die Objekt- und die logische Negation die Subjektposition in der zu S isomorphen Relation

$L = [P, N]$

einnimmt, kontrastiert darin die Selbstgegebenheit des Objektes mit der Selbstreflexivität des Subjektes, d.h. das Subjekt ist transzendental, das Objekt ist es nicht, und somit besteht wiederum ein "Reflexionsgefälle" zwischen Objekt und Subjekt bzw. Position und Negation, wie wir es zuvor zwischen subjektivem und objektivem Subjekt, d.h. Ich- und Du-Subjekt, hatten. Das bedeutet, daß wir erneut zwei mögliche Formen bekommen

$E(L) = [\Omega, [\Sigma]]$

oder

$E(L) = [[\Sigma], \Omega]$,

d.h. es ist $E(S) \cong E(L)$.

2.4. Schließlich bemerken wir, daß der Einbettungsoperator, der quasi ein relationales statt eines materialen Tertiums in S und L einführt, mit den beiden Paaren von Formen noch unvollständig angewandt ist, denn wir haben sowohl für E(S) als auch für E(L) jeweils ein Quadrupel von hierarchischen Austauschrelationen

$E(S) = [S_{ich}, [S_{du}]]$ $\quad$ $E(L) = [\Omega, [\Sigma]]$

$E(S) = [[S_{ich}], S_{du}]$ $\quad$ $E(L) = [[\Omega], \Sigma]$

$E(S) = [S_{du}, [S_{ich}]]$ $\quad$ $E(L) = [\Sigma, [\Omega]]$

$E(S) = [[S_{du}], S_{ich}]$ $\quad$ $E(L) = [[\Sigma], \Omega]$.

Da ferner die E(S) Subjektspezifikationen der einen S-Position in den E(L) sind, können wir somit die E(S) und die E(L) zu triadischen Einbettungsrelationen kombinieren

$E(L) = [\Omega, [[S_{ich}, [S_{du}]]]]$ $\quad$ $E(L) = [[\Omega], [S_{ich}, [S_{du}]]]$

$E(L) = [\Omega, [[[S_{ich}], S_{du}]]]$ $\quad$ $E(L) = [[\Omega], [[S_{ich}], S_{du}]]$

$E(L) = [\Omega, [[S_{du}, [S_{ich}]]]]$ $E(L) = [[\Omega], [S_{du}, [S_{ich}]]]$

$E(L) = [\Omega, [[[S_{du}], S_{ich}]]]$ $E(L) = [[\Omega], [[S_{du}], S_{ich}]]$

$E(L) = [[S_{ich}, [S_{du}]], [\Omega]]$ $E(L) = [[[S_{ich}, [S_{du}]]], \Omega]$

$E(L) = [[[S_{ich}], S_{du}], [\Omega]]$ $E(L) = [[[[S_{ich}], S_{du}]], \Omega]$

$E(L) = [[S_{du}, [S_{ich}]], [\Omega]]$ $E(L) = [[[S_{du}, [S_{ich}]]], \Omega]$

$E(L) = [[[S_{du}], S_{ich}], [\Omega]]$ $E(L) = [[[[S_{du}], S_{ich}]], \Omega]$.

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Ontische Einbettungen

1. Der in Toth (2014a, b) eingeführte Einbettungsoperator läßt sich natürlich auch auf die Ontik anwenden. Sei S = [x, y] ein logisch 2-wertiges Systems, dann besitzt dieses ohne Anwendung des Einbettungsoperators E lediglich die beiden folgenden Strukturen

$S_1 = [x, y]$

$S_2 = [y, x]$.

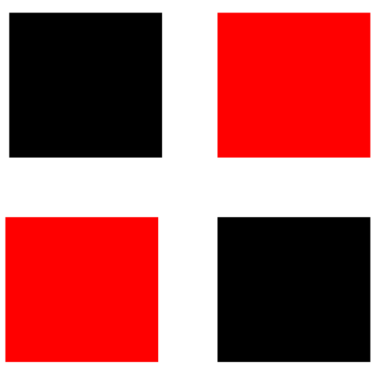
Mit Anwendung von E bekommen wir jedoch jeweils ein Quadrupel von Strukturen der Formen

$S_1 = [x, [y]]$ $\qquad$ $S_1 = [[y], x]$

$S_1 = [[x], y]$ $\qquad$ $S_1 = [y, [x]]$.

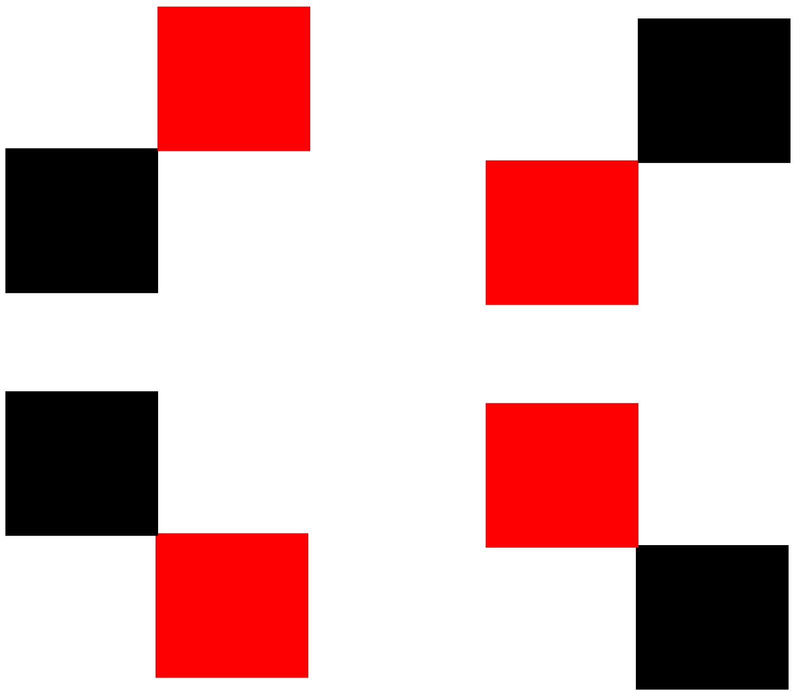
2.1. Links vs. Rechts

Diese Distinktion bleibt ohne E ontisch 2-wertig.

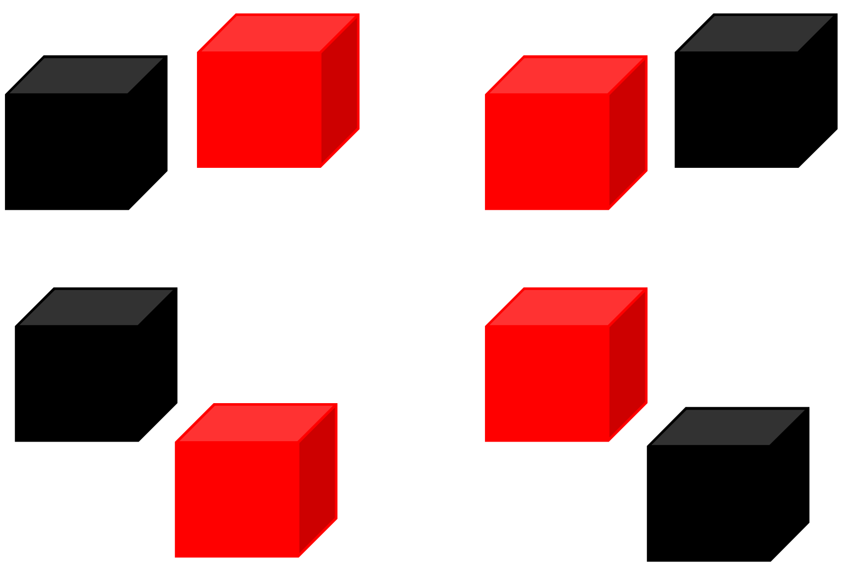


Dagegen zeigen die Distinktionen zwischen Unten und Oben sowie zwischen Vorn und Hinten unter Anwendung von E jeweils nicht nur 1 Paar, sondern 2 Paare ontisch dualer Strukturen.

2.2. Unten vs. Oben



2.3. Vorn vs. Hinten



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Toth, Alfred, Logische Einbettungen. In: Electronic Journal for Mathematical Semiotics, 2014b

Einbettungsstufen in der Semiotik

1. Bereits in Toth (2014a, b) wurde darauf hingewiesen, daß es keine semiotische Selbstdualität gibt, und zwar auch dann nicht, wenn man die 2-wertige aristotelische Basis der Semiotik beibehält. Die Definition der semiotischen Subrelationen

$S = <x.y>$ mit $x, y \in \{1, 2, 3)$

unterscheidet ja zwischen triadischem Hauptwert (x.) und trichotomischem Stellenwert (.y) und beruht daher nicht auf einer heterarchischen, sondern auf einer hierarchischen Austauschrelation. Daher kann man S auch in der Form

$S = [x, [y]]$

notieren, und damit gilt nicht nur für den Fall, daß $x \neq y$, sondern, falls $x = y$ ist

$\times[x, [y]] \neq [[y], x]]$,

d.h. in Sonderheit gelten die Ungleichungen

$\times(1, [1]] \neq [[1], 1]$

$\times(2, [2]] \neq [[2], 2]$

$\times(3, [3]] \neq [[3], 3]$

der hauptdiagonalen "genuinen" Subzeichen und

$\times[[[3, [1]], [2, [2]], [1, [3]]] \neq [[[3], 1], [[2], 2], [[1], 3]]]$

des nebendiagonalen "eigenrealen" Dualsystems.

Triadische und trichotomische Zeichenzahlen (vgl. Bense 1981, S. 17) befinden sich daher auf verschiedenen Einbettungsstufen.

2. Da nun die vollständige triadische Zeichenrelation durch Bense (1979, S. 53) durch

$ZR = (M \subset ((M \subset O) \subset (M \subset O \subset I)))$

definiert ist, d.h. weil

(M ⊂ O ⊂ I) (M)

(M ⊂ O) (O ⊃ M)

(M) (I ⊃ O ⊃ M)

gilt, folgt, daß sich der trichotomische Stellenwert von $S_n = <x, y>$ auf der gleichen Einbettungsstufe befinden muß wie der triadische Haupttwert von $S_{n+1} = <z, w>$. Damit bekommt man ein 4-stufiges semiotisches Einbettungsschema der benseschen Zeichenzahlen, von ihm selbst auch etwas unglücklicherweise "Primzeichen" genannt (Bense 1981, S. 17 ff.)

Z = [3.x, 2.y, 1.z]

1	3			
2		x 2		
3			y 3	
4				z

2. Es ist daher möglich, den zeicheninternen Zusammenhang zwischen den drei Subrelationen jeder Zeichen- und ihrer dual koordinierten Realitätsthematik mittels Identitätskoinzidenzen pro Einbettungsstufe darzustellen.

DS 1 = [3.1 2.1 1.1] × [1.1 1.2 1.3]
Ø Ø

DS 2 = [3.1 2.1 1.2] × [2.1 1.2 1.3]
Ø Ø

DS 3 = [3.1 2.1 1.3] × [3.1 1.2 1.3]
Ø Ø

DS 4 = [3.1 2.2 1.2] × [2.1 2.2 1.3]
Ø Ø Ø Ø

DS 5 = [3.1 2.2 1.3] × [3.1 2.2 1.3]
Ø Ø Ø Ø

DS 6 = [3.1 2.3 1.3] × [3.1 3.2 1.3]

Ø Ø Ø Ø

DS 7 = [3.2 2.2 1.2] × [2.1 2.2 2.3]

Ø Ø

DS 8 = [3.2 2.2 1.3] × [3.1 2.2 2.3]

Ø Ø

DS 9 = [3.2 2.3 1.3] × [3.1 3.2 2.3]

Ø Ø

DS 10 = [3.3 2.3 1.3] × [3.1 3.2 3.3]

Ø Ø Ø Ø

Bemerkenswerter- – und bislang auch unerklärterweise – weisen also nur 6 der 10 peirceschen Dualsysteme solche Einbettungskoinzidenzen auf.

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Toth, Alfred, Logische Einbettungen. In: Electronic Journal for Mathematical Semiotics, 2014b

Ränder eingebetteter Systeme und Umgebungen

1. Definiert man die selbsteinbettenden Definitionen des allgemeinen Systems und der allgemeinen Umgebung

$S^* = [S, U]$

$U^* = [U, S]$

durch den in Toth (2014a, b) eingeführten Einbettungsoperator E, dann bekommt man das Quadrupel

$S^* = [S, [U]]$ $\quad$ $S^* = [[U], S]$

$S^* = [U, [S]]$ $\quad$ $S^* = [[S], U]$.

2. Da Ränder ursprünglich durch das weitere Quadrupel

$S_1^{**} = [S, R[S, U], U]$

$S_2^{**} = [S, R[U, S], U]$

$U_1^{**} = [U, R[U, S], S]$

$U_2^{**} = [U, R[S, U], S]$

definiert wurden, muß dieses durch den Einbettungsoperator ebenfalls redefiniert werden. Durch Anwendung von E erhält man

$S_1^{**} = [S, R[S, [U]], [U]]$

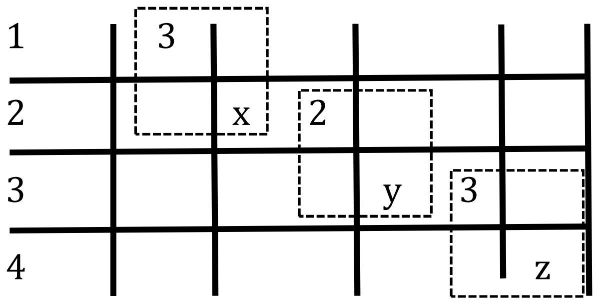
$S_2^{**} = [[S], R[U, [S]], U]$

$U_1^{**} = [U, R[U, [S]], [S]]$

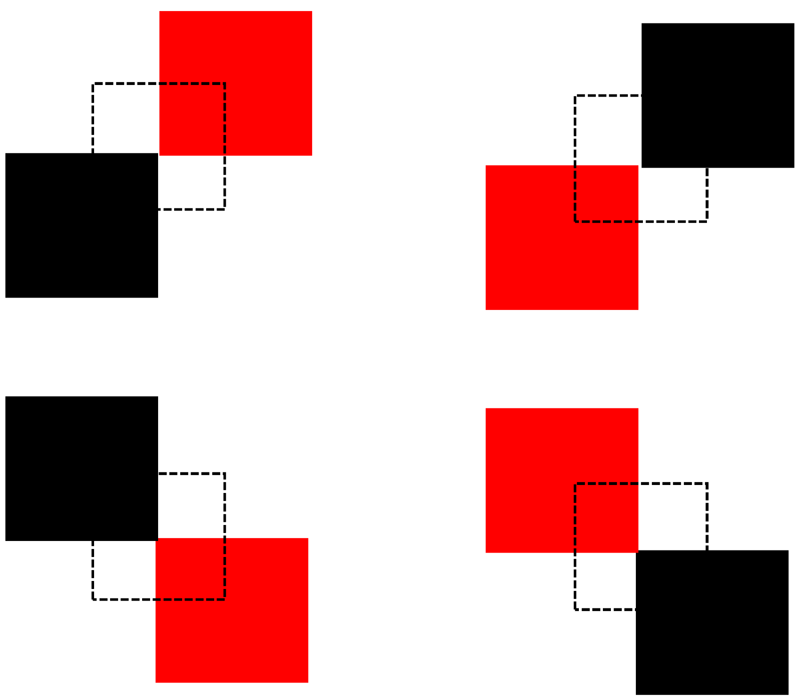
$U_2^{**} = [[U], R[S, [U]], S]$.

3. Wendet man diese Ränder eingebetteter Systeme und Umgebung auf die semiotischen Einbettunsstufen (vgl. Toth 2014c) an, so erhält man als Ränder die drei im folgenden Schema markierten triadisch-trichotomischen Bereiche

$Z = [3.x, 2.y, 1.z]$



4. In der Ontik werden solche Ränder durch Objekte wie z.B. Stufen, Treppen, Leitern, Brücken, usw. bewerkstelligt, vgl. die allgemeinen Schemata aus Toth (2014d)



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Toth, Alfred, Ontische Einbettungen. In: Electronic Journal for Mathematical Semiotics, 2014b

Ränder und Permutationen

1. Während eine geordnete Menge von 2 Elementen, wie z.B.

$S = <a, b>$

durch Anwendung des in Toth (2014a) eingeführten Einbettungsoperators auf 2 mal 2 mögliche Strukturen

$S_1 = [a, [b]]$ $\quad$ $S_2 = [[b], a]$

$S_3 = [[a], b]$ $\quad$ $S_4 = [b, [a]]$

abgebildet werden kann, kann eine Menge von 3 Elementen wie z.B.

$T = <a, b, c>$

natürlich zunächst auf 2 mal 3 mögliche Einbettunsgstrukturen abgebildet werden

$T_1 = [[a], b, c]$ $\quad$ $T_2 = [c, b, [a]]$

$T_3 = [a, [b], c]$ $\quad$ $T_4 = [c, [b], a]$

$T_5 = [a, b, [c]]$ $\quad$ $T_6 = [[c], b, a]$,

allerdings nur dann, wenn nicht alle 3 Elemente paarweise in einer hierarchischen Austauschrelationen stehen. Es ist daher leicht zu sehen, daß das obigen Sextupel der Einführung von E widerspricht, weil T_1 ... T_6 gemischte hierarchisch-heterarchische Austauschrelationen aufweisen.

2. Stattdessen müssen wir also ausgehen von

$T_1 = [[[a], b], c]$ $\quad$ $T_2 = [c, [b, [a]]]$

und erhalten weitere Einbettungsstrukturen ohne Bildung zusätzlicher Ränder nur durch Permutation solcher "verschachtelter" hierarchischer Relationen, d.h. wir bekommen für geordnete Tripel 3! = 6 Paare von Einbettungsstrukturen, neben dem soeben hingeschriebenen Paar also noch die folgenden weiteren fünf Paare

$T_3 = [[[a], c], b]$ $T_4 = [b, [c, [a]]]$

$T_5 = [[[b], a], c]$ $T_6 = [c, [a, [a]]]$

$T_7 = [[[b], c], a]$ $T_8 = [a, [c, [b]]]$

$T_9 = [[[c], a], b]$ $T_{10} = [b, [a, [c]]]$

$T_{11} = [[[c], b], a]$ $T_{12} = [a, [b, [c]]]$.

Es besteht also offenbar eine Beziehung zwischen n-tupeln, durch E erzeugten hierarchischen Austauschrelationen und Permutationen der Elemente von Mengen, insofern man durch Anwendung von E genau die doppelte Anzahl von Strukturen von n-tupeln erhält. (Man kann leicht beweisen, daß dies ein Satz der Arithmetik ist.)

3. Es dürfte dem Lesenden nicht entgangen sein, daß unsere Menge T, notiert in der Ordnung

$T = [a, [b, [c]]]$,

der Ordnungsstruktur der von Bense (1979, S. 53) definierten Zeichenrelation isomorph ist, die wir in der Form

$Z = [M, [O, [I]]]$

schreiben können. Da nun die M, O und I vermöge der von Bense (1975, S. 101) eingeführten semiotischen Matrix qua kartesische Produkte als semiotische Subrelationen definiert sind, stellen also M, O und I jeweils wiederum dyadische Strukturen der Form $S = \langle a.b \rangle$ mit $E(S) = \{[[a], b], [b, [a]], [[b], a], [a, [b]]\}$ dar (vgl. Toth 2014b), d.h. a, b und c im verdoppelten Sextupel permutierter Ränder in unserem obigen Schema können in jeder dieser 12 Strukturen wiederum auf 4-fache Weise als Sub-Randrelationen erscheinen. Das bedeutet also, daß man im folgenden verdoppelten Sextupel

$Z_1 = [[[I], O], M]$ $Z_2 = [M, [O, [I]]]$

$Z_3 = [[[I], M], O]$ $Z_4 = [O, [M, [I]]]$

$Z_5 = [[[O], I], M]$ $Z_6 = [M, [I, [O]]]$

Z_7 = [[[O], M], I] Z_8 = [I, [M, [O]]]

Z_9 = [[[M], I], O] Z_{10} = [O, [I, [M]]]

Z_{11} = [[[M], O], I] Z_{12} = [I, [O, [M]]].

jeweils gemäß den Abbildungen

α: $(M \rightarrow O)$

β: $(O \rightarrow I)$

α°β°: $(I \rightarrow M)$

folgende Quadrupel von Sub-Randrelationen einsetzen kann

[M, [O]]	[[O], M]	[O, [I]]	[[I], O]
[[M], O]	[O, [M]]	[[O], I]	[I, [O]]

[I, [M]]	[[M], I]
[[I], M]	[M, [I]].

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n-adizität und Einbettungsstufe von Zeichenrelationen

1. In Toth (2014a) war gezeigt worden, daß die peirce-bensesche 3-adische Zeichenrelation

Z = [3.a, 2.b, 1.c]

qua Anwendung des Einbettungsoperators E(Z) auf alle Subrelationen zu einem 4-stufigen Einbettungsschema

1	3			
2		x 2		
3			y 3	
4				z

führt, d.h. daß n-adizität dieser Relation und Einbettungsgrad ihrer Subrelationen nicht übereinstimmen.

2. Greifen wir daher auf die in Toth (2014b) vorgeführte Erweiterung von Z zurück. Dort war argumentiert worden, daß der Interpretantenbezug in Z natürlich nur das logische Ich-Subjekt repräsentieren kann, da Z auf der 2-wertigen aristotelischen Logik basiert, die eben nur dieses Subjekt kennt. Somit muß bereits in dem von Bense (1971, S. 39 ff.) eingeführten semiotischen Kommunikationsschema der Objektbezug nicht nur das kommunikative Referenzobjekt, d.h. die Nachricht, sondern auch das Du-Subjekt des Senders repräsentieren, denn der Empfänger wird mehr oder weniger arbiträr durch den Interpretantenbezug repräsentiert, und dieser kann ohne Verletzung des logischen Drittensatzes nicht zwei deiktisch geschiedene Subjekte repräsentieren. Ferner war gezeigt worden, daß eine Ich-Du-Deixis ohne Er-Deixis unvollständig ist. Diese aus der Linguistik längst bekannte Unterscheidung zwischen sprechender, angesprochener und besprochener Person wird jedoch bislang weder von der Logik noch von der Semiotik reflektiert. Führen wir also neben dem Ich-Interpretanten noch den Du- und den Er-Interpretanten in Z ein, so erhalten wir

Z* = [5.a, 4.b, 3.c, 2.d, 1.e]

1	5					
2		a 4				
3			b 3			
4				c 2		
5					d 1	
6						e

Da Z* die minimale Zeichenrelation mit vollständiger Repräsentation aller drei logischen Subjekte darstellt, stellt also auch das 6-stufige Einbettungsschema das minimale Einbettungsschema einer subjektdeiktisch vollständigen Zeichenrelation dar. Daraus erhalten wir das als semiotischen Satz formulierbare Ergebnis, daß die Anzahl von Einbettungsstufen einer n-adischen Zeichenrelation stets (n+1) Stufen umfaßt.

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Austauschrelationen und Einbettungsrelationen

1. Ich möchte im Anschluß an Toth (2014a) nochmals auf den folgenden Paragraphen im "Tractatus" Wittgensteins (5.5352) zurückkommen: "Ebenso wollte man 'Es gibt keine Dinge' ausdrücken durch '$\neg(\exists x). x = x$'. Aber selbst wenn dies ein Satz wäre, - wäre er nicht auch wahr, wenn es zwar 'Dinge gäbe', aber diese nicht mit sich selbst identisch wären?"

2. Ein Fundamentaldefekt der Semiotik (vgl. bereits Toth 2014b) besteht darin, daß zwar die semiotische Dichotomie von Objekt und Zeichen

$A = [O, Z]$

der logischen von Position und Negation

$L = [P, N]$

folgt, daß diese Austauschrelation A aber weder für die Subzeichen

$S = <a.b>$

noch für die aus ihnen konkatenierten Zeichenklassen gilt (vgl. Bense 1979, S. 53)

$Zkl = <M, <<M, O>, <M, O, I>>>$,

denn sowohl bei S als auch bei Zkl handelt es sich im Gegensatz zu $A \cong L$ nicht um Austausch-, sondern um Einbettungsrelationen. Wir haben damit also für $S = <a.b>$

$S_1 = [a, [b]] \qquad S_2 = [[b], a]$

$S_3 = [[a], b] \qquad S_4 = [b, [a]]$,

und für Zkl gilt somit

$Zkl = [M, [[M, [O]], [M, [[O], [I]]]]]$.

3. Die Frage, die sich nun stellt, ist allerdings die: Gelten wirklich nur für die Semiotik, nicht aber für die Logik hierarchische Einbettungsrelationen anstatt heterarchischer Austauschrelationen? Setzt man nämlich für die S_i die beiden

logischen Wahrheitswerte W und F ein, bekommt man ebenfalls ein Quadrupel logischer Strukturen

$L_1 = [W, [F]]$ $L_2 = [[F], W]$

$L_3 = [[W], F]$ $L_4 = [F, [W]]$.

Abwegig ist diese Idee keineswegs, denn bekanntlich ist in L = [P, N] nur die Position designiert – und zwar durch W -, während die Negation als durch F nicht-designiert erscheint. Günther sprach daher von einem "Reflexionsgefälle" zwischen designierten und nicht-designierten logischen Werten. Das Problem besteht allerdings darin, daß wir in den L_i zwar immer noch zwei Werte haben, daß aber der in Toth (2014c) eingeführte Einbettungsoperator qua Einbettung quasi ein relationales Tertium einführt, das nicht nur die Über- bzw. Unterordnung zwischen W und F, sondern auch deren lineare Ordnung erwirkt, denn die L_i stehen selbstverständlich paarweise in Ungleichheitsrelation, und kein L_i ist isomorph mit den Relationen [W, F] oder [F, W]. Wir stehen somit vor den Grundlagen einer völlig neuen Logik, die weder einen materialen dritten Wert annimmt noch ein polykontexturales Verbundsystem 2-wertiger Logiken wie die Günther-Logik (vgl. Günther 1976-80) darstellt. Eine solche Logik wäre qua ($L_i \cong S_i$) mit der Semiotik isomorph. Ferner stehen die bereits in früheren Arbeiten vorgeschlagenen Definitionen durch Selbsteinbettung

$Z^* = [Z, \Omega]$

$\Omega^* = [\Omega, Z]$

nun nicht mehr im Widerspruch mit den Definitionen von S und von Zkl, denn es ist

$Z^*_1 = [Z, [\Omega]]$ $Z^*_2 = [[\Omega], Z]$

$Z^*_3 = [\Omega, [Z]]$ $Z^*_4 = [[Z], \Omega]$.

$\Omega^*_1 = [\Omega, [Z]]$ $\Omega^*_2 = [[Z], \Omega]$

$\Omega^*_3 = [Z, [\Omega]]$ $\Omega^*_4 = [[\Omega], Z]$

mit

$Z^*_1 = \Omega^*_3$

$Z^*_2 = \Omega^*_4$

$Z^*_3 = \Omega^*_1$

$Z^*_4 = \Omega^*_2$.

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Das Objekt als Grenze des semiotischen Universums

1. Nach Toth (2014a) kann man die Tatsache, daß sich Zeichen und Objekt wechselseitig transzendent sind, durch die beiden Austauschrelationen

$A = [Z, \Omega]$

$A^{-1} = [\Omega, Z]$

ausdrücken, die man genauso wenig zur Deckung bringen kann wie z.B. im dreidimensionalen Raum die rechte und die linke Hand. Während aber bei Objekten eine zusätzliche Raumdimension genügt, um Chiralität zu überwinden, erfordert die Aufhebung der Kontexturgrenze zwischen Zeichen und Objekt, wie besonders Kronthaler (1992) gezeigt hatte, die Aufgabe der Grundgesetze des Denkens, welche das Fundament der 2-wertigen aristotelischen Logik bilden, in Sonderheit des logischen Drittensatzes. Somit hat in einer Semiotik, die auf der aristotelischen Logik beruht, das Objekt genauso keinen Platz wie das Zeichen in einer 2-wertigen aristotelischen Ontik keinen Platz hat. Das Objekt bildet somit eine Grenze des semiotischen Universums und das Zeichen bildet somit eine Grenze des ontischen Universums.

2. Allerdings kann man, wie in Toth (2014b) gezeigt, Zeichen und Objekt so in funktionale Abhängigkeit voneinander setzen, daß sie nicht mehr, wie in A und in A^{-1} , einander koordiniert, sondern einander sub- bzw. superordiniert sind. Durch Anwendung eines Einbettungsoperators erhält man aus A und A^{-1} das folgende Quadrupel von Einbettungsrelationen von Z und von Ω

$A_1 = [Z, [\Omega]]$ $\qquad A_1^{-1} = [[\Omega], Z]$

$A_2 = [\Omega, [A]]$ $\qquad A_2^{-1} = [[A], \Omega]$,

d.h. der Einbettungsoperator erwirkt zwar kein Tertium in der Form eines dritten, neben Z und Ω besehenden "Wertes", aber ein relationales Tertium, indem er die zwei Austauschrelationen A und A^{-1} in die vier Einbettungsrelationen A_1, A_1^{-1}, A_2 und A_2^{-1} transformiert. Sowohl Objekt als auch Zeichen, die sich zueinander wie These und Antithese verhalten, gehören somit nun einem System an, das wie eine Synthese sie beide enthält und die man abgekürzt durch

$Z^* = [Z, \Omega]$

$\Omega^* = [\Omega, Z]$

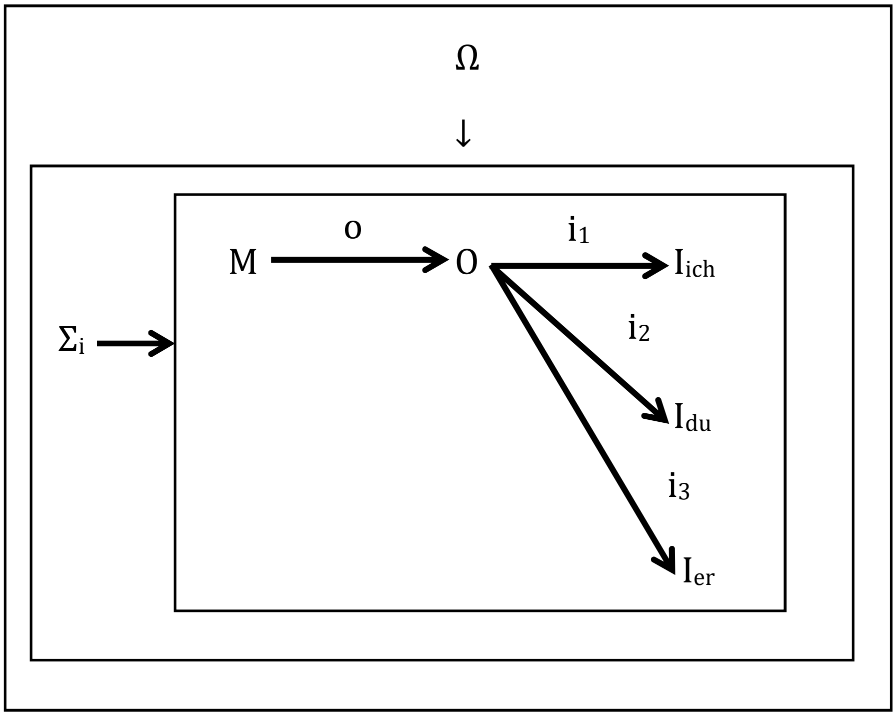
definieren kann. Dadurch verwandelt sich also der semiotische Automat, der der peirceschen Zeichendefinition korrespondiert (vgl. Bense 1975, S. 42 f.)

$$\Omega \qquad \boxed{\quad M \xrightarrow{\;o\;} O \xrightarrow{\;i\;} I \quad}$$

in einen semiotischen Automaten der Form

$$\boxed{\begin{array}{c} \Omega \\ \downarrow \\ \boxed{\quad M \xrightarrow{\;o\;} O \xrightarrow{\;i\;} I \quad} \end{array}}$$

3. Man beachte, daß diese Transformation vermöge Toth (2014a) unabhängig von der Existenz eines oder mehrerer Beobachter-Subjekte ist, denn die Relation zwischen Beobachtersubjekt und semiotischem Universum ist keineswegs transzendent, in Sonderheit verläuft also keine Kontexturgrenze zwischen beiden, denn das Beobachtersubjekt kann bei vollständiger Ich-Du-Er-Deixis auch nur wiederum ein Er-deiktisches sein. Deswegen ist es möglich, das Beobachtersubjekt ins semiotische Universum einzuschließen und ein weiteres beobachtetes System zu konstruieren, usw. Wir haben somit folgendes Modelle für ein kybernetisches semiotisches System 1. Ordnung (analog dazu für Systeme 2. Ordnung, vgl. Toth 2014a).



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Logische Designation und Einbettung

1. Die aristotelische Logik beruht auf vier sog. Grundgesetzen des Denkens – zu den folgenden drei, welche die 2-Wertigkeit dieser Logik verbürgen, kommt noch der Satz vom Grunde, der dazu dient, Zirkelschlüsse auszuschließen und für unser Anliegen im folgenden ohne Belang ist.

Satz der Identität: $p \equiv p$.

Satz vom Widerspruch: $\neg (p \wedge \neg p)$

Satz vom ausgeschlossenen Dritten: $p \vee \neg p$.

2. Man kann somit die 2-wertige aristotelische Logik als System der folgenden Form notieren (mit P = Position und N = Negation)

$L = [P, N]$

notieren. Nun gilt jedoch für P und N: "Beide Werte einer solchen Logik aber sind metaphysisch äquivalent. Das heißt, man kann sie beliebig miteinander vertauschen. Sie verhalten sich zueinander in einer totalen logischen Disjunktion, wie rechts und links. Es gibt keinen theoretischen Grund, welche Seite rechts und welche Seite links von der Zugspitze ist. Die Benennung beruht auf einer willkürlichen Entscheidung, und wenn man seinen Standpunkt wechselt, sind die rechte und die linke Seite miteinander vertauscht" (Günther 2000, S. 230 f.). Diese Ansicht wird selbst von Wittgenstein vertreten: "p und $\neg$p haben entgegengesetzten Sinn, aber es entspricht ihnen eine und dieselbe Wirklichkeit" (Tractatus, 4.0621). Das bedeutet also, daß die folgende Gleichung gilt

$L = [P, N] = [N, P]$.

3. Um zu vermeiden, daß somit $L = L^{-1}$ ist, behilft sich die 2-wertige Logik einer Art von Trick: "Die klassische Tradition der Logik setzt voraus, daß eine totale Disjunktion zwischen Subjekt und Objekt, zwischen Bewußtseinsprozeß und Bewußtseinsinhalt, also zwischen Reflexion und reflexionslosem Sein existiert. Dieser Dichotomie entspricht die zweiwertige Logik, der der eine Wert als positiv und der andere als negativ betrachtet wird. Diese Unterscheidung fällt in der klassischen Theorie mit der zwischen designierendem und

designationsfreiem Wert zusammen. Der positive Wert ist immer zugleich der designierende. Und der designationsfreie Wert ist der Index der Subjektivität, die sich aus dem Bild dieses Seins ausgeschlossen hat" (Günther 1980, S. 140).

4. Die Abbildung logischer Designation auf P und logischer Nicht-Designation auf N ist somit innerhalb der aristotelischen Logik völlig willkürlich. Man kann sie allerdings formal einwandfrei definieren, und zwar mit Hilfe des in Toth (2014) eingeführten Einbettungsoperators E. Angewandt auf L, definiert dieser das "thematische Gefälle" zwischen Designation und Nicht-Designation als Einbettungsrelation.

$E_1(L) = [P, [N]]$.

$E_2(L) = [[P], N]$

Aus den linearen Ordnungen L = [P, N] und L = [N, P] sind damit zwei nicht-lineare Ordnungen entstanden. Wegen der Möglichkeit, daß P und N zusätzlich ihre Positionen vertauschen können, ist dieses Paar jedoch nicht vollständig, denn wir bekommen zusätzlich

$E_3(L) = [N, [P]]$

$E_4(L) = [[N], P]$,

d.h. die pseudo-heterarchischen[7] Ordnungen L = [P, N] und L = [N, P], die in Wahrheit wegen der Abbildung der positiven Designation auf P und der negativen Nicht-Designation auf N selbst eine Nicht-Linearität in linearem Gewande darstellt, werden nun auf ein Quadrupel der Form

$E_1(L) = [P, [N]]$ $\quad$ $E^{-1}{}_1(L) = [[N], P]$

$E_2(L) = [[P], N]$ $\quad$ $E^{-1}{}_2(L) = [N, [P]]$

[7] Günther argumentiert umgekehrt: Er geht wegen der Bijektion der Abbildung von designiertem Wert W → P und nicht-designiertem Wert F → N davon aus, daß L = [P, N] und L = [N, P] hierarchische Ordnungen sind. Tatsächlich sind sie aber rein formal gesehen natürlich heterarchisch, weil L ja als Zwillingsordnung auftritt, dies liegt wiederum daran, daß die Designationsabbildung in der aristotelischen Logik ja nicht formal eingeführt wird. Dieser Unterschied der Argumentation ist aber für die unsere völlig ohne Belang.

Damit stellt wegen $W \rightarrow P$ und $F \rightarrow N$ auch das entsprechende System der Ordnung der Wahrheitswerte $V = [W, F]$ und $V = [F, W]$ ein Quadrupel dar

$E_1(L) = [W, [F]]$ $\quad\quad$ $E^{-1}{}_1(L) = [[F], W]$

$E_2(L) = [[W], F]$ $\quad\quad$ $E^{-1}{}_2(L) = [F, [W]]$.

Wir haben somit ohne Einführung eines dritten Wertes neben W und F, d.h. ohne einen der drei die 2-wertige Logik garantierenden Grundgesetze durch Anwendung von E auf L diese Grundgesetze selbst außer Kraft gesetzt, denn in Sonderheit gelten nun zwar

$\neg\, [W, [F]] = [[F], W]$

$\neg\, [[W], F] = [F, [W]]$

aber es ist

$\neg\, [W, [F]] \neq [F, [W]]$

$\neg\, [[W], F] \neq [[F], W]$.

Da somit alle vier Einbettungsrelationen paarweise verschieden sind, sind mit dem Drittensatz auch der Identitätssatz und der Widerspruchssatz außer Kraft gesetzt. Der Einbettungsoperator E führt somit eine Art von relationalem Tertium in die 2-wertige aristotelische Logik ein, ohne ein materiales Tertium als dritten Wert zu benötigen.

5. Wir stehen somit vor einer völlig neuen Logik, denn wir haben nun eine Abbildung

e: $\quad [W, F] \rightarrow [[W, [F]], [[F], W], [[W], F], [F, [W]]]$,

und somit müssen nicht nur die monadischen, sondern auch alle höheren Wahrheitswertfunktoren, in Sonderheit also die 16 dyadischen (vgl. z.B. Menne 1991, S. 34 f.) redefiniert werden. Z.B. muß die Konjunktionsfunktion

p	q	$p \wedge q$
W	W	W
W	F	F
F	W	F
F	F	F

durch die folgende neue Konjunktionsfunktion ersetzt werden

p	q	$p \wedge q$
W	[W]	
[W]	W	
W	[F]	
[W]	F	
F	[W]	
[F]	W	
F	[F]	
[F]	F	

wobei die Werte von ($p \wedge q$) vorderhand noch völlig unklar sind, denn jede Argumentstruktur der nunmehr 8 anstatt 4 Ordnungen besagt ja zweierlei: 1. welcher der beiden Werte W und F designiert und welcher nicht-designiert ist, und 2. in welcher Ordnung designierte und nicht-designierte Werte stehen, und nur das Letztere wird durch die 4 Ordnungen der aristotelischen Wahrheitswertfunktionen ausgedrückt. Z.B ist also in den Strukturen [W, [W]] und [[W], W]] W zwar in beiden Fällen im Widerspruch zur 2-Wertigkeit der aristotelischen Logik sowohl designiert als auch nicht-designiert, aber im ersten Falle ist designiertes W dem nicht-designierten W hierarchisch über-, im zweiten Falle aber untergeordnet.

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Mesozeichenklassen

1. Der Begriff des Mesozeichens stammt von Bense und wurde von ihm innerhalb seiner Theorie einer morphogenetischen Semiotik eingeführt (vgl. Bense 1983, S. 81 ff.). Mesozeichen sollen die relationale Differenz zwischen triadisch fungierenden semiotischen und dyadischen fungierenden metasemiotischen Entitäten überbrücken (vgl. Bense 1983, S. 87). Wenn dann Bense allerdings als stellvertretend für letztere die ebenfalls dyadischen entitätischen Realitäten von Realitätsthematiken mit ihren dualen Zeichenklassen vermittelt, die Mesozeichen also lediglich als nichtleere Schnittmengen zwischen Zeichenklassen und Realitätsthematiken bestimmt, die trivialerweise gar nicht leer sein können, da beide in Dualrelationen stehen, dann läuft eine auf einem dergestalt trivialen Begriff einer "Chreode" basierende "morphogenetische Semiotik" auf ein System von Trivialitäten hinaus.

2. Im folgenden wird deshalb ein alternatives System präsentiert, das nicht nur auf Meso-Subzeichen bzw. Paaren von solchen, sondern auf triadischen Mesozeichenrelationen basiert. Zunächst werden hier für sämtliche 3! = 6 Permutationen der Menge der Zeichenzahlen P = (1, 2, 3) zugelassen, d.h. also nicht nur die kanonische Ordnung der P innerhalb von Zeichenklassen in der sog. retrosemiosischen Ordnung ($3 > 2 > 1$), die vermutlich auf einem Mißverständnis von Peirces "pragmatischer Maxime" beruht.[8] Wenn wir als allgemeine Form einer Zeichenklasse

Zkl = (3.a, 2.b, 1.c)

setzen, dann bekommen wir also zunächst 6 mögliche Dualsysteme, d.h. Zkln mitsamt ihren koordinierten Realitätsthematiken (Rthn).

[8] Vgl. dazu Benses Bemerkung: "Man sieht also, wie weit die 'pragmatische Maxime' von Peirce, deren heuristische Schlußfigur, semiotisch fixiert, rekursiv, degenerierend ist, als sie vom Interpretanten auf das Mittel, von der Drittheit auf die Erstheit zurückgreift, in der Interpretantenkonzeption der Mathematik, wie sie Curry entwickelt, wirksam ist" (1975, S. 165). Ist es nicht vielmehr so, daß hiermit lediglich die sog. Gebrauchsfunktion des Zeichens ($I \rightarrow M$) gemeint ist?

(3.a, 2.b, 1.c) × (c.1, b.2, a.3)

(3.a, 1.c, 2.b) × (b.2, c.1, a.3)

(2.b, 3.a, 1.c) × (c.1, a.3, b.2)

(2.b, 1.c, 3.a) × (a.3, c.1, b.2)

(1.c, 3.a, 2.b) × (b.2, a.3, c.1)

(1.c, 2.b, 3.a) × (a.3, b.2, c.1)

3. Nun lassen sich aber sämtliche Subrelationen von Zkl×Rth der Form

S = <a.b>

vermöge des in Toth (2014) eingeführten Einbettungsoperators

E(S) =[[a, [b]], [[b], a], [[a], b], [b, [a]]]

auf das folgende Quadrupel abbilden

S_1 = [a, [b]] S_2 = [[b], a]

S_3 = [b, [a]] S_4 = [[b], a].

Dadurch enthalten wir statt 6 permutierten semiotischen Dualsystemen nun Paaren von solchen

[3[a], 2[b], 1[c]] × [[c]1, [b]2, [a]3]]

[[3]a, [2]b, [1]c] × [c[1], b[2], a[3]]

[3[a], 1[c], 2[b]] × [[b]2, [c]1, [a]3]]

[[3]a, [1]c, [2]b] × [b[2], c[1], a[3]]

$[2[b], 3[a], 1[c]] \times [[c]1, [a]3, [b]2]]$

$[[2]b, [3]a, [1]c] \times [c[1], a[3], b[2]]$

$[2[b], 1[c], 3[a]] \times [[a]3, [c]1, [b]2]]$

$[[2]b, [1]c, [3]a] \times [a[3], c[1], b[2]]$

$[1[c], 3[a], 2[b]] \times [[b]2, [a]3, [c]1]]$

$[[1]c, [3]a, [2]b] \times [b[2], a[3], c[1]]$

$[1[c], 2[b], 3[a]] \times [[a]3, [b]2, [c]1]]$

$[[1]c, [2]b, [3]a] \times [a[3], b[2], c[1]]$

Wir haben somit ein Vermittlungssystem aus 3 Haupt- und je zwei Teil-Systemen, das für konstantes a, b und c das semiotische Dualsystem

$DS = [[3.a, 2.b, 1.c] \times [c.1, b.2, a.3]]$

zwischen seinem Vorgängersystem

$DS_V = [[3.(a\text{-}1), 2.b(\text{-}1), 1.c(\text{-}1)] \times [(c\text{-}1).1, (b\text{-}1).2, (a\text{-}1).3]]$

und seinem Nachfolgersystem

$DS_N = [[3.(a+1), 2.b(+1), 1.c(+1)] \times [(c+1).1, (b+1).2, (a+1).3]]$

auf 12-fache Weise vermittelt. Diese Vermittlung funktioniert allerdings nur dann ohne Überspringung der strukturellen Möglichkeiten von DS, wenn man sich nicht nur auf die 10 peirceschen Dualsysteme beschränkt, sondern die Gesamtmenge aller $3^3 = 27$ über P erzeugbaren Dualsysteme nimmt. Das letztere stellt denn zusammen mit seinem großen strukturellen und ordnungstheoretischen Reichtum von $12 \times 27 = 324$ Dualsystemen das vollständige relationale Organon dar, auf dem eine nicht-triviale morphogenetische Semiotik gegründet werden sollte.

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Semiotische Einbettung und Dualität

1. Wendet man den in Toth (2014a) definierten Einbettungsoperator E auf die allgemeine Form von semiotischen Subrelationen

$S = \langle x.y \rangle$ mit $x, y \in \{1, 2, 3\}$

an, so bekommt man bekanntlich (vgl. Toth 2014b) das folgende Quadrupel von S-Strukturen

$S_1 = [x, [y]]$

$S_2 = [[x], y]$

$S_3 = [y, [x]]$

$S_4 = [[y], x]$,

d.h. es ist

$\times[S_1 = [x, [y]] = [S_4 = [[y], x]]$

$\times[S_2 = [[x], y]] = [S_3 = [y, [x]]]$.

Die einfache Dualität von S

$\times S = \times\langle x.y \rangle = \langle y.x \rangle$

wird somit durch ein Paar von Dualrelationen ersetzt, so wie eine Subrelation durch ein Quadrupel von Subrelationen ersetzt wird.

2. Die Einführung von E in die Semiotik führt bekanntlich dazu, dazu es keine Dualidentität der Form

$\times(x.x) = (x.x)$

mehr gibt, denn $x = y$ wird nun nicht mehr anders behandelt als $x \neq y$, da wir haben

$\times[x, [x]] \neq [[x], x]$.

Nehmen wir als Beispiel die Subrelation (1.3). Durch Anwendung von E bekommen wir

E(1.3) = [[1, [3]], [[1], 3], [3, [1]], [[3], 1]],

d.h. es gilt nicht nur

[1, [3]] $\neq$ [[1], 3],

sondern auch

[1, [3]] $\neq$ [[3], 1],

denn genau diese beiden Ungleichungen erscheinen in der peirceschen Identitätssemiotik als Identitäten vermöge

[1, [3]] = (1.3) [3, [1]] = (3.1)

[[3], 1] = (1.3) [[1], 3] = (3.1),

d.h. aber, die 2-wertige Semiotik abstrahiert zwar nicht von der Einbettung der die Subzeichen konstituierenden Primzeichen, aber von deren Lage innerhalb der als kartesische Produkte von Primzeichen definierten geordneten Paare.

3. Diese letztere Feststellung, daß nicht nur die Einbettung, sondern die ebenfalls durch den Einbettungsoperator E bewirkte Relativierung der Lage eingebetteter Zeichenzahlen semiotisch relevant ist, bedeutet nun gleichzeitig eine Relativierung der Dualrelation von Paaren semiotischer Subrelationen, d.h. die 2-wertige Trennung der allgemeinen Form eines semiotischen Dualsystems

DS = Zkl $\times$ Rth = (3.a, 2.b, 1.c) $\times$ (c.1, b.2, a.3)

wird insofern aufgehoben, als jede Subrelation S $\subset$ Zkl auch als S $\subset$ Rth und jede Subrelation S $\subset$ Rth auch als S $\subset$ Zkl erscheinen kann, d.h. daß wir z.B. neben einem Dualsystem wie

DS = (3.1, 2.1, 1.3) $\times$ (3.1, 1.2, 1.3)

auch Dualsysteme wie die folgenden

$DS = (3.1, 1.2, 1.3) \times (3.1, 2.1, 1.3)$

$DS = (1.3, 2.1, 3.1) \times (1.3, 1.2, 3.1)$, usw.

haben können. Etwas vereinfachend ausgedrückt, besagt dies, daß sich Zeichenthematik und Realitätsthematik gegenseitig durchdringen können, d.h. aber, daß die die Subjektposition repräsentierende Zeichenthematik und die die Objektposition repräsentierende Realitätsthematik relativ zur Subjekt-Objekt-Dichotomie und damit diese Dichotomie selbst, als semiotisch vermittelte, relativiert wird. Von hier aus führt somit ein direkter Weg zu der von Günther bereits früh vorgeschlagenen 4-fachen Subkategorisierung der 2-wertigen logisch-erkenntnistheoretischen Opposition von Objekt und Subjekt in subjektives und objektives Objekt sowie in objektives und subjektives Subjekt (vgl. Günther 1976, S. 336 ff.).

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Semiomorphogenetische Homöostase

1. Jede semiotische Subrelation der Form

S = <a.b>

läßt sich vermöge des in Toth (2014a) eingeführten Einbettungsoperators

E(S) =[[a, [b]], [[b], a], [[a], b], [b, [a]]]

auf das folgende Quadrupel abbilden

S_1 = [a, [b]]　　　S_2 = [[b], a]

S_3 = [b, [a]]　　　S_4 = [[b], a].

Dadurch enthalten wir statt 3! = 6 aus dem einen semiotischen Dualsystem der Form

DS = (3.a, 2.b, 1.c) × (c.1, b.2, a.3)

permutierbaren semiotischen Dualsystemen nun Paare von solchen

[3[a], 2[b], 1[c]] × [[c]1, [b]2, [a]3]]

[[3]a, [2]b, [1]c] × [c[1], b[2], a[3]]

[3[a], 1[c], 2[b]] × [[b]2, [c]1, [a]3]]

[[3]a, [1]c, [2]b] × [b[2], c[1], a[3]]

[2[b], 3[a], 1[c]] × [[c]1, [a]3, [b]2]]

[[2]b, [3]a, [1]c] × [c[1], a[3], b[2]]

[2[b], 1[c], 3[a]] × [[a]3, [c]1, [b]2]]

[[2]b, [1]c, [3]a] × [a[3], c[1], b[2]]

$$[1[c], 3[a], 2[b]] \times [[b]2, [a]3, [c]1]]$$

$$[[1]c, [3]a, [2]b] \times [b[2], a[3], c[1]]$$

$$[1[c], 2[b], 3[a]] \times [[a]3, [b]2, [c]1]]$$

$$[[1]c, [2]b, [3]a] \times [a[3], b[2], c[1]]$$

Wir haben somit ein 12-teiliges Vermittlungssystem aus 3 Haupt- und je 2 Teil-Systemen, die für konstantes a, b und c das semiotische Dualsystem

$$DS = [[3.a, 2.b, 1.c] \times [c.1, b.2, a.3]]$$

zwischen seinem Vorgängersystem

$$DS_V = [[3.(a\text{-}1), 2.b(\text{-}1), 1.c(\text{-}1)] \times [(c\text{-}1).1, (b\text{-}1).2, (a\text{-}1).3]]$$

und seinem Nachfolgersystem

$$DS_N = [[3.(a+1), 2.b(+1), 1.c(+1)] \times [(c+1).1, (b+1).2, (a+1).3]]$$

auf 12-fache Weise vermitteln (vgl. Toth 2014b, c). Neben dem hierdurch generierten enormen ordnungstheoretischen Strukturreichtum der Semiotik, der vom Standpunkt der peirce-benseschen Identitätssemiotik aus nicht einmal erahnbar ist, dürfte die ebenfalls hierdurch entdeckte Homömostatik solcher Mesozeichen-Dualsysteme eine der bedeutendsten semiotischen Errungenschaften sein, welche durch die Einführung von Einbettungsoperatoren ermöglicht wird.

[3[a],	2[b],	1[c]] × [	[c]1,	[b]2,	[a]3]]
[[3]a,	[2]b,	[1]c] × [	c[1],	b[2],	a[3]]
[3[a],	1[c],	2[b]] × [	[b]2,	[c]1,	[a]3]]
[[3]a,	[1]c,	[2]b] × [	b[2],	c[1],	a[3]]
[2[b],	3[a],	1[c]] × [	[c]1,	[a]3,	[b]2]]
[[2]b,	[3]a,	[1]c] × [	c[1],	a[3],	b[2]]
[2[b],	1[c],	3[a]] × [	[a]3,	[c]1,	[b]2]]
[[2]b,	[1]c,	[3]a] × [	a[3],	c[1],	b[2]]
[1[c],	2[b],	3[a]] × [	[a]3,	[b]2,	[c]1]]
[[1]c,	[2]b,	[3]a] × [	a[3],	b[2],	c[1]]
[1[c],	3[a],	2[b]] × [	[b]2,	[a]3,	[c]1]]
[[1]c,	[3]a,	[2]b] × [	b[2],	a[3],	c[1]]

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Eigenreale und kategorienreale Homöostase

1. Im Anschluß an Toth (2014a-d) verstehen wir unter semiotischer Homöostase die Selbstregulierung semiotisch-morphogenetischer Systeme (vgl. Bense 1983, S. 81 ff., Toth 2007). In der peirce-benseschen Identitätssemiotik gibt es eine solche Homöostase nur kraft dem als dualidentisch bestimmten sog. eigenrealen Dualsystem in Form des "determinantensymmetrischen Dualitätssystems", das wir in der Form, wie es in Bense (1992, S. 76) erscheint, hier wiedergeben.

Zkl			Rth			Rpw	
3.1	2.1	1.1	1.1	1.2	1.3	9	
3.1	2.1	1.2	2.1	1.2	1.3	10	Mittel
3.1	2.1	1.3	3.1	1.2	1.3	11	
3.1	2.2	1.2	2.1	2.2	1.3	11	
3.2	2.2	1.2	2.1	2.2	2.3	12	Objekt
3.2	2.2	1.3	3.1	2.2	2.3	13	
3.1	2.3	1.3	3.1	3.2	1.3	13	
3.2	2.3	1.3	3.1	3.2	2.3	14	Interpretant
3.3	2.3	1.3	3.1	3.2	3.3	15	
3.1	2.2	1.3	3.1	2.2	1.3	12	Eigenrealität

In Sonderheit gibt es also in einer 2-wertigen Semiotik kein dem eigenrealen korrespondierendes "kategorienreales" Dualitätssystem, auch wenn Bense die Kategorienrealität als "Eigenrealität schwächerer Repräsentation" (1992, S. 40) bezeichnet hatte. Führt man jedoch den Einbettungsoperator E in die Semiotik ein und wendet ihn auf ein Subzeichen der allgemeinen Form

S = <x.y>

an, so ergibt

E(S) =[[a, [b]], [[b], a], [[a], b], [b, [a]]], d.h. wir bekommen das folgende Quadrupel

$S_1 = [a, [b]]$ $S_2 = [[b], a]$

$S_3 = [b, [a]]$ $S_4 = [[b], a]$.

E verändert somit nicht nur den Einbettungsgrad jeder Zeichenzahl, sondern auch deren Position innerhalb der geordneten Paare. Darauf folgt in Sonderheit die Aufhebung des die logische 2-Wertigkeit garantierenden Identitätssatzes für die Semiotik vermöge

$\times <x.x> \neq [[x, [x]], [[x], x]]$,

d.h. also nicht nur, daß die sog. genuinen, bislang als identitive Morphismen aufgefaßten Subrelationen der semiotischen Hauptdiagonalen nicht mehr selbst-identisch sind, sondern daß auch die die semiotische Nebendiagonale bildende Zeichenklasse bzw. Realitätsthematik nicht mehr eigenreal ist. Im folgenden soll jedoch gezeigt werden, daß das in Toth (2014d) eingeführte 12-Tupel semiomorphogenetischer Dualitätssysteme eine neue Form von Identität, nun allerdings von morphogenetisch vermittelter Identität, in die Semiotik bringt, und das (nicht ohne weiteres zu erwartende) wichtigste Ergebnis besteht darin, daß diese Systeme vermittelter Teilsysteme nicht nur eine neue Form von eigenrealer, sondern auch von kategorienrealer Homöostase erzeugen.

2.1. Eigenreale Homöostase

[3[1],	2[2],	1[3]] × [[3]1,	[2]2,	[1]3]]
[[3]1,	[2]2,	[1]3] × [3[1],	2[2],	1[3]]
[3[1],	1[3],	2[2]] × [[2]2,	[3]1,	[1]3]]
[[3]1,	[1]3,	[2]2] × [2[2],	1[3],	1[3]]
[2[2],	3[1],	1[3]] × [[3]1,	[1]3,	[2]2]]
[[2]2,	[3]1,	[1]3] × [1[3],	1[3],	2[2]]
[2[2],	1[3],	3[1]] × [[1]3,	[3]1,	[2]2]]
[[2]2,	[1]3,	[3]1] × [1[3],	3[1],	2[2]]
[1[3],	2[2],	3[1]] × [[1]3,	[2]2,	[3]1]]
[[1]3,	[2]2,	[3]1] × [1[3],	2[2],	3[1]]
[1[3],	3[1],	2[2]] × [[2]2,	[1]3,	[3]1]]
[[1]3,	[3]1,	[2]2] × [2[2],	1[3],	3[1]]

2.2. Kategorienreale Homöostase

[3[3],	2[2],	1[1]] × [[1]1,	[2]2,	[3]3]]
[[3]3,	[2]2,	[1]1] × [1[1],	2[2],	3[3]]
[3[3],	1[1],	2[2]] × [[2]2,	[1]1,	[3]3]]
[[3]3,	[1]1,	[2]2] × [2[2],	1[1],	3[3]]
[2[2],	3[3],	1[1]] × [[1]1,	[3]3,	[2]2]]
[[2]2,	[3]3,	[1]1] × [1[1],	3[3],	2[2]]
[2[2],	1[1],	3[3]] × [[3]3,	[1]1,	[2]2]]
[[2]2,	[1]1,	[3]3] × [3[3],	1[1],	2[2]]
[1[1],	2[2],	3[3]] × [[3]3,	[2]2,	[1]1]]
[[1]3,	[2]2,	[3]3] × [3[3],	2[2],	1[1]]
[1[1],	3[3],	2[2]] × [[2]2,	[3]3,	[1]1]]
[[1]1,	[3]3,	[2]2] × [2[2],	3[3],	1[1]]

Man beachte, daß das System der kategorienrealen Homöostase irreduzibel ist, obwohl sich Quadrupel für den Fall, daß x = y ist, natürlich im Prinzip auf Paare reduzieren lassen. Was die Reduktion des obigen Systems jedoch verhindert, ist wiederum die relative Position von Teildualitäten innerhalb der Teilsysteme des Gesamtsystems. Dieser Sachverhalt führt nun dazu, daß die homöostatischen Strukturen von Eigen- und Kategorienrealität unter Beseitigung identitätssemiotischer 2-Wertigkeit sogar identisch sind. Anders gesagt: Eine Rückabbildung der beiden homöostatisch-morphogenetischen semiotischen Systeme auf die Identitätssemiotik schließt eine gemeinsame Homöostase von Eigen- und Kategorienrealität wieder aus. Benses Vermutung, es könnte sich bei der Kategorienrealität um eine abgeschwächte Form von Eigenrealität handeln, bewahrt also ihre Gültigkeit nach Aufhebung des Identitässatzes nicht nur, sondern bestätigt diese Vermutung sogar in überraschender Weise. Man könnte abschließend folgende Vermutung aufstellen: Während die eigenreale Homöostase den semiotischen Zureichenden Grund für das bensesche semiotische Universum (vgl. Bense 1983) repräsentiert, repräsentiert die kategorienreale Homöostase das semiotische Universum selbst, d.h. beide Homöostasen zusammen, die ja außerdem strukturell nicht nur isomorph, sondern identisch sind, etablieren das System der Theoretischen Semiotik im Sinne eines im modelltheoretischen Sinne abgeschlossenen Universums.

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Eigenrealität mit und ohne Transzendenz

1. In gewissem Sinne als Zusammenfassung – oder mindestens als Zwischenbilanz – seines letzten semiotischen Buches kann man den folgenden Passus Benses über die semiotische Eigenrealität verstehen: "Wenn nun das kosmologische Sein, das Universum im Sinne eines verknüpften 'unteilbaren Seins', als ein einseitiges (im Prinzip als stets zusammenhängendes oder wenigstens verknüpfbares) Sein aufgefaßt werden muß, dann kann es auch nur als Eigenrealität ohne Transzendenz repräsentierbar sein und als System der triadisch-kategorialen Realitäten-Relationen ontologisch existent und unserem rational funktionierenden Bewußtsein in produzierbaren triadisch geordneten Zeichen, Zahlen und ästhetischen Zuständen zugänglich werden" (Bense 1992, S. 51).

2. Die peirce-bensesche Semiotik ist konzipiert als ein semiotisches "Universum" (Bense 1983) im Sinne von modelltheoretischer Abgeschlossenheit, d.h. es enthält in Sonderheit nur die ihre Objekte bezeichnenden Zeichen, aber nicht die Objekte selbst. Diese Konzeption steht allerdings in Widerspruch zu Benses eigener Definition des Zeichens als "Metaobjekt" (Bense 1967, S. 9). Objekte bilden danach die Domänen von Abbildungen, deren Codomänen die Zeichen sind. Gehört also die "Zuordnung" (ibd.) von Zeichen zu Objekten zur Semiotik, so müssen auch die Domänen der Abbildung zur Semiotik gehören. Nun ist aber die Semiotik – wie sämtliche übrigen Wissenschaften – auf die zweiwertige aristotelische Logik gegründet, und somit ist die Dichotomie von Objekt und Zeichen isomorph derjenigen der logischen Position und Negation. Das Zeichen nimmt somit die Rolle der Negativität und damit des logischen Subjektes ein, während das Objekt diejenige der Position und damit des logischen Objektes einnimmt. Kronthaler (1992) hat also sehr recht, wenn er feststellt, daß innerhalb der Identitätssemiotik Objekt und Zeichen einander "ewig transzendent" sind.

3. Allerdings ist es, wie bereits in Toth (2014a) gezeigt, möglich, mittels der Systemtheorie eine gemeinsame Basis sowohl für die Semiotik als Universum der Zeichen als auch für die Ontik als Universum der Objekte zu konstruieren, oder vielleicht besser: zu rekonstruieren. Man setzt

$$Z^* = [Z, \Omega]$$

$\Omega^* = [\Omega, Z]$.

Da $Z^* = \Omega^{*-1}$ und also auch $\Omega^* = Z^{*-1}$ ist, folgt, daß sich die Definitionen Z^* und Ω^* nur in Bezug auf den jeweiligen Einbettungsgrad von Z und Ω unterscheiden, d.h. wir haben

$Z^* = [[Z, [\Omega]], [[Z], \Omega]$

$\Omega^* = [[\Omega, [Z]], [[\Omega], Z]$.

Damit ist – beinahe unversehens – der logische Identitätssatz für die Semiotik aufgehoben, denn die neuen Definitionen von Z^* und von Ω^* unterscheiden sich nicht nur durch Einbettung bzw. Nicht-Einbettung von Z und Ω, sondern auch durch deren Ordnung innerhalb der vier geordneten Paare, die paarweise dual zueinander sind. Während also in der Identitätssemiotik Subzeichen der Form

$S = \langle x.y \rangle$ mit $x, y \in \{1, 2, 3\}$

im Falle von $x = y$ als identisch erscheinen, vgl.

$\times(1.1) = (1.1)$

$\times(2.2) = (2.2)$

$\times(3.3) = (3.3)$,

ist dies nun nicht mehr der Fall, da ja

$\times \langle x.x \rangle \neq [[x, [x]], [[x], x]]$

ist. Daraus folgt unmittelbar, daß auch die Eigenrealität, welche die Nicht-Transzendenz des semiotischen Universums garantiert und die zweiwertig durch die Dualidentität

$\times(3.1, 2.2, 1.3) = (3.1, 2.2, 1.3)$

verbürgt ist, wegen

$(3.1) \neq [[3.[1]], [[3].1], [1.[3]], [[1].3]]$

$(2.2) \neq [[2.[2]], [[2].2]]$

$(1.3) \neq [[1.[3]], [[1].3], [3.[1]], [[3].1]]$

hinfällig wird. Wir haben somit allein durch den in Toth (2014b) eingeführten Einbettungsoperator E, d.h. ohne die materielle logische Wertigkeit der aristotelischen semiotischen Basis aufzugeben, die Semiotik in ein System mit Transzendenz transformiert, denn die jeweils dualen verdoppelten Einbettungsrelationen in

$Z^* = [[Z, [\Omega]], [[Z], \Omega]$

$\Omega^* = [[\Omega, [Z]], [[\Omega], Z].$

sind ja, wie wir soeben demonstriert haben, nicht nur für die Dichotomie zwischen Objekt und Zeichen, d.h. zeichenextern, sondern auch für die paarweisen Dichotomien zwischen M, O und I, d.h. zeichenintern, gültig. Anders ausgedrückt, wird also die präsemiotische Basisdichotomie, die der logischen Dichotomie von Position und Negation isomorph ist, vermittels des Einbettungsoperators von den semiotischen Subrelation und damit von den Zeichenklassen und ihren dualen Realitätsthematiken "mitgeführt".

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Semiotische Einbettungsrelationen und Abbildungstypen

1. Im Anschluß an Toth (2014), wo wir durch Anwendung des Einbettungsoperators E auf die allgemeine Form semiotischer Subrelationen

S = [x, y]

E(S) = [[x, [y], [[y], x], [[x], y], [y, [x]]]

(mit x, y ∈ {1, 2, 3}) erhalten hatten, können wir nun sämtliche 9 semiotischen Subrelationen auf Quadrupel bzw. Paare von semiotischen Einbettungsrelationen abbilden.

(1.1) → [[1, [1]], [[1], 1]]

(1.2) → [[1, [2]], [[2], 1], [[1], 2], [2, [1]]]

(1.3) → [[1, [3]], [[3], 1], [[1], 3], [3, [1]]]

(2.1) → [[2, [1]], [[1], 2], [[2], 1], [1, [2]]]

(2.2) → [[2, [2]], [[2], 2]]

(2.3) → [[2, [3]], [[3], 2], [[2], 3], [3, [2]]]

(3.1) → [[3, [1]], [[1], 3], [[3], 1], [1, [3]]]

(3.2) → [[3, [2]], [[2], 3], [[3], 2], [2, [3]]]

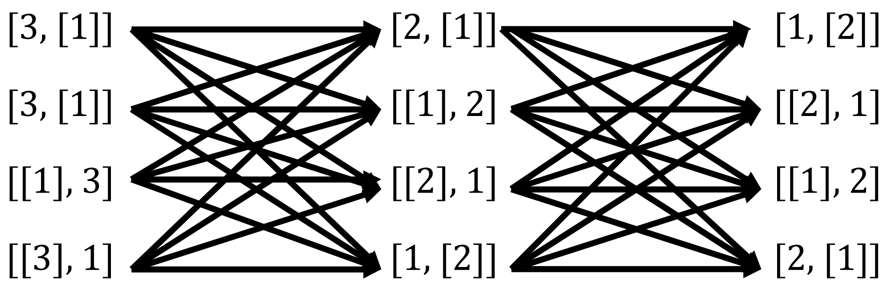
(3.3) → [[3, [3]], [[3], 3]]

Wie man sieht, haben natürlich nur die sog. genuinen Subzeichen aus der Hauptdiagonalen der semiotischen Matrix Paare statt Quadrupel als Codomänen. Von diesen erscheint (1.1) nur bei einem einzigen Dualsystem. (3.3) erscheint ferner innerhalb der die Hauptdiagonale der semiotischen Matrix bildenden Kategorienklasse.

2.1. 1. Abbildungstypus

Zu diesem Abbildungstypus gehören alle Dualsysteme, welche keine genuinen Subrelationen aufweisen.

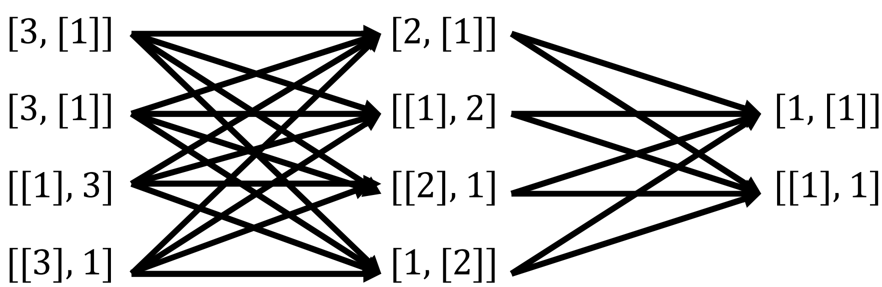
(3.1, 2.1, 1.2) × (2.1, 1.2, 1.3) →



2.2. 2. Abbildungs-Typus

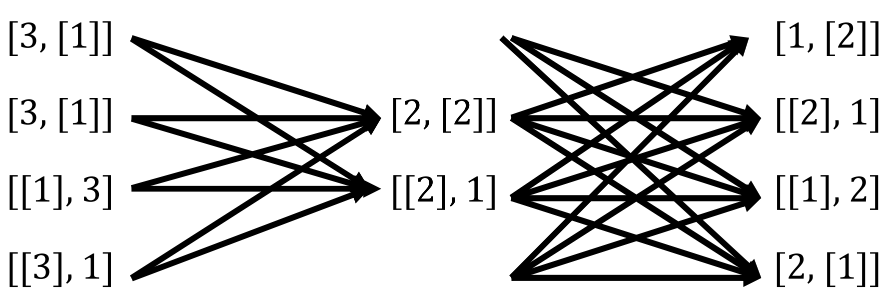
Zu diesem Abbildungstypus gehört nur das folgende Dualsystem.

(3.1, 2.1, 1.1) × (1.1, 1.2, 1.3) →



2.3. 3. Abbildungstypus

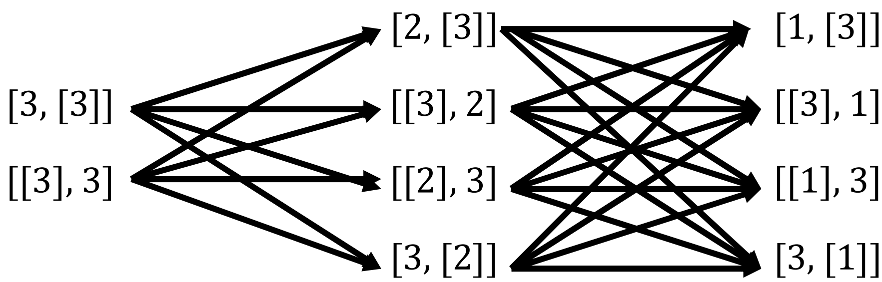
(3.1, 2.2, 1.2) × (2.1, 2.2, 1.3) →



2.4. 4. Abbildungstypus

Zu diesem Abbildungstypus gehört nur das folgende Dualsystem.

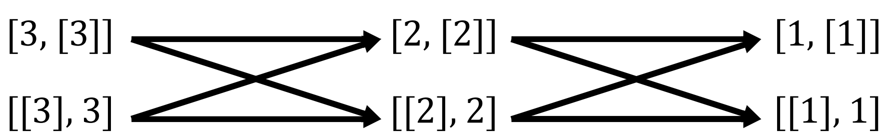
(3.3, 2.3, 1.3) × (3.1, 3.2, 3.3) →



2.5. 5. Abbildungstypus

Zu diesem Abbildungstypus gehört nur das folgende Dualsystem, das jedoch nicht zu den regulären semiotischen Dualsystemen zählt.

(3.3, 2.2, 1.1) × (1.1, 2.2, 3.3) →



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Das semiotische Dodekagon

1. Die Menge der in Toth (2014a) eingeführten Mesozeichen-Relationen, einschließlich ihrer dualen Relationen, lassen sich, wie im folgenden gezeigt wird, als drei Teilsysteme chiastischer Tetragone darstellen. Diese wiederum sind als System von Teilsystemen isomorph dem in Toth (2014b) eingeführten homöostatischen morphogenetisch-semiotischen System.

2.1. Erstes chiastisches Tetragon

[3[a], 2[b], 1[c]]	[[3]a, [2]b, [1]c]
[[c]1, [b]2, [a]3]]	[c[1], b[2], a[3]]
×	
[3[a], 1[c], 2[b]]	[[3]a, [1]c, [2]b]
[[b]2, [c]1, [a]3]]	[b[2], c[1], a[3]]

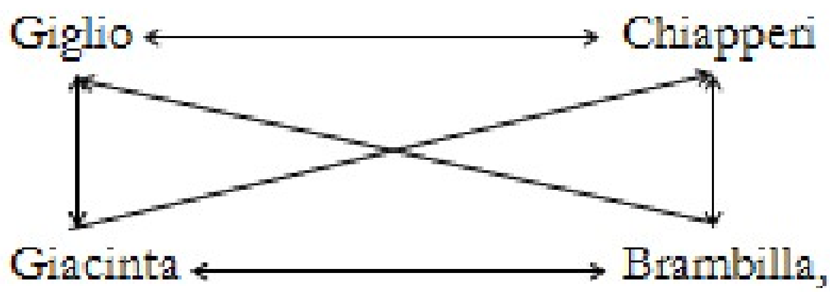
2.2. Zweites chiastisches Tetragon

[2[b], 3[a], 1[c]]	[[2]b, [3]a, [1]c]
[[c]1, [a]3, [b]2]]	[c[1], a[3], b[2]]
×	
[2[b], 1[c], 3[a]]	[[2]b, [1]c, [3]a]
[[a]3, [c]1, [b]2]]	[a[3], c[1], b[2]]

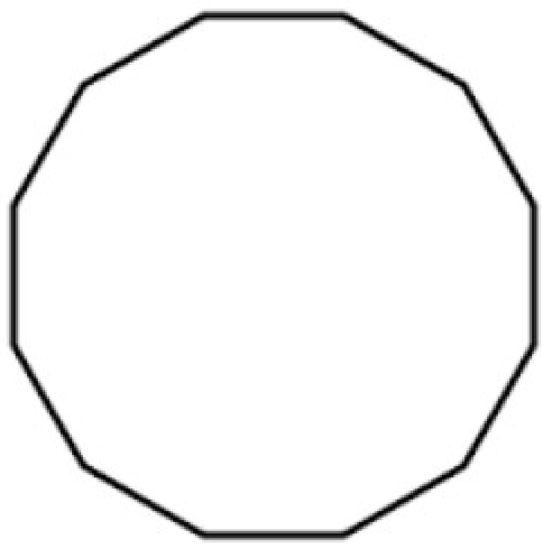
2.3. Drittes chiastisches Tetragon

[1[c], 3[a], 2[b]]	[[1]c, [3]a, [2]b]
[[b]2, [a]3, [c]1]]	[b[2], a[3], c[1]]
×	
[1[c], 2[b], 3[a]]	[[1]c, [2]b, [3]a]
[[a]3, [b]2, [c]1]]	[a[3], b[2], c[1]]

Das wohl bekannteste ciastische Tetragon ist dasjenige, das in Toth (2006) für die vier wechselweise identischen Hauptfiguren von E.T.A. Hoffmanns "Prinzessin Brambilla" bestimmte wurde.



Diese drei Tetragone können nun zu einem semiotischen Dodekagon der folgenden allgemeinen Form angeordnet werden, wobei die dualen semiotischen Relationen dia-gonal entgegengesetzte Ecken des Deodekagons einnehmen.



Den 12 Ecken des semiotischen Dodekagons korrespondieren die 24 Felder des in Toth (2014b) konstruierten homöostatischen semiomorphogenetischen Systems, das in der folgenden Darstellungsweise übrigens einen Hausdorff-Raum darstellt.

[3[a],	2[b],	1[c]] × [[c]1,	[b]2,	[a]3]]
[[3]a,	[2]b,	[1]c] × [c[1],	b[2],	a[3]]
[3[a],	1[c],	2[b]] × [[b]2,	[c]1,	[a]3]]
[[3]a,	[1]c,	[2]b] × [b[2],	c[1],	a[3]]
[2[b],	3[a],	1[c]] × [[c]1,	[a]3,	[b]2]]
[[2]b,	[3]a,	[1]c] × [c[1],	a[3],	b[2]]
[2[b],	1[c],	3[a]] × [[a]3,	[c]1,	[b]2]]
[[2]b,	[1]c,	[3]a] × [a[3],	c[1],	b[2]]
[1[c],	2[b],	3[a]] × [[a]3,	[b]2,	[c]1]]
[[1]c,	[2]b,	[3]a] × [a[3],	b[2],	c[1]]
[1[c],	3[a],	2[b]] × [[b]2,	[a]3,	[c]1]]
[[1]c,	[3]a,	[2]b] × [b[2],	a[3],	c[1]]

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Ränder und Einbettungsstufen

1. Die 2-wertige logische Basisrelation

$L = [p, \neg p]$

wird von sämtlichen Logiken – die günthersche polykontexturale Logik eingeschlossen – dahingehend interpretiert, daß zwischen den beiden Werten p und $\neg p$ eine Kontexturgrenze verläuft, d.h. daß alle Objekte, auf die p zutrifft, nicht- $\neg p$ sind und alle Objekte, auf die $\neg p$ zutrifft, nicht-p sind, daher gilt auch die doppelte Negation $\neg\neg p \equiv p$. Die drei Grundgesetze des Denkens, d.h. der Satz der Identität, der Satz des verbotenen Widerspruchs und der Satz des ausgeschlossenen Dritten, können daher paarweise durch einander definiert werden, da sie alle die gleiche logische Aussagen machen, daß nämlich $p \neq \neg p$ ist.

2. Die Frage ist jedoch, wie man diese Aussage

$p \neq \neg p$

logisch begründet. Erstens gibt es neben der Identität $\neg\neg p \equiv p$ noch eine Gleichheit, die allerdings nicht ein, sondern zwei Objekte voraussetzt, daher ist die Ungleichheit in $p \neq \neg p$ offenbar eine Negation von Identität und nicht von Gleichheit und widerspricht sich selbst, da diese Ungleichheit nur dann sinnvoll ist, wenn von einem einzigen Objekt die Rede ist. Zweitens aber hält eine Logik der Form L überhaupt keine Handhabe bereit, um eine solche Ungleichung aufzustellen. Es wurde zwar, wie allgemein bekannt ist, auf zahlreiche Weise versucht, logische Identität zu definieren (vgl. Menne 1992, S. 65 ff.), aber das Verhältnis von Nicht-Identität und Nicht-Gleichheit liegt in tiefstem formalem (und auch inhaltlichem) Dunkel. Sobald zwei Objekte auch nur in einem Merkmal nicht übereinstimmen, sind sie ungleich, wann aber sind sie gleich? Gibt es auf ontischer Ebene – und von nichts anderem als von Objekten ist ja auch in der Logik die Rede – überhaupt eine Unterscheidung von Gleichheit und Identität? Nehmen wir einmal an, es gibt Gleichheit und es gibt Ungleichheit, dann muß es ein Drittes geben, welches überhaupt die Möglichkeit einräumt, daß p nicht gleich p, sondern gleich nicht-p ist. In anderen Worten: Die stillschweigende Voraussetzung $p \neq \neg p$, auf der die Grundgesetze des Denkens ruhen, unter ihnen also der Drittensatz, lautet, daß es ausgerechnet ein solches Drittes geben muß, um die beiden Fälle $p = \neg p$ und $p \neq \neg p$ voneinander zu unterscheiden. In einer Logik, die keinen dritten Wert neben p und $\neg$ p zuläßt, können diese beiden Werte ja nur Spiegelungen von einander sein, vgl. dazu bereits Günther (2000, S. 230): "Beide Werte einer solchen Logik aber sind metaphysisch äquivalent".

3. Der auch von der polykontexturalen Logik gezogene Schluß, daß zwischen p und $\neg$ p in L eine Kontexturgrenze verläuft, ist also falsch – und übrigens im Falle der güntherschen Logik auch unverständlicherweise falsch, da Günther und seine Nachfolger die metaphysische Äquivalenz von p und $\neg$ p ja gesehen haben. Da dies nun so ist, folgt, daß in L $p = \neg p$ ist, es sei denn, es gibt trotzdem ein Tertium, welches die Differenz zwischen p und $\neg$ p einführt. Eine solche Möglichkeit wurde in Toth (2014) eingeführt. Da die 2-wertige Logik (wie auch die polykontexturale, die 2-wertige Logiken als Teilsysteme enthält) zwischen designierten und nicht-designierten Werten unterscheidet, wobei merkwürdigerweise in beiden Logiken immer p und also niemals $\neg$ p als der designierte Wert auftritt, ist es möglich, nicht-designierte Werte durch einen Einbettungsoperator E auf eine andere Einbettungsstufe zu setzen. Das bedeutet, daß

$E(L) = [p, [p]]$

oder

$E(L) = [[p], p]$

ist. Wie man sieht, braucht man nun auch die Negation nicht mehr, d.h. man kommt im Falle von L mit einem einzigen Wert aus. Ob man diesen als Wahr oder als Falsch bezeichnet, ist eine Frage der Semiotik und keine der Logik, oder wie Günther sich ausdrückte: "Es gibt keinen theoretischen Grund, welche Seite rechts und welche Seite links von der Zugspitze ist. Die Benennung beruht auf einer willkürlichen Entscheidung, und wenn man seinen Standpunkt wechselt, sind die rechte und die linke Seite miteinander vertauscht" (2000, S. 230 f.).

Man kann ferner E iterieren und erhält auf diese Weise Hierarchien von Einbettungsstufen

$E(L) = [p, [p]]$

$E(E(L)) = [p, [[p]]]$

$E(E(E(L))) = [p, [[[p]]]], ...$

oder

$E(L) = [[p], p]$

$E(E(L)) = [[[p]], p]$

$E(E(E(L))) = [[[[p]]], p], ...$

Setzt man nun wahlweise Ω (Objekt) oder Σ (Subjekt) für p ein, so bekommt man also theoretisch unendlich tiefe Einbettungsstufen sowohl für das Objekt als auch für das Subjekt, d.h. nicht nur Subjekte – wie in der polykontexturalen Logik angenommen –, sondern auch Objekte haben "Reflexionstiefen". Daß gerade die Ontik an solchen objektalen Reflexionsstrukturen interessiert ist, dürfte kaum verwundern angesichts der Tatsache, daß die Ontik davon ausgeht, daß es keine absoluten Objekte und Subjekte gibt, d.h. daß jedem Objekt ein Subjektanteil und jedem Subjekt ein Objektanteil inhäriert. Dies ist übrigens eine notwendige Folgerung aus der Tatsache, daß in der 2-wertigen Logik L ohne Annahme eines selbstwidersprüchlichen Tertiums $p = \neg p$ gilt.

4. Transformiert man nun also

$\tau \quad [L = [p, \neg p]] \rightarrow [p, [p]] \,/\, [[p], p],$

so fungiert als "Tertium" der Operator E, und es gilt selbstverständlich

$p \neq [p],$

und zwar ohne einen dritten Wert zwischen oder außerhalb der Werte von L annehmen zu müssen.

Allerdings folgt aus $p \neq [p]$ weiterhin, daß die beiden sich durch ihre Einbettungsstufe unterscheidenden Werte Ränder haben, d.h. daß entweder

$R[p, [p]] = \emptyset$

oder

$R[p, [p]] \neq \emptyset$

ist. Trifft der letztere Fall zu, so folgt außerdem, daß

$R[p, [p]] \neq R[[p], p]$,

etwa in der Weise, wie der Blick aus einem Fenster oder in ein Fenster ebenfalls perspektivisch geschieden sind. Für Hierarchien von Einbettungen gilt also der erstere Fall in Sonderheit dann, wenn sich die beiden Glieder eines Randes durch mehr als eine Einbettungsstufe unterscheiden, vgl.

$L_1 = [p, [p]]$	$L_1^{-1} = [[p], p]$
p	p
R ---------------	R ---------------
p	p,

aber

$L_2 = [p, [[p]]]$	$L_2^{-1} = [[[p]], p]$
p	p
R ---------------	R ---------------
---------------	---------------
p	p

Um wiederum ein impressionistisches Beispiel zur Illustration heranzuziehen: Die Decke meines Büros ist zwar der untere Teil eines vertikalen Teilsystemrandes, dessen oberer Teil der Fußboden des Büros eines Kollegen ist, nicht aber derjenige des Fußbodens des Kollegen zwei oder mehr Stockwerke über mir.

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Possession, Copossession und Einbettung

1. Bereits in Toth (2014a) wurde gezeigt, daß die aus Bense (1979, S. 53, 67) herleitbare, von ihm selbst als "Relation über Relationen" bezeichnete Zeichenrelation durch

$$ZR = (M \subset ((M \subset O) \subset (M \subset O \subset I))),$$

d.h. als ein System mit Selbstenthaltung, welche das Fundierungsaxiom der Zermelo-Fraenkelschen Mengentheorie außer Kraft setzt, definiert werden kann.

2. Nun hatten wir in Toth (2014b) ferner gezeigt, daß man, statt von einer 2-wertigen Dichotomie

$$S^* = [S, U]$$

auszugehen, Systeme, wie natürlich alle dichotomischen Relationen, dadurch definieren kann, daß man zuerst einen der beiden Werte durch den anderen definiert

$$S_1^* = [S, U[S]] \qquad S_2^* = S_1^{*-1} = [U[S], S]$$

$$S_3^* = [[S], U[S]] \qquad S_4^* = S_3^{*-1} = [U[S], [S]],$$

und dann die Wertedifferenz durch Einführung eines Einbettungsoperators eliminiert

$$S_1^* = [S, [S]]$$

$$S_2^* = S_1^{*-1} = [[S], S].$$

3. Nimmt man die in 1. und 2. gewonnenen Ergebnisse zusammen, folgt daraus, daß man die Zeichenrelation nun auf dreifache Weise mit jeweils einer der drei Fundamentalkategorien definieren kann

$$M^* = [M, [M, [M]]]$$

$$O^* = [O, [O, [O]]]$$

$$I^* = [I, [I, [I]]].$$

Daraus folgt weiter, daß die obige lineare Definition von ZR nicht die einzige ist, sondern daß dazu noch eine vertikale besteht, so daß lineare und vertikale Systemdefinitionen jeweils Paare orthogonaler Relationen bilden. D.h. wir haben für das Zeichen neben

$M^* = [M \subset [M \subset [M]]]$

$$\begin{array}{ccccccc} M & & & & & & M \\ & M & & \text{bzw.} & & M & \\ & & M & & M & & \end{array}$$

$O^* = [O \subset [O \subset [O]]]$

$$\begin{array}{ccccccc} O & & & & & & O \\ & O & & \text{bzw.} & & O & \\ & & O & & O & & \end{array}$$

$I^* = [I \subset [I \subset [I]]]$

$$\begin{array}{ccccccc} I & & & & & & I \\ & I & & \text{bzw.} & & I & \\ & & I & & I & & \end{array}$$

Die diesen linearen und vertikalen copossessiven Inklusionsketten bzw. –hierarchien korrespondierenden possessiven erhält man durch Dualisation, welche im Falle der Semiotik mit Konversion zusammenfällt, also z.B. $M^{*-1} = [[[M] \supset M] \supset M]$ und Reflexion der vertikalen Orthogonalrelationen.

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Zeichenzahlen als imaginäre Zahlen

1. Nach einem Vorschlag von Bense (1976, S. 60) kann man die Menge der Zeichenzahlen

$S = (1.1, 1.2, 1.3, 2.1, 2.2, 2.3, 3.1, 3.2, 3.3)$

in einem kartesischen Koordinatensystem wie folgt darstellen

Dabei gilt also

$(P_{td} \neq P_{tt}) = (x.) \neq (.x)$ für $(x.) \in P_{td}$ und $(.x) \in P_{tt}$.

2. Zeichenzahlen als Elemente von S unterscheiden sind also von den von Bense (1981, S. 17 ff.) auch als Primzeichen bezeichneten Zeichenzahlen als Elemente von $P = (1, 2, 3)$, insofern die letzteren die Peanoaxiome erfüllen (vgl. Bense 1975, S. 167 ff.), die ersteren aber nicht, sondern den doppelt positiven Quadranten eines gaußschen Zahlenfeldes bilden, wobei man somit entweder P_{td} oder P_{tt} als imaginäre Achse auffassen kann. Rein formal könnte man somit für jede Zeichenzahl der Form $S = \langle a.b \rangle$ vier reell-imaginäre kartesische Produkte definieren

$\langle a.b \rangle \qquad \langle a.b_i \rangle$

$\langle a_i.b \rangle \qquad \langle a_i.b_i \rangle$.

Nun hatten wir allerdings bereits in Toth (2014a) gezeigt, daß wir wegen $(P_{td} \neq P_{tt})$ für jedes $S = \langle a.b \rangle$ ein Quadrupel der Form

$S_1 = [a, [b]] \qquad S_2 = [[b], a]$

$S_3 = [[a], b]$ $S_4 = [b, [a]]$

mit $S_2 = S_1^{-1}$ und $S_4 = S_3^{-1}$

bekommen. Wenn wir also annehmen, daß wir die Imaginarität entweder von P_{td} oder von P_{tt} ebenfalls durch Anwendung des Einbettungsoperators E definieren dürfen, dann wird vermöge der reell-imaginären kartesischen Produkte aus dem Quadrupel ein Octupel

$S_1 = [a, [b]]$ $S_2 = [[b], a]$

$S_3 = [[a], b]$ $S_4 = [b, [a]]$

$S_5 = [a, b]$ $S_6 = [b, a]$

$S_7 = [[a], [b]]$ $S_8 = [[b], [a]]$

(mit $S_6 = S_5^{-1}$ und $S_8 = S_7^{-1}$). Wie man allerdings zeigen kann (vgl. Toth 2014b-d), sind die beiden zusätzlichen Paare S_5/ S_6 und S_7/ S_8 redundant, da das erste Paare keine Einbettung und das zweite Paar eine redundante Einbettung enthält. Daraus folgt also, daß sich die Imaginarität von P_{td} oder von P_{tt} allein durch das Paar

$S_1 = [a, [b]]$

$S_2 = [[a], b]$

sowie eines Konversionsoperators K darstellen läßt, d.h. wir können die Menge S von Zeichenzahlen durch das Tripel

$S = (S_1, S_2, K)$

definieren.

3. Man beachte, daß die Definition $S = (S_1, S_2, K)$ auch ausreicht, um quaternionäre Zeichenzahlen zu definieren. Dabei ist auszugehen von

$S^* = [a, [b, [c, [d]]]]$,

worin a reell und b, c, d imaginär sind. Bei den 4! = 24 Permutationen

$S_1^* = [a, [b, [c, [d]]]]$

$S_2^* = [b, [a, [c, [d]]]]$

$S_3^* = [c, [b, [a, [d]]]]$

$S_4^* = [d, [b, [c, [a]]]]$, usw.,

die man erhält, kann man wegen der für komplexe Zeichenzahlen möglichen Reduktion der Quadrupel auf Paare, wie sie oben gezeigt wurde, auf die zu S_1^* ... S_{24}^* konversen quaternionären Zeichenzahlen verzichten. Da Imaginarität ja durch Anwendung des Einbettungsoperators E definiert wurde, sind die drei quaternionären Zeichenzahlen jeweils paarweise als Einbettungen von Einbettungen definierbar, d.h. es ändert sich gegenüber den komplexen Zeichenzahlen überhaupt nichts.

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Dyadenpaare als bikomplexe Zeichenzahlen

1. Wie zuletzt in Toth (2014) gezeigt, kann man die allgemeine Form von Subzeichen der kleinen Matrix (vgl. Bense 1975, S. 100 ff.)

$S = <x.y>$

mit $x, y \in \{1, 2, 3\}$

zunächst als Quadrupel der Form

$S_1 = [x, [y]]$ $\qquad$ $S_2 = S_1^{-1} = [[y], x]$

$S_3 = [[x], y]$ $\qquad$ $S_4 = S_3^{-1} = [y, [x]]$

und vermöge

$S = (S_1, S_2, K)$,

d.h. durch Einführung eines Konversionsoperators, als Paar der Form

$S_1 = [x, [y]]$

$S_2 = [[x], y]$

darstellen.

2. Da der Einbettungsoperator nach Toth (2014) die Position der imaginären Zahlen in den komplexen Zeichenzahlen S_1 und S_2 markiert, kann man nun die allgemeine Form von Subzeichen der großen Matrix (vgl. Bense 1975, S. 105)

$S^* = <<x.y>, <z.w>>$

als bikomplexe Zahl einführen, so daß

$<x.y> \in S$

$<z.w> \in S$

gilt, d.h. jedes Dyadenpaar der Form S^* tritt in den folgenden 4 möglichen Formen auf

$S_1^* = [[x, [y]], [x, [y]]]$ $\qquad$ $S_3^* = [[[x], y], [[x], y]]$

$S_2^* = [[x, [y]], [[x], y]]$ $\qquad$ $S_4^* = [[[x], y], [x, [y]]]$.

3. Einen Sonderfall stellen die von Arin (1981, S. 220 ff.) eingeführten determinierten Subzeichen dar, deren allgemeine Form man wie folgt darstellen kann

$S^{**} = <<x.y>, <<a.b>, <<c.d>, <e.f>>>>$,

bei denen also folgende Determinationen gelten

3.1. $<x.y> \leftarrow <<a.b>, <<c.d>, <e.f>>>$

3.2. $<a.b> \leftarrow <c.d>$

3.3. $<c.d> \leftarrow <e.f>$,

d.h. eine vollständige triadische Zeichenrelation determiniert ein Subzeichen, wobei allerdings $<<a.b>, <<c.d>, <e.f>>>$ nicht in der triadischen Ordnung von Zeichenklassen, d.h. (I, O, M) erfolgen muß, sondern alle 3! = 6 möglichen Permutationen zugelassen sind. Da sich gemäß Walther (1979, S. 79) Triaden als Konkatenationen von Dyaden darstellen lassen, kann also jede Triade als Dyadenpaar dargestellt und vermöge unserer Ergebnisse in Kap. 2 in der Form der S_i^* dargestellt werden. Allerdings bedingt die Determination der Arinschen Zeichenzahlen weitere Anwendungen des Einbettungsoperators E, d.h. wir haben z.B.

$S_{11}^* = [[x, [y]], [[x, [y]]]]$ $\qquad$ $S_{12}^* = [[[x, [y]]], [x, [y]]]$

$S_{21}^* = [[x, [y]], [[[x], y]]]$ $\qquad$ $S_{22}^* = [[[x, [y]]], [[x], y]]$

$S_{31}^* = [[[x], y], [[[x], y]]]$ $\qquad$ $S_{32}^* = [[[[x], y]], [[x], y]]$

$S_{41}^* = [[[x], y], [[x, [y]]]]$ $\qquad$ $S_{41}^* = [[[[x], y]], [x, [y]]]$

für jedes Paar bikomplexer Zeichenzahlen.

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Kann es konjugierte Zeichenzahlen geben?

1. In Toth (2014a) hatten wir gezeigt, daß es sinnvoll ist, bei komplexen Zeichenzahlen der Form

$S = <a.b>$

mit $a \in P_{td}$ und $b \in P_{td}$ sowie $a, b \in \{1, 2, 3\}$ die folgenden vier Möglichkeiten imaginärer Zeichenzahlanteile zu unterscheiden

$<a.b> \qquad <a.b_i>$

$<a_i.b> \qquad <a_i.b_i>$.

Nach Toth (2014b) sind diese den folgenden vier Paaren eingebetteter semiotischer Subrelation äquivalent

$<a.b> = [a, b]$

$<a.b_i> = [a, [b]]$

$<a_i.b> = [[a], b]$

$<a_i.b_i> = [[a], [b]]$.

Dabei scheiden allerdings gemäß der Definition des semiotischen Einbettungsoperators (vgl. Toth 2014c) [a, b] und [[a], [b]] aus, so daß komplexe Zeichenzahlen auf die beiden folgenden semiotischen Einbettungsstrukturen reduzierbar sind

$<a.b_i> = [a, [b]]$ mit $<a.b_i>^{-1} = [[b], a]$

$<a_i.b> = [[a], b]$ mit $<a_i.b>^{-1} = [b, [a]]$.

2. Konkret gesprochen, bedeutet dies, daß

$\times(a.b) \neq (b.a)$,

d.h. daß z.B. die Dualisation eines Symbols (1.3) zwar quantitativ, aber nicht qualitativ mit dem Rhema (3.1) zusammenfällt. In Sonderheit folgt daraus, daß fürdie sogenannte dualidentische Zeichenklasse der Eigenrealität

$\times$ (3.1, 2.2, 1.3) =

(3.1, 2.2, 1.3)

die beiden identisch erscheinenden (und hier untereinander geschriebenen) Paare von Subrelationen nicht-identisch sind, denn das Rhema (3.1) der Zeichenthematik ist das dualisierte Legizeichen der Realitätsthematik – et vice versa. Und selbst beim "genuinen" Index (2.2) besteht keine Identität, denn nach dem oben Gesagt gilt selbstverständlich

$\times$[3,[1]] = [[1], 3]

$\times$[[3], 1] = [1, [3]]

und also [3, [1]] $\neq$ [[3], 1]] sowie [[1], 3] $\neq$ [1, [3]].

$\times$[2, [2]] = [[2], 2]

$\times$[[2], 2] = [2, [2]]

und also [2, [2]] $\neq$ [[2], 2] sowie [[2], 2] $\neq$ [2, [2]].

Daraus folgt also, daß im abstrakten Paar komplexer Zeichenzahlen

$<a.b_i>$ = [a, [b]] mit $<a.b_i>^{-1}$ = [[b], a]

$<a_i.b>$ = [[a], b] mit $<a_i.b>^{-1}$ = [b, [a]]

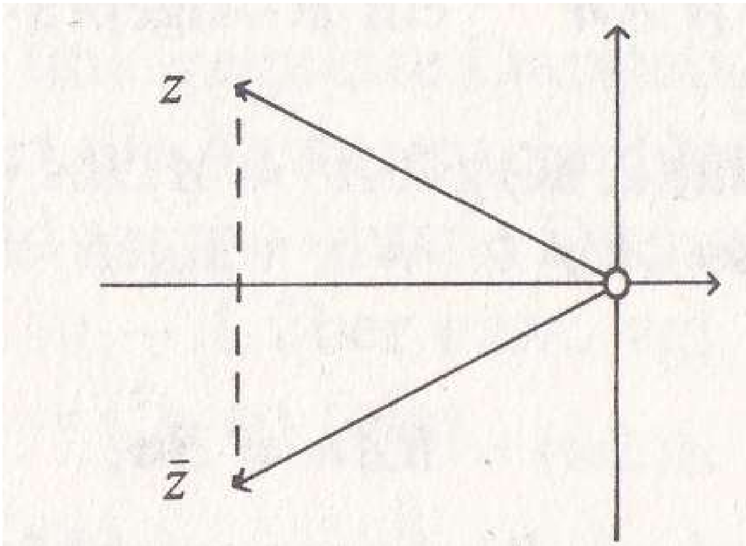
sich die beiden dualen Paare in qualitativer Konjugationsrelation befinden. Anders gesagt, für jedes der beiden Paare, d.h. für [a, [b]] und [[a], b] einerseits sowie für [[b], a] und [b, [a]] andererseits gilt die aus der Mathematik natürlich bekannte Spiegelungstransformation einer komplexen Zahl der abstrakten Form

$z = a + bi$

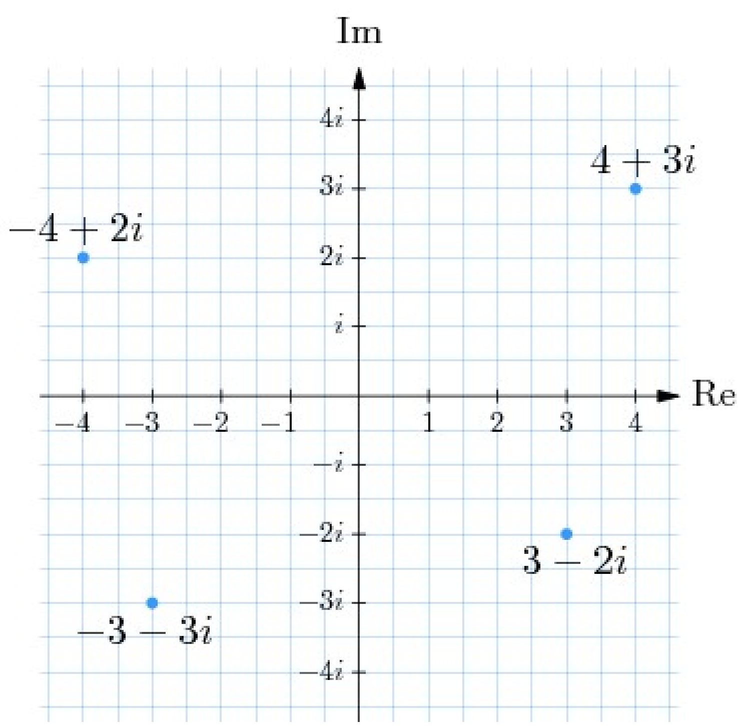
und ihrer zugehörigen Konjugierten der abstrakten Form

$\bar{z} = a - bi$,

deren geometrisches Verhältnis am einfachsten durch das folgende Diagramm aus Ebbinghaus et al. (1992, S. 58) darstellbar ist.



Sie gilt allerdings nicht nur für zwei, sondern für alle vier Quadranten der Gaußschen Zahlenebene



In anderen Worten: Es folgt aus unseren Betrachtungen nichts Geringeres als die Existenz negativer Zeichen, eine Vermutung, die ich, allerdings vor einem vollkommen anderen Hintergrund, bereits anfangs der 1990er Jahre anläßlich eines Semiotikkongresses in Stuttgart geäußert hatte und die damals, nach dem Tode Max Benses, auf sehr großes Unverständnis gestoßen war.

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Abbildungen komplexer Zeichenzahlen

1. Im Anschluß an Toth (2014a) gehen wir aus von der algebraischen Struktur komplexer Zeichenzahlen

$Z = (\langle a.b\rangle, \times, *)$

mit a, b ∈ {1, 2, 3},

darin <a.b> die allgemeine Form der benseschen semiotischen Subrelationen, × die bensesche Dualrelation und * die in Toth (2014b) eingeführte Operation der Einbettungsreflexion darstellt. Wir können dann wie folgt ein komplexes semiotisches Zahlenfeld konstruieren.

$-z = -a + bi$			$z = a + bi$		
[[1], 1]	[[2], 1]	[[3], 1]	[1, [1]]	[2, [1]]	[3, [1]]
[[1], 2]	[[2], 2]	[[3], 2]	[1, [2]]	[2, [2]]	[3, [2]]
[[1], 3]	[[2], 3]	[[3], 3]	[1, [3]]	[2, [2]]	[3, [3]]
$-\bar{z} = -a - bi$			$\bar{z} = a - bi$		
[1, [1]]	[1, [2]]	[1, [3]]	[[1], 1]	[[1], 2]	[[1], 3]
[2, [1]]	[2, [2]]	[2, [3]]	[[2], 2]	[[2], 2]	[[2], 3]
[3, [1]]	[3, [2]]	[3, [3]]	[[3], 1]	[[2], 2]	[[3], 3].

2. Wir können also genau 6 Abbildungen komplexer Zeichenzahlen unterscheiden.

2.1. [x, [y]] → [[y], x] ≅ $[z = a + bi] \rightarrow [\bar{z} = a - bi]$

2.2. [x, [y]] → [[x], y] ≅ $[z = a + bi] \rightarrow [-z = -a + bi]$

2.3. [x, [y]] → [y, [x]] ≅ $[z = a + bi] \rightarrow [-\bar{z} = -a - bi]$

2.4. $[[y], x] \rightarrow [[x], y] \quad \cong \quad [\bar{z} = a - bi] \rightarrow [-z = -a + bi]$

2.5. $[[y], x] \rightarrow [y, [x]] \quad \cong \quad [\bar{z} = a - bi] \rightarrow [-\bar{z} = -a - bi]$

2.6. $[[x], y] \rightarrow [y, [x]] \quad \cong \quad [-z = -a + bi] \rightarrow [-\bar{z} = -a - bi]$

Bedient man sich zur Darstellung dieser Abbildungen der semiotischen Basisoperationen $\times$ und *, dann haben wir

2.1. $[x, [y]] \rightarrow [[y], x] \quad = \quad \times[x, [y]]$

2.2. $[x, [y]] \rightarrow [[x], y] \quad = \quad *[x, [y]]$

2.3. $[x, [y]] \rightarrow [y, [x]] \quad = \quad \times*[x, [y]]$

2.4. $[[y], x] \rightarrow [[x], y] \quad = \quad \times*[x, [y]]$

2.5. $[[y], x] \rightarrow [y, [x]] \quad = \quad *[x, [y]]$

2.6. $[[x], y] \rightarrow [y, [x]] \quad = \quad \times[x, [y]]$,

d.h. wir bekommen drei Paare von operational gleichen Abbildungen

2.1. $[x, [y]] \rightarrow [[y], x] \quad = \quad \times[x, [y]]$

2.6. $[[x], y] \rightarrow [y, [x]] \quad = \quad \times[x, [y]]$

2.2. $[x, [y]] \rightarrow [[x], y] \quad = \quad *[x, [y]]$

2.5. $[[y], x] \rightarrow [y, [x]] \quad = \quad *[x, [y]]$

2.3. $[x, [y]] \rightarrow [y, [x]] \quad = \quad \times*[x, [y]]$

2.4. $[[y], x] \rightarrow [[x], y] \quad = \quad \times*[x, [y]]$.

3. Verwendet man schließlich den in Toth (2013c) eingeführten Einbettungs-oprator E, welcher reelle in komplexe Zeichenzahlen transformiert, haben wir

$E_x([x, y]) = [[x], y]$

$E_y([x, y]) = [x, [y]]$

$E_x([y, x]) = [y, [x]]$

$E_y([x, y]) = [x, [y]]$.

Literatur

Toth, Alfred, Reelle und imaginäre ontische Strukturen. In: Electronic Journal for Mathematical Semiotics, 2014a

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Paarzahlen und Quadrupel-Zahlen

1. Peano-Zahlen sind, wie allgemein bekannt ist, linear durch die 5 Peano-Axiome geordnet

P = 1, 2, 3, ..., n.

Benses Absicht der Einführung seiner "Primzeichen" genannten Zeichenzahlen (Bense 1981, S. 17 ff.) setzt daher den, ebenfalls von Bense geführten, Nachweis der Isomorphie von Peano-Zahlen und Zeichenzahlen voraus (vgl. Bense 1975, S 167 ff.).

2. Ordnet man die Zeichen in einer der sechs Permutationen der Primzeichen-Relation

Z = (1, 2, 3)

an, d.h. in den Ordnungen (1, 2, 3), (1, 3, 2), (2, 1, 3), (2, 3, 1), (3, 1, 2) oder (3, 2, 1), so ändert sich natürlich nichts an deren Linearität, die wir durch

$\boxed{1} \quad \boxed{2} \quad \boxed{3}$

veranschaulichen können. Nur hatten wir bereits in Toth (2014) nachgewiesen, daß die drei dyadischen Teilrelationen von Z, d.h. (1, 2), (2, 3) und (1, 3), um deren Haupt- und Stellenwerte zu unterscheiden, in Form von Einbettungen notiert werden müssen. Damit erscheint jede abstrakte dyadische Relation der Form S = [x.y] in vier möglichen Formen

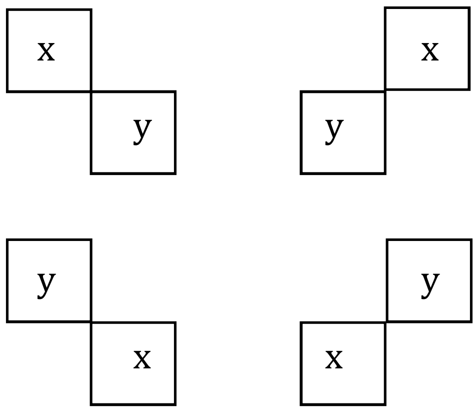
[x, [y]]	[[y], x]
[[x], y]	[y, [x]],

so daß wir also für die drei dyadischen Teilrelationen von Z ein Tripel von Quadrupeln

[1, [2]]	[[2], 1]	[2, [3]]	[[3], 2]
[[1], 2]	[2, [1]]	[[2], 3]	[3, [2]]
[1, [3]]	[[3], 1]		

[[1], 3]   [3, [1]]

erhalten. Nun sind diese eingebetteten Relationen zueinander dualer Paare natürlich nicht mehr linear, sondern orthogonal, und wir können sie durch



darstellen. Dabei gilt also nicht mehr nur

[x.y] ≠ [y.x],

sondern wir bekommen statt eines Paares ein Quadrupel von Ungleichungen

[x, [y]]   ≠   [[y], x]

≠   ≠

[[x], y]   ≠   [y, [x]].

Literatur

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Bense, Max, Axiomatik und Semiotik. Baden-Baden 1981

Toth, Alfred, Einbettungsoperatoren. In: Electronic Journal for Mathematical Semiotics, 2014

Zeichen und Einbettungsstufen

1. Ordnet man die Zeichen in einer der sechs Permutationen der Primzeichen-Relation

Z = (1, 2, 3)

an, d.h. in den Ordnungen (1, 2, 3), (1, 3, 2), (2, 1, 3), (2, 3, 1), (3, 1, 2) oder (3, 2, 1), so ändert sich natürlich nichts an deren Linearität, die wir durch

$\boxed{1}$ $\boxed{2}$ $\boxed{3}$

veranschaulichen können. Nur hatten wir bereits in Toth (2014) nachgewiesen, daß die drei dyadischen Teilrelationen von Z, d.h. (1, 2), (2, 3) und (1, 3), um deren Haupt- und Stellenwerte zu unterscheiden, in Form von Einbettungen notiert werden müssen. Damit erscheint jede abstrakte dyadische Relation der Form S = [x.y] in vier möglichen Formen

[x, [y]] [[y], x]

[[x], y] [y, [x]],

so daß wir also für die drei dyadischen Teilrelationen von Z ein Tripel von Quadrupeln

[1, [2]]	[[2], 1]	[2, [3]]	[[3], 2]
[[1], 2]	[2, [1]]	[[2], 3]	[3, [2]]
[1, [3]]	[[3], 1]		
[[1], 3]	[3, [1]]		

erhalten. Nun sind diese eingebetteten Relationen zueinander dualer Paare natürlich nicht mehr linear, sondern orthogonal, und wir können sie durch

x				x
	y		y	

y				y
	x		x	

darstellen. Dabei gilt also nicht mehr nur

$[x.y] \neq [y.x]$,

sondern wir bekommen statt eines Paares ein Quadrupel von Ungleichungen

$$\begin{array}{ccc} [x, [y]] & \neq & [[y], x] \\ \neq & & \neq \\ [[x], y] & \neq & [y, [x]]. \end{array}$$

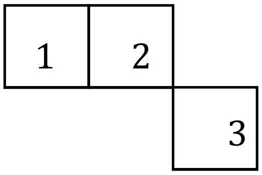
2. Diese Zusammenfassung aus Toth (2015) sei nun als Ausgangspunkt dafür genommen, den in Toth (2014) eingeführten Einbettungsoperator auch auf P = (1, 2, 3) selbst, d.h. nicht nur auf dyadische Teilrelationen, sondern auf triadische Relationen anzuwenden. Dabei sind natürlich drei Einbettungsstufen zu unterscheiden.

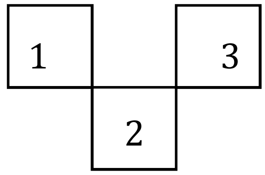
2.1. Eine Einbettungsstufe

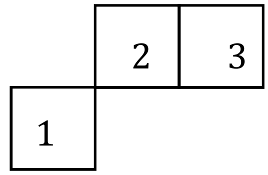
Dies ist der lineare Fall der mit den Peanozahlen isomorphen Zeichenzahlen bzw. Primzeichen.

1	2	3		2	1	3		3	1	2
1	3	2		2	3	1		3	2	1

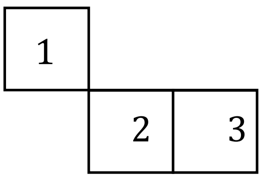
2.2. Zwei Einbettungsstufen

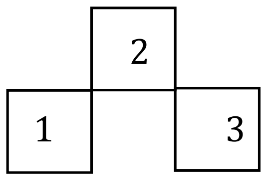


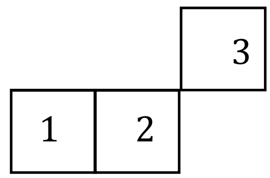




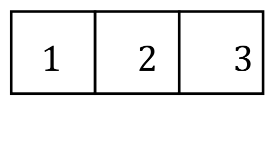
Z = [1, 2, [3]] Z = [1, [2], 3] Z = [[1], 2, 3]

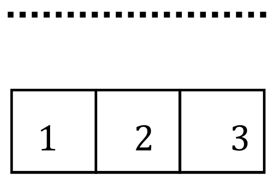






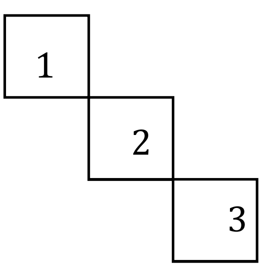
Z = [1, [2, 3]] Z = [[1], 2, [[3]] Z = [[1, 2], 3]

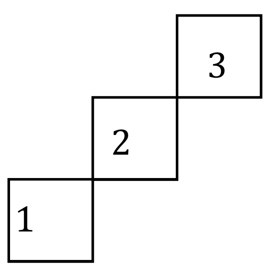




Z = [1, 2, 3] Z = [[1, 2, 3]]

2.3. Drei Einbettungsstufen





Für alle in 2.1. bis 2.3. dargestellten Einbettungsstrukturen können natürlich jeweils die 6 permutationellen Ordnungen von P eingesetzt werden.

Literatur

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Bense, Max, Axiomatik und Semiotik. Baden-Baden 1981

Toth, Alfred, Einbettungsoperatoren. In: Electronic Journal for Mathematical Semiotics, 2014

Toth, Alfred, Paarzahlen und Quadrupelzahlen. In: Electronic Journal for Mathematical Semiotics, 2015

Zu einer Arithmetik eingebetteter semiotischer Relationen

1. Zur Einleitung vgl. Toth (2014, 2015a, b).

2. Jede abstrakte dyadische semiotische Relation der Form S = [x.y] mit x, y ∈ {1, 2, 3} kann in vier möglichen Einbettungsstrukturen erscheinen

$$\begin{array}{ll} [x, [y]] & [[y], x] \\ [[x], y] & [y, [x]]. \end{array}$$

Es gilt für E[x.y] = [x.y]

[x.y] ≠ [y.x],

und für E[x.y] ≠ [x.y]

$$\begin{array}{ccc} [x, [y]] & \neq & [[y], x] \\ \neq & & \neq \\ [[x], y] & \neq & [y, [x]]. \end{array}$$

2.1. Addition

[x, [y]] + [x, [y]] = [x, [y]]

[x, [y]] + [[y], x] = [(x + [y]), ([y] + x)]

[x, [y]] + [[x], y] = [(x + [x]), ([y] + y)]

[x, [y]] + [y, [x]] = [(x + y), ([y] + [x]), usw.

2.2. Subtraktion

[x, [y]] –[x, [y]] = [x, [y]]

[x, [y]] – [[y], x] = [(x – [y]), ([y] – x)]

[x, [y]] – [[x], y] = [(x – [x]), ([y] – y)]

[x, [y]] – [y, [x]] = [(x – y), ([y] – [x]), usw.

3. Jede abstrakte triadische semiotische Relation der Form Z = [x, y, z] mit x, y, z ∈ {1, 2, 3} kann in neun möglichen Einbettungsstrukturen erscheinen

Z = [x, y, z]	Z = [[x, y, z]]	
Z = [[x], y, z]	Z = [x, [y], z]	Z = [x, y, [z]]
Z = [[x, y], z]	Z = [x, [y, z]]	Z = [[x], y, [[z]]
Z = [x, [y, [z]]		

3.1. Addition

[x, y, z] + [[x, y, z]] = [(x, y, z) + [x, y, z]]]

[x, y, z] + [[x], y, z] = [(x, y, z) + [[x], y, z]]

[x, y, z] + [x, [y], z] = [(x, y, z) + [x, [y], z]]

[x, y, z] + [x, y, [z]] = [(x, y, z) + [x, y, [z]]]

[x, y, z] + [[x, y], z] = [(x, y, z) + [[x, y], z]]

[x, y, z] + [x, [y, z]] = [(x, y, z) + [x, [y, z]]]

[x, y, z] + [[x], y, [[z]] = [(x, y, z) + [[x], y, [[z]]]

[x, y, z] + [x, [y, [z]] = [(x, y, z) + [x, [y, [z]]]

3.2. Subtraktion

[x, y, z] – [[x, y, z]] = [(x, y, z) – [x, y, z]]]

[x, y, z] – [[x], y, z] = [(x, y, z) – [[x], y, z]]

[x, y, z] – [x, [y], z] = [(x, y, z) – [x, [y], z]]

[x, y, z] – [x, y, [z]] = [(x, y, z) – [x, y, [z]]]

[x, y, z] – [[x, y], z] = [(x, y, z) – [[x, y], z]]

[x, y, z] – [x, [y, z]] = [(x, y, z) – [x, [y, z]]]

[x, y, z] – [[x], y, [[z]] = [(x, y, z) – [[x], y, [[z]]]

[x, y, z] – [x, [y, [z]] = [(x, y, z) – [x, [y, [z]]]

Literatur

Toth, Alfred, Einbettungsoperatoren. In: Electronic Journal for Mathematical Semiotics, 2014

Toth, Alfred, Paarzahlen und Quadrupelzahlen. In: Electronic Journal for Mathematical Semiotics, 2015a

Toth, Alfred, Zeichen und Einbettungsstufen. In: Electronic Journal for Mathematical Semiotics, 2015b

Einbettungstheoretische Semiotik I

1. Im Anschluß an Toth (2015a-c) definieren wir im folgenden eine Semiotik, welche im Gegensatz zu derjenigen von Peirce und Bense über eingebettete Zeichenzahlen verfügt (vgl. Toth 2014). Sie enthält die peirce-bensesche Semiotik, geht aber weit über sie hinaus.

2.1. Definition der Zeichenrelation

$Z = [(1, 2, 3), E]$,

darin E der Einbettungsoperator ist mit

$E(1) = [1]$

$E(2) = [2]$

$E(3) = [3]$

2.2. Jede dyadische Subrelation der Form $S = [x.y]$ (mit, $x, y \in \{1, 2, 3\}$) läßt sich durch Anwendung von E in vier möglichen Formen notieren

$[x, [y]] \qquad [[y], x]$

$[[x], y] \qquad [y, [x]]$.

Damit steht der paarweisen Ungleichung nicht-eingebetteter Subrelationen

$[x.y] \neq [y.x]$

das Quadrupel von Ungleichungen

$$\begin{array}{ccc} [x, [y]] & \neq & [[y], x] \\ \neq & & \neq \\ [[x], y] & \neq & [y, [x]] \end{array}$$

gegenüber.

2.3. Das vollständige System semiotischer Subrelationen

[1, [1]]	[[1], 1]	[1, [2]]	[[2], 1]	[1, [3]]	[[3], 1]
[[1], 1]	[1, [1]]	[[2], 1]	[1, [2]]	[[3], 1]	[1, [3]]
[2, [1]]	[[2], 1]	[2, [2]]	[[2], 2]	[2, [3]]	[[3], 2]
[[2], 1]	[1, [2]]	[[2], 2]	[2, [2]]	[[3], 2]	[2, [3]]
[3, [1]]	[[1], 3]	[3, [2]]	[[2], 3]	[3, [3]]	[[3], 3]
[[3], 1]	[1, [3]]	[[2], 3]	[3, [2]]	[[3], 3]	[3, [3]]

3. Die triadische Relation der Form Z = [x, y, z] kann auf 4 Einbettungsstufen erscheinen.

3.1. 0 Einbettungsstufen

Z = [1, 2, 3]	Z = [2, 1, 3]	Z = [3, 1, 2]
Z = [1, 3, 2]	Z = [2, 3, 1]	Z = [3, 2, 1]

3.2. 1 Einbettungsstufe

Z = [1, 2, [3]]	Z = [2, 1, [3]]	Z = [3, 1, [2]]
Z = [1, 3, [2]]	Z = [2, 3, [1]]	Z = [3, 2, [1]]
Z = [1, [2], 3]	Z = [2, [1], 3]	Z = [3, [1], 2]
Z = [1, [3], 2]	Z = [2, [3], 1]	Z = [3, [2], 1]
Z = [[1], 2, 3]	Z = [[2], 1, 3]	Z = [[3], 1, 2]
Z = [[1], 3, 2]	Z = [[2], 3, 1]	Z = [[3], 2, 1]

$Z = [[1, 2, 3]]$	$Z = [[2, 1, 3]]$	$Z = [[3, 1, 2]]$
$Z = [[1, 3, 2]]$	$Z = [[2, 3, 1]]$	$Z = [[3, 2, 1]]$

3.3. 2 Einbettungsstufen

$Z = [1, [2, 3]]$	$Z = [2, [1, 3]]$	$Z = [3, [1, 2]]$
$Z = [1, [3, 2]]$	$Z = [2, [3, 1]]$	$Z = [3, [2, 1]]$

$Z = [[1], 2, [3]]$	$Z = [[2], 1, [3]]$	$Z = [[3], 1, [2]]$
$Z = [[1], 3, [2]]$	$Z = [[2], 3, [1]]$	$Z = [[3], 2, [1]]$

$Z = [[1, 2], 3]$	$Z = [[2, 1], 3]$	$Z = [[3, 1], 2]$
$Z = [[1, 3], 2]$	$Z = [[2, 3], 1]$	$Z = [[3, 2], 1]$

3.4. 3 Einbettungsstufen

$Z = [1, [2, [3]]]$	$Z = [2, [1, [3]]]$	$Z = [3, [1, [2]]]$
$Z = [1, [3, [2]]]$	$Z = [2, [3, [1]]]$	$Z = [3, [2, [1]]]$

$Z = [[[1], 2], 3]$	$Z = [[[2], 1], 3]$	$Z = [[[3], 1], 2]$
$Z = [[[1], 3], 2]$	$Z = [[[2], 3], 1]$	$Z = [[[3], 2], 1]$

Weitere Einbettungsstufen setzen die Iteration von $E \rightarrow \{E^2, ..., E^n\}$ voraus, dann erhält man eingebettete Zeichenrelation wie z.B. $Z = [[[1], 2], 3], [1, [[2], [[3]]], [[[2], [3]], 1]$, usw. Ferner kann man iterierte Einbettungsstufen natürlich miteinander kombinieren. Dadurch wird die einbettungstheoretische Semiotik zu einem System von kaum vorstellbarer Komplexität.

4. Operatoren

Für die nicht-einbettungstheoretische Semiotik gelten seit Beckmann (1976) die verbandstheoretische Vereinigung $\sqcup$ und der Durchschnitt $\sqcap$ anstelle der Peano-Addition und –Subtraktion. Sie gilt selbstverständlich darüber hinaus für alle Zeichenrelationen der gleichen Einbettungsstufe.

4.1. Addition von Subrelationen

[1, [2] + [1, [3]] = [1, [3]]

[[2], 1] + [[3], 1] = [[3], 1]

[[1], 2] + [[2], 1] = [[2], 1]]

[2, [1]] + [3, [1]] = [3, [1]]

4.2. Subtraktion von Subrelationen

[1, [3] – [1, [2]] = [1, [2]]

[[3], 1] – [[2], 1] = [[2], 1]

[[1], 3] – [[1], 2] = [[1], 2]]

[3, [1]] – [2, [1]] = [2, [1]]

4.3. Dagegen können Subrelationen ungleicher Einbettungsstufen nicht addiert bzw. subtrahiert werden.

[1, [2]] + [[2], 1] = [(1 + [2]), ([2] + 1)]

[1, [2]] + [[1], 2] = [(1 + [1]), ([2] + 2)]

[1, [2]] + [2, [1]] = [(1 + 2), ([2] + [1])], usw.

[1, [2]] – [[2], 1] = [(1 – [2]), ([2] – 1)]

[1, [2]] – [[1], 2] = [(1 – [1]), ([2] – 2)]

[1, [2]] – [2, [1]] = [(1 – 2), ([2] – [1])], usw., d.h. statt Addition (Vereinigung) bzw. Subtraktion (Durchschnitt) tritt hier eine neue Operation ein, die wir Determination nennen können, insofern jedes Paar der Form [[x.y], E] durch die beiden möglichen Fälle

$$[[x.y], E] \pm [[z.w], E] = [[[x.y], E] \leftarrow [[z.w], E]]$$

$$[[x.y], E] \pm [[z.w], E] = [[[x.y], E] \rightarrow [[z.w], E]]$$

darstellbar ist.

Literatur

Beckmann, Peter, Verbandstheoretische Darstellung der Subzeichen und Zeichenklassen. In: Semiosis 2, 1976, S 31-35

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Toth, Alfred, Zu einer Arithmetik eingebetteter semiotischer Relationen. In: Electronic Journal for Mathematical Semiotics, 2015c

Grundlegung einer Arithmetik kontexturierter Objekte

Ne tudd meg, hogy én egyedül
Mit beszélek majd a Halállal.

Endre Ady, "Halálvirág: a csók"[9]

1. Die Mathematik der Qualitäten (vgl. Kronthaler 1986), die auf der polykontexturalen Logik und Ontologie Gotthard Günthers beruht (vgl. Günther 1976-80, 1991), kann man als ein Vermittlungssystem 2-wertiger Logiken in Subjektfunktion definieren (vgl. Toth 2015). Das bedeutet dreierlei: 1. Polykontextural ist lediglich die Vermittlung zwischen den Logiken, die weiterhin 2-wertig bleiben. 2. Es gibt somit keine Vermittlung zwischen den beiden einzigen Werten der 2-wertigen Logiken. 3. Die Erweiterung der Mono- zur Polykontexturalität verdankt sich einzig der Iterierbarkeit des Subjektes, denn nur dieses ist kontexturell relevant. Eine kontexturelle Relevanz des "toten" Objektes wird diesem somit explizit abgesprochen. Das Objekt ist damit in den Permutationszyklen bzw. Permutogrammen immer konstant (vgl. Thomas 1985).

Vor dem Hintergrund der theoretischen Ontik, die wir in den letzten Jahren der theoretischen Semiotik von Peirce und Bense zur Seite gestellt haben, ist die kontexturelle Irrelevanz des Objektes jedoch aus zwei Gründen falsch. 1. Der Objektbegriff, welcher der Ontik zugrunde liegt, ist der des subjektiven Objektes, da wir Objekte nur durch die Filter unserer Sinne wahrnehmen können und die Vorstellung eines objektiven, d.h. absoluten bzw. apriorischen Objektes damit zum Phantasma wird. 2. Die Falschheit der Annahme, daß Objekte nicht kontexturiert sein können, folgt direkt aus der Isomorphie von Objekt und Zeichen (vgl. Toth 2014a), die übrigens bereits zu Recht von der Semiotik von Georg Klaus postuliert worden war (vgl. Klaus 1973).

2. Logische Existenz kann nach einem genialen Vorschlag Albert Mennes durch Selbstidentität definiert werden (vgl. Menne 1991, S. 107). Damit sind auch ontisch nicht-existente Objekte wie der Pegasus, die Meerjungfrau und Frau

[9] "Du sollst nicht wissen, was ich allein dem Tod noch sagen werde". Vgl. zum hier vorausgesetzten theoretischen Hintergrund Rudolf Kaehr, Über Todesstruktur, Maschine und Kenogrammatik. In: Spuren (Hamburg), Nr. 38, Okt. 1991, S. 47-53.

Holle logisch existent. Daraus folgt allerdings, daß Existenz unter völliger Absehung des Objektbegriffes, und zwar durch die logische, d.h. nicht-ontische und nicht-semiotische Eigenschaft der Widerspruchsfreiheit definiert wird. Die bemerkenswerte Möglichkeit, solche ontisch nicht-existenten Objekte als Zeichen zu repräsentieren, zeigt somit, daß die Menge subjektiver Objekte bedeutend größer ist als diejenige objektiver Objekte, d.h. daß der Subjektanteil im subjektiven Objekt nicht nur reduktiv[10], sondern gleichzeitig produktiv wirkt, und zwar im Sinne der von Bense (1992, S. 16) festgestellten "Seinsvermehrung". Damit erhebt sich allerdings die Frage, was die Bedeutung des arithmetischen Satzes ist, daß in der quantitativen Mathematik nur mit "Gleichem" operiert werden könne (vgl. Kronthaler 1990), denn Gleichheit und Ungleichheit müssen ja ebenfalls über Selbstidentität definiert werden. Beispielsweise sind die beiden Gleichungen

1 Apfel + 1 Apfel = 2 Äpfel

1 Birne + 1 Birne = 2 Birnen,

wie man sieht, lösbar, da jeweils beide Summanden "gleich" sind, wogegen die Gleichung

1 Apfel + 1 Birne = ?

unlösbar ist, da die beiden Summanden "ungleich" sind. Die "Lösung" 2 Früchte zeigt allerdings, daß nur scheinbar Objekte addiert werden, denn die folgenden beiden Gleichungen

1 Frucht + 1 Frucht = 2 Früchte

1 + 1 = 2

besagen genau dasselbe wie die ersten beiden Gleichungen, d.h. alle vier Gleichungen sind wegen ihrer Objektunabhängigkeit quantitativ. Objekte sind hingegen per definitionem qualitativ, d.h. es gibt keine nicht-qualitativen

[10] Hierher gehört die (mir allerdings nicht lokalisierbare) Äußerung Kafkas, daß der, welcher imstande wäre, alle Eigenschaften von Objekten mit seinen Sinnen zu erfassen, bereits beim Übertreten der Schwelle seines Hauses tot zusammenbrechen müßte.

Objekte. Somit sind in Sonderheit Zahlen keine Objekte, und damit müssen sie Zeichen sein. Wenn wir also "1 Apfel + 1 Apfel" hinschreiben, dann bezeichnet dieser Ausdruck keine ontischen Äpfel, sondern ihre Anzahlen als Zeichen n Form von Zahlen. Merkwürdigerweise gilt aber für Zahlen die Bedingung der Gleichheit von Operanden nicht, denn eine Gleichung wie z.B.

$1 + 2 = 3$

ist lösbar, obwohl die Summanden ungleich sind. Daraus folgt, daß der Satz, daß nur Gleiches operierbar ist, nicht nur sinnlos, sondern falsch ist. Sinnlos ist er deshalb, weil alle vier obigen Gleichungen dasselbe besagen, falsch ist er, da verschiedene Zahlen, d.h. reine Quantitäten operiert werden können. Man braucht nur die beiden folgenden Gleichungen hinzuschreiben, um sich von der Richtigkeit dieser Folgerung zu überzeugen

1 Apfel + 2 Äpfel = 3 Äpfel

1 Apfel + 2 Birnen = ?.

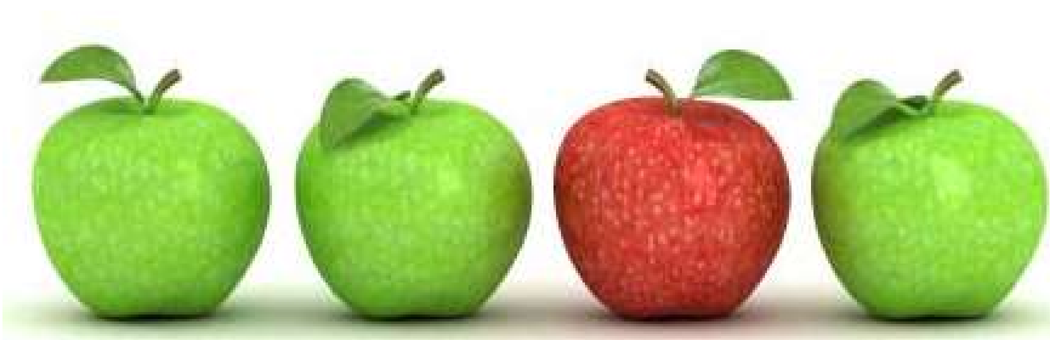
Es gibt allerdings noch einen dritten Grund, warum der Satz, daß nur Gleiches operierbar ist, falsch ist, denn vgl. z.B. die folgende Gleichung

1 Jonathan-Apfel + 1 Cox Orange-Apfel = ?

Hier kommt nun die Objektinvariante der Sortigkeit ins Spiel, d.h. die Tatsache, daß jedes Objekt einer bestimmten Sorte angehört. Von hier aus ist es ferner ein kleiner Schritt zur Einsicht, daß es keine zwei identischen Objekte gibt, und dies ist selbst bei Zwillingen mit identischer DNS der Fall, denn niemand wird bezweifeln, daß die beiden Menschen trotzdem Individuen sind. Daraus folgt, daß jedes Objekt, genauso wie jedes Subjekt, selbstidentisch sein muß, und daraus wiederum folgt, daß selbst eine so harmlos aussehende Gleichung wie

1 Apfel + 1 Apfel = 2 Äpfel

möglicherweise falsch, aber ganz bestimmt sinnlos ist, denn wir wissen nicht, welcher Sorte diese Äpfel angehören, und wir wissen mit Bestimmtheit, daß die beiden Summanden niemals den gleichen Apfel bezeichnen können, d.h. daß die Referenzobjekte der beiden Summanden verschieden sind. Somit wird bereits in der Addition 1 Apfel + 1 Apfel Ungleiches operiert. Da gemäß unserer obigen Feststellung diese Addition dasselbe besagt wie 1 + 1, stellt sich ferner die Frage, ob die beiden Einsen nicht ebenfalls ungleich sind. Logisch ist ja, wie Menne (1992, S. 38 ff.) festgestellt hatte, zwischen "sign event" und "sign structure" zu unterscheiden, d.h. zwischen dem Zeichen 1 als konkreter Instanz und dem Zeichen 1 als abstraktem Typus. Diese dyadische Unterscheidung, die sich auch in der Semiotik von Georg Klaus findet, war allerdings bereits als triadische Unterscheidung zwischen Tone, Token und Type von Peirce eingeführt worden und betrifft die Notwendigkeit, zwischen Zeichen als Qualizeichen, Sinzeichen und Legizeichen, d.h. als Qualität, Quantität und Norm zu unterscheiden. Somit kann bereits die anscheinend unverdächtige Addition 1 + 1 dreideutig sein, denn die beiden Summanden können paarweise auf dreifache Weise verschieden sein, und somit folgt wieder die Möglichkeit ihrer Ungleichheit, die dazu führt, daß nicht einmal die Gleichung 1 + 1 = ? lösbar ist, da wir ja nicht wissen, ob die Summanden Tones, Tokens oder Types und dabei gleich oder verschieden sind. Die einzige wissenschaftlich haltbare Aussage, die wir über die auf dem folgenden Photo abgebildeten Äpfel machen können,



ist somit: "Wir sehen 4 Äpfel". Wesentlich ist dabei die durch "wir" induzierte Subjektabhängigkeit der Apfel-Objekte. Wir können ferner festellten, daß es sich auf dem Bild um 2 Sorten von Äpfeln handelt und daß alle 4 Äpfel paarweise ungleich, d.h. Tones sind. Vor allem aber folgt aus dem Gesagten, daß es unwissenschaftlich ist, die Situation auf dem Bild in der Form einer Gleichung 1 + 1 + 1 + 1 = 4 auszudrücken.

3. Objekte sind damit genauso selbstidentisch wie es Subjekte sind, d.h. es gibt nicht nur Subjekt-Individuen, sondern auch Objekt-Individuen, und vor allem gibt es nur individuelle Objekte und Subjekte. Identität ist damit gleichbe-

deutend mit Selbstidentität, und Gleichheit wird dreideutig, insofern zwischen Tones, Tokens und Types zu unterscheiden ist. Wegen der Subjektabhängigkeit von Objekten, die ja ontisch und semiotisch nur als wahrgenommene existieren, sind damit nicht nur Subjekte, sondern auch Objekte kontexturell relevant. Die polykontexturale Logik, die lediglich die Kontexturalität von Subjekten, nicht aber diejenige von Objekten anerkennt, ist damit defizient. Wie eine Arithmetik kontexturierter Objekte aussehen könnte, wird im folgenden formal aufgezeigt. Wegen der Isomorphie von Objekten und Zeichen genügt es dabei, die Kontexturiertheit von Zeichen zu bestimmen. Entsprechend der von Bense eingeführten Dreiteilung des Zeichensbegriffes in Primzeichen, Subzeichen und Zeichen unterscheiden wir damit zwischen Primobjekten, Subobjekten und Objekten. Zur Vereinfachung der folgenden Darstellung setzen wir fest, daß die Anzahl der Kontexturen, in denen ein Objekt oder Zeichen auftreten kann, der Anzahl der Objekte oder Zeichen entspricht. Das bedeutet also nicht, daß ein Objekt oder Zeichen nicht gleichzeitig in mehreren Kontexturen auftreten kann – das Gegenteil ist der Fall (vgl. Toth 2009) –, sondern daß zunächst nur so viele Kontexturen angenommen werden, wie es Objekte bzw. Zeichen gibt. Diese Annahme ist allerdings keineswegs zwingend, da wegen der Isomorphie von Objekt und Zeichen nicht nur kein Zeichen, sondern auch kein Objekt isoliert auftritt und somit immer vor dem Hintergrund theoretisch unendlich vieler Kontexturen aufscheint.

3.1. Primzeichen und Primobjekte

$$P = (1, 2, 3) = \begin{cases} P = (1_1, 2_1, 3_1) \\ P = (1_2, 2_2, 3_2) \\ P = (1_3, 2_3, 3_3) \end{cases}$$

$P = (1_1, 2_1, 3_2)$ $\quad$ $P = (1_2, 2_2, 3_1)$

$P = (1_1, 2_2, 3_1)$ $\quad$ $P = (1_2, 2_1, 3_2)$

$P = (1_2, 2_1, 3_1)$ $\quad$ $P = (1_1, 2_2, 3_2)$

$P = (1_1, 2_1, 3_3)$ $P = (1_3, 2_3, 3_1)$

$P = (1_1, 2_3, 3_1)$ $P = (1_3, 2_1, 3_3)$

$P = (1_3, 2_1, 3_1)$ $P = (1_1, 2_3, 3_3)$

$P = (1_1, 2_1, 3_3)$ $P = (1_3, 2_3, 3_1)$

$P = (1_1, 2_3, 3_1)$ $P = (1_3, 2_1, 3_3)$

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$P = (1_2, 2_2, 3_3)$ $P = (1_3, 2_3, 3_2)$

$P = (1_2, 2_3, 3_2)$ $P = (1_3, 2_2, 3_3)$

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$P = (1_1, 2_2, 3_3)$ $P = (1_3, 2_2, 3_1)$

$P = (1_1, 2_3, 3_2)$ $P = (1_2, 2_3, 3_1)$

$P = (1_2, 2_1, 3_3)$ $P = (1_3, 2_1, 3_2)$

3.2. Subzeichen und Subobjekte

$S \subset (P \times P)$

Hier wird nach der Aufhebung der polykontexturalen Defizienz der Nicht-Kontexturiertheit der Objekte die zweite der beiden eingangs festgestellten Defizienzen, diejenige der Nicht-Vermitteltheit der logischen Werte in jeder 2-wertigen Logik, eliminiert, und zwar durch die Einführung von Rändern zwischen den Elementen von Dichotomien, die der logischen 2-wertigen Basisdichotomie isomorph sind. Formal geschieht dies durch den in Toth (2014b) definierten Einbettungsoperator.

$S = [x, [y]]$ $S = [[y], x]$

$S = [[x], y]$ $S = [y, [x]]$

$S = [x_i, [y_i]]$ $S = [[y_i], x_i]$ $S = [[x_i], y_i]$ $S = [y_i, [x_i]]$

$S = [x_j, [y_j]]$ $S = [[y_j], x_j]$ $S = [[x_j], y_j]$ $S = [y_j, [x_j]]$

$S = [x_i, [y_j]]$ $S = [x_j, [y_i]]$

$S = [[y_i], x_j]$ $S = [[y_j], x_i]$

$S = [[x_i], y_j]$ $S = [[x_j], y_i]$

$S = [y_i, [x_j]]$ $S = [y_j, [x_i]]$

3.3. Zeichen und Objekte

$Z = [[3.x], [2.y], [1.z]] \times [[z.1], [y.2], [x.3]]$

3.3.1. Einbettungstransformationen

$[3.x] \rightarrow ([3, [x]], [[x], 3], [[3], x], [x, [3]])$

$[2.y] \rightarrow ([2, [y]], [[y], 2], [[2], y], [y, [2]])$

$[1.z] \rightarrow ([1, [z]], [[z], 1], [[1], z], [z, [1]])$

3.3.2. Kontexturierungstransformationen

$[3.x] \rightarrow ([3, [x]], [[x], 3], [[3], x], [x, [3]])$

$[3, [x]] \rightarrow ([3_i, [x_i]], [3_j, [x_j]], [3_i, [x_j]], [3_j, [x_i]])$

$[[x], 3] \rightarrow ([[x_i], 3_i], [[x_j], 3_j], [[x_i], 3_j], [[x_j], 3_i])$

$[[3], x] \rightarrow ([[3_i], x_i], [[3_j], x_j], [[3_i], x_j], [[3_j], x_i])$

$[x, [3]] \rightarrow ([x_i, [3_i]], [x_j, [3_j]], [x_i, [3_j]], [x_j, [3_i]])$

$[2.y] \to ([2, [y]], [[y], 2], [[2], y], [y, [2]])$

$[2, [y]] \to ([2_i, [y_i]], [2_j, [y_j]], [2_i, [y_j]], [2_j, [y_i]])$

$[[y], 2] \to ([[y_i], 2_i], [[y_j], 2_j], [[y_i], 2_j], [[y_j], 2_i])$

$[[2], y] \to ([[2_i], y_i], [[2_j], y_j], [[2_i], y_j], [[2_j], y_i])$

$[y, [2]] \to ([y_i, [2_i]], [y_j, [2_j]], [y_i, [2_j]], [y_j, [2_i]])$

$[1.z] \to ([1, [z]], [[z], 1], [[1], z], [z, [1]])$

$[1, [z]] \to (([1_i, [z_i]], ([1_j, [z_j]], ([1_i, [z_j]], ([1_j, [z_i]])$

$[[z], 1] \to ([[z_i], 1_i], [[z_j], 1_j], [[z_i], 1_j], [[z_j], 1_i])$

$[[1], z] \to ([[1_i], z_i], [[1_j], z_j], [[1_i], z_j], [[1_j], z_i])$

$[z, [1]] \to ([z_i, [1_i]], [z_j, [1_j]], [z_i, [1_j]], [z_j, [1_i]])$

3.3.3. Permutationstransformationen

$[[3.x], [2.y], [1.z]] \times [[z.1], [y.2], [x.3]] \to$

$([[3.x], [2.y], [1.z]] \times [[z.1], [y.2], [x.3]]$

$[[3.x], [1.y], [2.z]] \times [[z.2], [y.1], [x.3]]$

$[[2.x], [3.y], [1.z]] \times [[z.1], [y.3], [x.2]]$

$[[2.x], [1.y], [3.z]] \times [[z.3], [y.1], [x.2]]$

$[[1.x], [3.y], [2.z]] \times [[z.2], [y.3], [x.1]]$

$[[1.x], [2.y], [3.z]] \times [[z.3], [y.2], [x.1]]$

Anschließend wiederum Anwendung von 2.3.1. und 2.3.2., d.h. die drei Transformationen bilden einen Algorithmus. Da das System der 10 peirce-benseschen Dualsysteme eine Teilmenge der Gesamtmenge der über der Zeichenform $Z = [3.x, 2.y, 1.z]$ durch Filterung der Inklusionsordnung $x \leqq y \leqq z$ mit $x, y, z \in P$ möglichen $3^3 = 27$ Dualsysteme ist, gehen wir dabei von den letzteren aus, d.h. der Algorithmus ist auf das folgende Gesamtsystem anzuwenden. Das Ergebnis ist, wie man leicht feststellen kann, ein enorm komplexes System, das eines ganzen Buches zur vollständigen Darstellung bedürfte.

$Z_1 = [[3.1, 2.1, 1.1] \times [1.1, 1.2, 1.3]]$

$Z_2 = [[3.1, 2.1, 1.2] \times [2.1, 1.2, 1.3]]$

$Z_3 = [[3.1, 2.1, 1.3] \times [3.1, 1.2, 1.3]]$

$Z_4 = [[3.1, 2.2, 1.1] \times [1.1, 2.2, 1.3]]$

$Z_5 = [[3.1, 2.2, 1.2] \times [2.1, 2.2, 1.3]]$

$Z_6 = [[3.1, 2.2, 1.3] \times [3.1, 2.2, 1.3]]$

$Z_7 = [[3.1, 2.3, 1.1] \times [1.1, 3.2, 1.3]]$

$Z_8 = [[3.1, 2.3, 1.2] \times [2.1, 3.2, 1.3]]$

$Z_9 = [[3.1, 2.3, 1.3] \times [3.1, 3.2, 1.3]]$

$Z_{10} = [[3.2, 2.1, 1.1] \times [1.1, 1.2, 2.3]]$

$Z_{11} = [[3.2, 2.1, 1.2] \times [2.1, 1.2, 2.3]]$

$Z_{12} = [[3.2, 2.1, 1.3] \times [3.1, 1.2, 2.3]]$

$Z_{13} = [[3.2, 2.2, 1.1] \times [1.1, 2.2, 2.3]]$

$Z_{14} = [[3.2, 2.2, 1.2] \times [2.1, 2.2, 2.3]]$

$Z_{15} = [[3.2, 2.2, 1.3] \times [3.1, 2.2, 2.3]]$

$Z_{16} = [[3.2, 2.3, 1.1] \times [1.1, 3.2, 2.3]]$

$Z_{17} = [[3.2, 2.3, 1.2] \times [2.1, 3.2, 2.3]]$

$Z_{18} = [[3.2, 2.3, 1.3] \times [3.1, 3.2, 2.3]]$

$Z_{19} = [[3.3, 2.1, 1.1] \times [1.1, 1.2, 3.3]]$

$Z_{20} = [[3.3, 2.1, 1.2] \times [2.1, 1.2, 3.3]]$

$Z_{21} = [[3.3, 2.1, 1.3] \times [3.1, 1.2, 3.3]]$

$Z_{22} = [[3.3, 2.2, 1.1] \times [1.1, 2.2, 3.3]]$

$Z_{23} = [[3.3, 2.2, 1.2] \times [2.1, 2.2, 3.3]]$

$Z_{24} = [[3.3, 2.2, 1.3] \times [3.1, 2.2, 3.3]]$

$Z_{25} = [[3.3, 2.3, 1.1] \times [1.1, 3.2, 3.3]]$

$Z_{26} = [[3.3, 2.3, 1.2] \times [2.1, 3.2, 3.3]]$

$Z_{27} = [[3.3, 2.3, 1.3] \times [3.1, 3.2, 3.3]]$

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Einbettungsstufen von Seinsfunktionen

1. Nach Bense (1976, S. 26) läßt sich ein 4-stufiges relationales System von Seinsfunktionen gemäß dem folgenden Schema aufstellen

Gegenstand	0-stellige Seinsfunktion
Zeichen	1-stellige Seinsfunktion
Bewußtsein	2-stellige Seinsfunktion
Kommunikation	3-stellige Seinsfunktion

(zur Definition von Objekten als 0-stellige Relationen vgl. ferner Bense 1975, S. 65 f.).

2. Bekanntlich kann man ferner die Peanozahlen (die darüber hinaus mit den benseschen Primzeichen, d.h. den von uns so geanannten Zeichenzahlen, isomorph sind, vgl. Bense 1975, S. 168 ff.) durch die beiden Elemente Ø und {Ø} wie folgt definieren (vgl. Wiener 1917).

$$\begin{aligned}
0 &:= \emptyset \\
1 &:= \{0\} = \{\emptyset\} \\
2 &:= \{0, 1\} = \{\emptyset, \{\emptyset\}\} \\
3 &:= \{0, 1, 2\} = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}
\end{aligned}$$

Unter Berücksichtigung der Ergebnisse in Toth (2015) erhalten wir damit für das Bewußtsein als 2-stellige Seinsfunktion

L = [Ø, {Ø}]

ein Quadrupel von Einbettungsrelationen

L_1 = [Ø, [{Ø}]] L_2 = [[{Ø}], Ø]

L_3 = [[Ø], {Ø}] L_4 = [{Ø}, [Ø]].

Für die Kommunikation als 3-stellige Seinsfunktion

L = [Ø, {Ø}, {Ø, {Ø}}]

bekommt man ein 13-tupel von Einbettungsrelationen

$L_1 = [Ø, \{Ø\}, [\{Ø, \{Ø\}\}]]$ $L_2 = [[\{Ø, \{Ø\}\}], \{Ø\}, Ø]]$

$L_3 = [Ø, [\{Ø\}], \{Ø, \{Ø\}\}]$ $L_4 = [\{Ø, \{Ø\}\}, [\{Ø\}], Ø]$

$L_5 = [[Ø], \{Ø\}, \{Ø, \{Ø\}\}]$ $L_6 = [\{Ø, \{Ø\}\}, \{Ø\}, [Ø]]$

$L_7 = [Ø, [\{Ø\}, \{Ø, \{Ø\}\}]]$ $L_8 = [[\{Ø, \{Ø\}\}, \{Ø\}], Ø]$

$L_9 = [[Ø], \{Ø\} [\{Ø, \{Ø\}\}]]$ $L_{10} = [[\{Ø, \{Ø\}\}], \{Ø\}, [Ø]]$

$L_{11} = [[Ø, \{Ø\}], \{Ø, \{Ø\}\}]$ $L_{112} = [\{Ø, \{Ø\}\}, [\{Ø\}, Ø]]$

$L_{13} = [[Ø, \{Ø\}, \{Ø, \{Ø\}\}]]$

Während es also, wie ebenfalls bereits in Toth (2015) festgestellt, bei 2-stelligen Seinsfunktionen nicht etwa 2, sondern 4 nicht-leere Ränder gibt, gibt es bei 3-stelligen Seinsfunktionen nicht weniger als 13 nicht-leere Ränder, die, ohne einen dritten Wert neben Ø und {Ø} einzuführen, als tertia non dantur fungieren.

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Multiset-Relationen

1. In Cantor-Mengentheorien sind Mengen wie z.B.

$M_1 = \{0, 1, 2, 3\}$

$M_2 = \{0, 0, 1, 2, 2, 2, 3\}$

gleich, d.h. es ist $M_1 = M_2$, da mehrfach auftretende Elemente irrelevant sind. In der "Multi-Mengentheorie" (Theory of Multisets) gilt hingegen $M_1 \neq M_2$ (vgl. z.B. Singh et al. 2007). Im folgenden wird gezeigt, daß die Einführung von Multisets besonders für Dichotomien mit Einbettungsoperatoren von großem Nutzen sind (vgl. zuletzt Toth 2015a, b).

2. Unterscheidbarkeit und Nicht-Unterscheidbarkeit bei Cantor-Relationen

Die logische Basisdichotomie

$L = [0, 1]$,

die also eine Relation über der als Wahrheitswerte interpretierten Menge $M = (0, 1)$ definiert, ist hinsichtlich ihrer Relata nicht-unterscheidbar, da sie einen leeren Rand haben

$R[0, 1] = \emptyset$.

Diese Leerheit des Randes gehört zum für die 2-wertige Logik bindenden Gesetz des Tertium non datur, denn z.B. kann ein dritter Wert in einer 3-wertigen Logik

$L = [0, ½, 1]$

als Rand $R[0, 1] = ½$ definiert werden.

Wie wir allerdings bereits in Toth (2014) gezeigt hatten, kann man Ränder auch ohne zusätzliche Werte einführen, und zwar durch Einbettungsoperatoren. Sei

$E: \emptyset \rightarrow [\emptyset]$,

dann bekommen wir für $L = [0, 1]$ das folgende Quadrupel von Relationen

$L_1 = [0, [1]]$ $\qquad$ $L_2 = [[1], 0]$

$L_3 = [[0], 1]$ $\qquad$ $L_4 = [1, [0]]$

mit $L_2 = L_1^{-1}$ und $L_4 = L_3^{-1}$ sowie $R[0, [1]] \neq R[[0], 1] \neq R[1, [0]] \neq R[[1], 0]$.

Wir haben hier also die Einbettung selbst, die als Drittes, freilich als Nicht-Wert, fungiert, d.h. eine Differenz anstatt eines Wertes, die zwischen den beiden Gliedern der Dichotomien vermittelt.

3. Unterscheidbarkeit und Nicht-Unterscheidbarkeit bei Multi-Relationen

In $L = [0, 1]$ kann der eine Wert nicht mehr als die Spiegelung des anderen Wertes sein, denn es gilt ja

$\neg 0 = 1$

$\neg 1 = 0$

und daher

$\neg\neg 0 = 1$

$\neg\neg 1 = 0$.

Man sieht allerdings bereits auf metasemiotischer Ebene, bei den sog. Litotes ("nicht uncool"), daß die Selbstaufhebung von Operatoren durch Iteration ein Unsinn ist. Entsprechend kann der Operator bei eingebetteten Relata nicht mehr funktionieren, d.h. es ist z.B. $\neg 0 \neq \neg[0] \neq \neg[[0]]$, usw. Wenn wir also die rein semiotische, nicht aber ontische Differenz zwischen den Werten 0 und 1 in L = [0, 1] beseitigen und stattdessen schreiben

$L = [0, 0]$

$L = [1, 1]$,

dann werden die Relata 0 und 1 doppeldeutig, insofern wir jeweils zwei Möglichkeiten haben

$L = [0, 0] = [0]$ $\quad\quad$ $L = [1, 1] = [1]$

$L = [0, 0] \neq [0]$ $\quad\quad$ $L = [1, 1] \neq [1]$,

d.h. im jeweils ersten Fall, wo also Gleichheit besteht, handelt es sich um Cantor-Relationen, aber im jeweils zweiten Fall, wo also Nicht-Gleichheit besteht, handelt es sich um Multi-Relationen. Wegen der Möglichkeit der Ungleichheit mehrfach auftretender Relata erhalten wir nun auch dort Quadrupel und nicht nur Paare von Relationen, wo nur ein Relatum anstatt zwei Relata vorliegen

$L_1 = [0, [0]]$ $\quad\quad$ $L_2 = [[0], 0]$

$L_3 = [[0], 0]$ $\quad\quad$ $L_4 = [0, [0]]$,

d.h. es gibt hier die Möglichkeiten

$L_1 = L_4$ oder $L_1 \neq L_4$

$L_2 = L_3$ oder $L_2 \neq L_3$.

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Toth, Alfred, Einbettungsstufen von Seinsfunktionen. In: Electronic Journal for Mathematical Semiotics, 2015b

Polykontexturalität und Pseudo-Polykontexturalität

1. Wie bereits in Toth (2015a) ausgeführt, ist die von Gotthard Günther inaugurierte polykontexturale Logik (vgl. Günther 1976-80, 1991) nur insofern polykontextural, als sie ein Vermittlungssystem (theoretisch unendlich vieler) 2-wertiger Logiken darstellt. Es findet aber weder zwischen den beiden Werten von L = [0, 1] eine Vermittlung statt, noch wird die Konstanz der Objektposition aufgehoben, d.h. es kann lediglich die Subjektposition iteriert werden, so daß sich die polykontexturale Logik von der monokontexturalen aristotelischen Logik lediglich darin unterscheidet, daß sie jedem Subjekt eine eigene, allerdings immer noch 2-wertige, Logik zugesteht.

2. Wie in Toth (2015b) gezeigt, kann man ein Tertium comparationis in L = [0, 1] einführen, ohne einen dritten logischen Wert einzuführen, d.h. ohne gegen die drei Grundgesetze der aristotelischen Logik zu verstoßen. Dies geschieht durch Definition eines Einbettungsoperators E, der einen logischen Wert dem anderen sub- oder superordinierz, der also mit anderen Worten die Juxtaposition der Werte L aufhebt, welche letztlich dafür verantwortlich ist, daß Position und Negation nichts mehr als Spiegelungen von einander sein können. Durch E wird L auf das folgende Quadrupel von Strukturen abgebildet

$$L = [0, 1] \rightarrow$$

$$\left\{ \begin{array}{ll} L_1 = [0, [1]] & L_2 = [[1], 0] \\ L_3 = [[0], 1] & L_4 = [1, [0]] \end{array} \right\}$$

Dieses Verfahren der Einführung von Rändern als tertia comparationis zwischen 0 und 1, die also keine Werte, sondern Einbettungsdifferenzen sind, funktioniert gemäß Toth (2015c) sogar dann, wenn man von einer 1-wertigen Logik ausgeht, vorausgesetzt, man faßt ihre Dichotomien als Multiset-Relationen auf.

$L = [0, 0]$ $\qquad$ $L = [1, 1]$

In diesem Fall gibt es nämlich jeweils zwei Möglichkeiten

$L = [0, 0] = [0]$ $\qquad$ $L = [1, 1] = [1]$

$L = [0, 0] \neq [0]$ $\quad\quad$ $L = [1, 1] \neq [1]$,

und man erhält sogleich

$L_1 = [0, [0]]$ $\quad\quad$ $L_2 = [[0], 0]$

$L_3 = [[0], 0]$ $\quad\quad$ $L_4 = [0, [0]]$

$L_1 = [1, [1]]$ $\quad\quad$ $L_2 = [[1], 1]$

$L_3 = [[1], 1]$ $\quad\quad$ $L_4 = [1, [1]]$.

3. Iteration des Objektes

Schwieriger ist die Beurteilung der Iteration der Objektposition, denn sie verstößt auf jeden Fall gegen die aristotelische Logik, da man in diesem Fall nicht umhin kommt, mindestens éinen weiteren logischen Wert einzuführen. In der 2-wertigen Logik gibt es nur die folgenden 4 möglichen Permutationen

0	0	1	1
0	1	0	1.

Bezeichne im folgenden 0 das Objekt und 1, 2 die beiden Subjekte einer 3-wertigen Logik. Dann gibt es einen Permutationszyklus von 3! = 6 möglichen Wahrheitswertfunktionen, bei denen somit das Objekt konstant bleibt. Umgekehrt kann man aber natürlich auch zwei beliebige Werte als Objekte bestimmen und den verbleibenden Wert als Subjekt setzen, ohne daß sich formal irgend etwas ändert

0	0	1	1	2	2
1	2	0	2	0	1
2	1	2	0	1	0.

Eine echte polykontexturale Logik ist somit erst eine solche, bei der sowohl Objekt- als auch Subjektposition iterierbar sind, d.h. eine minimal 4-wertige Logik mit einem Permutationszyklus von 4! = 24 Wahrheitswertfunktionen.

0_1	0_1	0_1	0_1	0_1	0_1	0_2	0_2	0_2	0_2	0_2	0_2
0_2	0_2	2	2	3	3	0_1	0_1	2	2	3	3
2	3	0_2	3	0_2	2	2	3	0_1	3	0_1	2
3	2	3	0_2	2	0_2	3	2	3	1	2	0_1.

Erst in einer solchen Logik wird also mit der güntherschen Vorstellung ernst gemacht, daß nicht nur zwischen Subjekt und Objekt zu scheiden ist, sondern daß diese durch subjektive Objekte einerseits und durch objektive Subjekte andererseits vermittelt sind (vgl. Günther 1976, S. 249 ff.)

	S	0
S	SS	SO
0	OS	OO

mit

OS = V(O, S)

SO = V(S, O).

Während für die Iteration der Subjektposition das einfache Beispiel zur Illustration genügt, daß dasselbe Objekt, von verschiedenen Subjekten betrachtet, jeweils ein anderes ist – eine Tatsache, die sich besonders durch die semiotischen Abbildungen des konstanten Objektes durch die nicht-konstanten Subjekte nachprüfen läßt, stellt der bekannte Satz des Heraklit, en potamoîs toîs autoîs embaínomen kai ouk embaínomen (in die gleichen Flüsse steigen wir hinein und steigen wir nicht hinen), falls es sich um das gleiche Subjekt handelt, eine Illustration für die Iteration der Objektposition dar. Das Wasser wechselt, da es sich ja um einen Fluß handelt, und das Subjekt, das hinein- und hinaussteigt, bleibt konstant. Eine echte polykontexturale Situation liegt somit dann vor, wenn mindestens zwei Subjekte in den gleichen Fluß hinein- und hinaussteigen. Zum Abschluß stellt sich allerdings die Frage, ob in einer solchen echten polykontexturalen Logik noch zwischen Logik und Ontologie unterschieden werden kann, wie dies Günther im Falle seiner pseudo-polykontexturalen Logik explizit tun konnte (vgl. Günther 1980, S. 146).

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Unvermittelte und vermittelte 2-wertige Permutationszyklen

1. Wie in Toth (2015a) gezeigt wurde, liegt der tiefste Grund dafür, daß die beiden einzigen Werte der 2-wertigen aristotelischen logischen Dichotomie

$L = [0, 1]$

nur Spiegelbilder voneinander sind, nicht an den drei Grundgesetzen des Denkens, d.h. den Sätzen der Identität, des Ausgeschlossenen Dritten und des Verbotenen Widerspruchs, sondern daran, daß sie juxtaponiert sind, d.h. sich auf der gleichen logischen sowie erkenntnistheoretischen Stufe befinden. Daher sind sie auch austauschbar, d.h. es ist

$L = [0, 1] \cong [1, 0]$,

so daß man also eine zur üblichen, auf dem positionalen Wahrheitsbegriff aufgebauten Logik isomorphe Logik konstruieren kann, die auf dem negationalen Falschheitsbegriff aufgebaut ist. (Diese Erkenntnis findet sich, vortheoretisch natürlich, m.W. zum erstenmal in Oskar Panizzas Erzählung "Die Kirche von Zinsblech" (1893).) In einer solchen Logik kann es natürlich nur einen Permutationszyklus von 2! = 4 Wahrheitswertfunktionen geben, deren Werte freilich wiederum juxtaponiert sind

0 0

0 1

1 0

1 1.

2. Führt man jedoch einen Einbettungsoperator E ein, der die Juxtaposition der Werte eliminiert und sie durch Sub- bzw. Superordination substituiert (vgl. Toth 2015b), so ergeben sich nicht 4, sondern 8 Wahrheitswertfunktionen, die sich wie folgt in Tableau-Schreibweise darstellen lassen.

2.1. $L_1 = [0, [0]]$

$$\begin{array}{ll} 0 & Ø \\ \hline Ø & 0 \end{array}$$

2.2. $L_2 = [[0], 0]$

$$\begin{array}{ll} Ø & 0 \\ \hline 0 & Ø \end{array}$$

2.3. $L_3 = [1, [1]]$

$$\begin{array}{ll} 1 & Ø \\ \hline Ø & 1 \end{array}$$

2.4. $L_4 = [[1], 1]$

$$\begin{array}{ll} Ø & 1 \\ \hline 1 & Ø \end{array}$$

2.5. $L_5 = [0, [1]]$

$$\begin{array}{ll} 0 & Ø \\ \hline Ø & 1 \end{array}$$

2.6. $L_6 = [[0], 1]$

$$\begin{array}{ll} Ø & 1 \\ \hline 0 & Ø \end{array}$$

2.7. $L_7 = [1, [0]]$

$$\begin{array}{ll} 1 & Ø \\ \hline Ø & 0 \end{array}$$

2.8. $L_8 = [[1], 0]$

$$\frac{\emptyset \quad 0}{1 \quad \emptyset}$$

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Logische Vermittlung durch Differenz

1. Bekanntlich verbietet das Gesetz des Ausgeschlossenen Dritten, daß die beiden Werte der 2-wertigen aristotelischen Basisdichotomie

$L = [0, 1]$

vermittelt sind, d.h. daß es einen Rand gibt, der eine Partizipationsrelation beider Werte darstellt. Logiken der Form

$L = [0, ½, 1]$

sind daher 3-wertig, und bei ihnen ist das Gesetz des Tertium comparationis durch ein Gesetz des Quartum comparations substituiert. Wie jedoch in Toth (2015a, b) gezeigt wurde, kann man logische Vermittlung durch nicht-wertige Differenz einführen, indem man einen Einbettungsoperator E definiert, welcher die Juxtaposition der beiden Werte in L = [0, 1] aufhebt. Dadurch erhält man ein Quadrupel der Form

$E(L) =$

$L_1 = [0, [1]]$ $\qquad$ $L_3 = [1, [0]]$

$L_2 = [[0], 1]$ $\qquad$ $L_4 = [[1], 0]$,

worin die logischen Relationen gleichzeitig als Ränder von L = [0, 1] fungieren.

2. Erst in einer solchen 2-wertigen Logik mit differentieller Vermittlung wird also mit der güntherschen Vorstellung ernst gemacht, daß nicht nur zwischen Subjekt und Objekt zu scheiden ist, sondern daß diese durch subjektive Objekte einerseits und durch objektive Subjekte andererseits vermittelt sind (vgl. Günther 1976, S. 249 ff.)

	S	0
S	SS	SO
0	OS	OO

mit

OS = V(O, S)

SO = V(S, O).

Wie in Toth (2015c) gezeigt, kann man die in L = [0, 1] möglichen 2 Permutationszyklen mit homogenen Wertfunktionen wie folgt durch 4 Tableaux darstellen.

2.1. L_1 = [0, [0]]

0	Ø
Ø	0

2.2. L_2 = [[0], 0]

Ø	0
0	Ø

2.3. L_3 = [1, [1]]

1	Ø
Ø	1

2.4. L_4 = [[1], 1]

Ø	1
1	Ø

Das bedeutet nun allerdings, daß die wiederum 4 möglichen Tableaux für die beiden Permutationszyklen mit heterogenen Wahrheitswertfunktionen genau den erkenntnistheoretischen Vermittlungen

OS = V(O, S)

SO = V(S, O)

korrespondieren.

2.5. $L_5 = [0, [1]]$

$$\begin{array}{cc} 0 & \emptyset \\ \hline \emptyset & 1 \end{array}$$

2.6. $L_6 = [[0], 1]$

$$\begin{array}{cc} \emptyset & 1 \\ \hline 0 & \emptyset \end{array}$$

2.7. $L_7 = [1, [0]]$

$$\begin{array}{cc} 1 & \emptyset \\ \hline \emptyset & 0 \end{array}$$

2.8. $L_8 = [[1], 0]$

$$\begin{array}{cc} \emptyset & 0 \\ \hline 1 & \emptyset. \end{array}$$

Literatur

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Toth, Alfred, Unvermittelte und vermittelte 2-wertige Permutationszyklen. In: Electronic Journal for Mathematical Semiotics, 2015c

Ontische Werte-Tableaux I

1. Bekanntlich erzeugt der in Toth (2014) eingeführte Einbettungsoperator E ein Quadrupel von Strukturen, welche die in der Basisdichotomie L = [0, 1] juxtaponierten Werte entweder durch sub- bzw. superordinierte oder durch prä- bzw. postponierte substituiert (vgl. Toth 2015a).

$$E(L) = \left(\begin{array}{ll} L_1 = [0, [1]] & L_3 = [1, [0]] \\ L_2 = [[0], 1] & L_4 = [[1], 0] \end{array} \right)$$

2. Wie bereits in Toth (2015b) angedeutet, kann man deswegen ontische Tableaux definieren, auf denen diese Werte zweidimensional durch die beiden Erscheinungsformen von E, d.h. als ⇅-Einbettung oder als ⇆-Einbettung, angeordnet werden können. Damit werden ontische, semiotische und logische Werte – und damit sämtliche Werte der vollständigen Objekthierarchie

$$H = \Omega \subset \{\Omega\} \subset \{\{\Omega\}\} \subset \{\{\{\Omega\}\}\}$$

(vgl. Toth 2015c) – mit Hilfe dieser Tableaux in Funktion von metaphysischen Orten gesetzt, allerdings nicht wie in der polykontexturalen Logik nur von subjektabhängigen Orten, sondern gleichfalls von objektabhängigen Orten (vgl. Toth 2015d), und ferner wird, da E lediglich als Differenz, aber nicht als zusätzlicher Wert zwischen den Werten von L = [0, 1] vermittelt, wegen Nicht-Verletzung des Tertium-Gesetzes die 2-wertige aristotelische Logik nicht aufgehoben.

2.1. Werte-Tableau für Null-Objekte

$$\begin{array}{ll} \varnothing & \varnothing \\ \varnothing & \varnothing \end{array}$$

2.2. Werte-Tableaux für 1-Objekte

$$\begin{array}{ll} \varnothing & 1 \\ \varnothing & \varnothing \end{array} \qquad \begin{array}{ll} 1 & \varnothing \\ \varnothing & \varnothing \end{array} \qquad \begin{array}{ll} \varnothing & \varnothing \\ 1 & \varnothing \end{array} \qquad \begin{array}{ll} \varnothing & \varnothing \\ \varnothing & 1 \end{array}$$

2.3. Werte-Tableaux für 2-Objekte

0	1	Ø	Ø	Ø	1	1	Ø	0	Ø	Ø	0
Ø	Ø	0	1	0	Ø	Ø	0	1	Ø	Ø	1

1	0	Ø	Ø	Ø	0	0	Ø	1	Ø	Ø	1
Ø	Ø	1	0	1	Ø	Ø	1	0	Ø	Ø	0

Beispielsweise ergibt sich für den semiotischen Wert $S = \langle 1.3 \rangle$, d.h. das Legizeichen

1	3	Ø	Ø	Ø	3	3	Ø	1	Ø	Ø	1
Ø	Ø	1	3	1	Ø	Ø	1	3	Ø	Ø	3

3	1	Ø	Ø	Ø	1	1	Ø	3	Ø	Ø	3
Ø	Ø	3	1	3	Ø	Ø	3	1	Ø	Ø	1,

d.h. nicht-ortsfunktional eingeführte Subzeichen und Zeichen können mit Hilfe der Werte-Tableaux auf 12-fache Weise differenziert werden, indem sie auf alle in einem 2-dimensionalen Raum möglichen Einbettungsstrukturen auf ihre metaphysischen Orte abgebildet werden.

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Ontische Werte-Tableaux II

1. Die in Teil I (vgl. Toth 2015a) präsentierten Werte-Tableaux, welche veranschaulichen, wie der in Toth (2014) eingeführte Einbettungsoperator E, allein durch Differenz, d.h. ohne einen zusätzlichen Wert einzuführen, die in der Basisdichotomie L = [0, 1] juxtaponierten Werte entweder durch sub- bzw. superordinierte oder durch prä- bzw. postponierte substituiert (vgl. Toth 2015b),

$$E(L) = \left(\begin{array}{ll} L_1 = [0, [1]] & L_3 = [1, [0]] \\ L_2 = [[0], 1] & L_4 = [[1], 0] \end{array} \right)$$

können natürlich wegen der Ergebnisse in Toth (2015b) sowohl mittels $E_{\uparrow\downarrow}$ als auch mittels $E_{\leftrightarrows}$ dargestellt werden, d.h. die in Toth (2015a) präsentierten Tableaux sind Abstraktionen von der Zweidimensionalität der durch E bewirkten Einbettungen. Für praktische Anwendungen auf reale Objekte mögen daher die im folgenden beigebrachten Differenzierungen von Nutzen sein.

2.1. Werte-Tableau für Null-Objekte

2.1.1. $E_{\uparrow\downarrow}$

$$\begin{array}{cc} \varnothing & \varnothing \\ \hline \varnothing & \varnothing \end{array}$$

2.1.2. $E_{\leftrightarrows}$

$$\begin{array}{c|c} \varnothing & \varnothing \\ \varnothing & \varnothing \end{array}$$

2.2. Werte-Tableaux für 1-Objekte

2.2.1. $E_{\uparrow\downarrow}$

$$\frac{\emptyset \quad 1}{\emptyset \quad \emptyset} \qquad \frac{1 \quad \emptyset}{\emptyset \quad \emptyset} \qquad \frac{\emptyset \quad \emptyset}{1 \quad \emptyset} \qquad \frac{\emptyset \quad \emptyset}{\emptyset \quad 1}$$

2.2.2. $E_{\leftrightarrows}$

$$\begin{array}{c|c} \emptyset & 1 \\ \emptyset & \emptyset \end{array} \qquad \begin{array}{c|c} 1 & \emptyset \\ \emptyset & \emptyset \end{array} \qquad \begin{array}{c|c} \emptyset & \emptyset \\ 1 & \emptyset \end{array} \qquad \begin{array}{c|c} \emptyset & \emptyset \\ \emptyset & 1 \end{array}$$

2.3. Werte-Tableaux für 2-Objekte

2.3.1. $E_{\uparrow\downarrow}$

$$\frac{0 \quad 1}{\emptyset \quad \emptyset} \qquad \frac{\emptyset \quad \emptyset}{0 \quad 1} \qquad \frac{\emptyset \quad 1}{0 \quad \emptyset} \qquad \frac{1 \quad \emptyset}{\emptyset \quad 0} \qquad \frac{0 \quad \emptyset}{1 \quad \emptyset} \qquad \frac{\emptyset \quad 0}{\emptyset \quad 1}$$

$$\frac{1 \quad 0}{\emptyset \quad \emptyset} \qquad \frac{\emptyset \quad \emptyset}{1 \quad 0} \qquad \frac{\emptyset \quad 0}{1 \quad \emptyset} \qquad \frac{0 \quad \emptyset}{\emptyset \quad 1} \qquad \frac{1 \quad \emptyset}{0 \quad \emptyset} \qquad \frac{\emptyset \quad 1}{\emptyset \quad 0}$$

2.3.2. $E_{\leftrightarrows}$

$$\begin{array}{c|c} 0 & 1 \\ \emptyset & \emptyset \end{array} \qquad \begin{array}{c|c} \emptyset & \emptyset \\ 0 & 1 \end{array} \qquad \begin{array}{c|c} \emptyset & 1 \\ 0 & \emptyset \end{array} \qquad \begin{array}{c|c} 1 & \emptyset \\ \emptyset & 0 \end{array} \qquad \begin{array}{c|c} 0 & \emptyset \\ 1 & \emptyset \end{array} \qquad \begin{array}{c|c} \emptyset & 0 \\ \emptyset & 1 \end{array}$$

$$\begin{array}{c|c} 1 & 0 \\ \emptyset & \emptyset \end{array} \qquad \begin{array}{c|c} \emptyset & \emptyset \\ 1 & 0 \end{array} \qquad \begin{array}{c|c} \emptyset & 0 \\ 1 & \emptyset \end{array} \qquad \begin{array}{c|c} 0 & \emptyset \\ \emptyset & 1 \end{array} \qquad \begin{array}{c|c} 1 & \emptyset \\ 0 & \emptyset \end{array} \qquad \begin{array}{c|c} \emptyset & 1 \\ \emptyset & 0 \end{array}$$

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Zählen mit Peanozahlen in ontischen Tableaux

1. Die in Toth (2015a) eingeführten ontischen Tableaux kann man als Strukturen für völlig neue Zählweisen von Peanozahlen anwenden. Zur Erinnerung sei wiederholt, daß die Anwendung des Einbettungsoperators auf die logische Basisdichotomie L = [0, 1] (sowie allen ihr isomorphen ontischen und semiotischen Dichotomien) für 2 Werte ein Quadrupel von $2^2 = 4$ Strukturen erzeugt, d.h. die Juxtaposition der Werte in L entweder in eine Relation von Subordination/Superordination oder in eine Relation von Präposition/ Postposition transformiert (vgl. Toth 2015b). Dadurch werden Paare von Werten durch Differenzen (und also nicht durch Einführung eines dritten Wertes, welcher dem Gesetz des Verbotenen Dritten widerspräche) vermittelt und sind also nicht mehr länger bloße Spiegelbilder voneinander, wie das in der klassischen aristotelischen Logik der Fall ist.

2.1. P = {0}

2.1.1. Vorwärtszählung

0 Ø

2.1.2. Rückwärtszählung

Ø 0

Streng genommen sind hier Vor- und Rückwärtszählung gar nicht unterscheidbar, vgl. die Nullstellen vor den Proto-, Deutero- und Tritozahlen in der Mathematik der Qualitäten (Kronthaler 1986).

2.2. P = {0, 1}

2.2.1. Vorwärtszählung

0	1	Ø	Ø	Ø	1	1	Ø	0	Ø	Ø	0
Ø	Ø	0	1	0	Ø	Ø	0	1	Ø	Ø	1

2.2.2. Rückwärtszählung

1	0	Ø	Ø	Ø	0	0	Ø	1	Ø	Ø	1
Ø	Ø	1	0	1	Ø	Ø	1	0	Ø	Ø	0

2.3. P = {0, 1, 2}

2.3.1. Kontinuierliche Zählung

2.3.1.1. Vorwärtszählung

0	Ø	Ø	Ø	0	Ø	Ø	Ø	0
1	Ø	Ø	Ø	1	Ø	Ø	Ø	1
2	Ø	Ø	Ø	2	Ø	Ø	Ø	2

0	1	2	Ø	Ø	Ø	Ø	Ø	Ø
Ø	Ø	Ø	0	1	2	Ø	Ø	Ø
Ø	Ø	Ø	Ø	Ø	Ø	0	1	2

0	Ø	Ø	Ø	Ø	0
Ø	1	Ø	Ø	1	Ø
Ø	Ø	2	2	Ø	Ø

2.3.1.2. Rückwärtszählung

2	Ø	Ø	Ø	2	Ø	Ø	Ø	2
1	Ø	Ø	Ø	1	Ø	Ø	Ø	1
0	Ø	Ø	Ø	0	Ø	Ø	Ø	0

2	1	0		Ø	Ø	Ø		Ø	Ø	Ø
Ø	Ø	Ø		2	1	0		Ø	Ø	Ø
Ø	Ø	Ø		Ø	Ø	Ø		2	1	0

2	Ø	Ø		Ø	Ø	2
Ø	1	Ø		Ø	1	Ø
Ø	Ø	0		0	Ø	Ø

2.3.2. Diskontinuierliche Zählung

2.3.2.1. Vorwärtszählung

0	Ø	1		0	Ø	Ø		0	Ø	Ø
Ø	2	Ø		1	Ø	2		Ø	Ø	Ø
Ø	Ø	Ø		Ø	Ø	Ø	...	Ø	1	2

2.3.2.2. Rückwärtszählung

2	Ø	1		2	Ø	Ø		2	Ø	Ø
Ø	0	Ø		1	Ø	0		Ø	Ø	Ø
Ø	Ø	Ø		Ø	Ø	Ø	...	Ø	1	0.

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Ontische Tableaux für komplexe Zahlen

1. In Toth (2015a) hatten wir gezeigt, wie man mit Hilfe der in Toth (2015b) eingeführten ontischen Tableaux Peano-Zahlen auf strukturell verschiedene Weise zählen kann. Im folgenden beschränken wir uns auf Teilmengen mit zwei Zahlen.

2. P = {0, 1}

2.1. Vorwärtszählung

0	1	Ø	Ø	Ø	1	1	Ø	0	Ø	Ø	0
Ø	Ø	0	1	0	Ø	Ø	0	1	Ø	Ø	1

2.2. Rückwärtszählung

1	0	Ø	Ø	Ø	0	0	Ø	1	Ø	Ø	1
Ø	Ø	1	0	1	Ø	Ø	1	0	Ø	Ø	0

3. Die in Kap. 2 präsentierten zwei Mal sechs Zählstrukturen unterscheiden allerdings nicht zwischen den beiden möglichen Einbettungstypen des Einbettungsoperator E, der sowohl als $E_{\uparrow\downarrow}$ als auch als $E_{\leftrightarrows}$ auftreten kann (vgl. Toth 2015c).

3.1. Subordinative und superordinative Einbettung

$$E_{\uparrow\downarrow} = \begin{pmatrix} L_1 = [0, [1]] & L_3 = [1, [0]] \\ L_2 = [[0], 1] & L_4 = [[1], 0] \end{pmatrix}$$

3.2. Präpositionale und postpositionale Einbettung

$$E_{\leftrightarrows} = \begin{pmatrix} L_1 = [0, \boxed{1}]] & L_3 = [1, \boxed{0}]] \\ L_2 = [\boxed{0}, 1] & L_4 = [\boxed{1}, 0] \end{pmatrix}$$

Die beiden Einbettungstypen sind jedoch durch eine einfache Transformation aufeinander abbildbar

τ: ⇅ → ⇆

mit

τ_1: $L_1 \rightarrow L_2$ τ_2: $L_3 \rightarrow L_4$

τ_1^{-1}: $L_2 \rightarrow L_1$ τ_2^{-1}: $L_4 \rightarrow L_3$,

d.h. die in Kap. 2. präsentierten Strukturen beinhalten bereits alle möglichen Fälle von P = {0, 1}. Allerdings ist es möglich, $E_{⇅}$ und E⇆ gleichzeitig auf P anzuwenden. Man erhält dann Tableaux, bei denen keine Ø-Stellen mehr auftreten, allerdings ist das Ergebnis in beiden möglichen Fällen doppeldeutig.

$$3.3.1.\ L_1 \cup L_2 = \left\{ \begin{array}{l} [0, 1] \\ [[0], [1]] \end{array} \right. = \begin{array}{cc} 0 & 1 \\ \hline 0 & 1 \end{array}$$

$$3.3.2.\ L_3 \cup L_4 = \left\{ \begin{array}{l} [1, 0] \\ [[1], [0]] \end{array} \right. = \begin{array}{cc} 0 & 1 \\ \hline 0 & 1 \end{array}$$

In beiden Fällen ist die resultante Struktur entweder koordinativ, d.h. die Differenzen zwischen Sub- und Superordination sowie zwischen Prä- und Potposition werden nicht nur unter sich, sondern auch zwischen einander aufgehoben ([0, 1] und [1, 0]), oder jede Zahl wird separat eingebettet ([[0], [1]) und [[1], [0]]).

3.4. Wegen dieser Einbettungsambiguität kann man nun ferner die durch die folgenden Matrizen definierbaren vier Arten von komplexen Zahlen (entsprechend den vier Quadranten des Gaußschen Zahlenfeldes)

$z =$		$\bar{z} =$		$-z =$		$-\bar{z} =$	
0	-1	0	1	-0	1	-0	-1
1	0	-1	0	-1	-0	1	-0

ebenfalls durch ontische Tableaux definieren. Ontisch handelt es sich bei ihnen also um doppelt eingebettete Objekte. Da man nun mathematische Komplexität durch ontische Copossessivität (und umgekehrt) definieren kann (vgl. Toth 2014), so kann man die vier Arten von komplexen Zahlen wie folgt ontisch charakterisieren: Bei den beiden nicht-negativen komplexen Zahlen weisen die beiden Diagonalen der Matrix gleiche Zahlenwerte auf, die sich nur in einer der beiden Diagonalen durch Negativität maximal einer Zahl auszeichnen. Es gibt somit mathematisch zwar Possessivität von Possessivität, aber keine Copossessivität von Copossessivität. Ontisch hingegen tritt die letztere Möglichkeit in der Form von verdoppelter Exessivität auf, vgl. das folgende Beispiel, in welchem sich ein exessives Fenster innerhalb einer Nische befindet.

Langackerstr. 18, 8057 Zürich

Bei den beiden negativen komplexen Zahlen hingegen treten negative Zahlen in beiden Diagonalen der Matrizen auf, allerdings gilt hier, daß nicht sowohl Possessivität von Possessivität als auch Copossessivität von Copossessivität gleichzeitig auftreten kann. In dieser Hinsicht decken sich also die mathematischen Tableaux mit den ontischen, denn es gibt keine Objekte, die gleichzeitig, d.h. ohne ein weiteres Referenzsystem oder ohne Perspektivenwechsel, exessiv und adessiv sind.

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Subjektive Objekte und objektive Subjekte

1. Die 2-wertige aristotelische Logik, welche die arithmetische Form L = [0, 1] hat, worin es also zwischen den beiden Zahlwerten 0 und 1 keine Vermittlung gibt, kennt natürlich weder subjektive Objekte noch objektive Subjekte, sondern nur Objekte und Subjekte, als deren Zahlwerte jeweils 0 oder 1 fungieren können. Wie in Toth (2014) gezeigt, ist es allerdings nicht nötig, einen dritten Zahlwert einzuführen und somit gegen das logische Grundgesetz des Tertium non datur zu verstoßen, um eine Vermittlung in L zu bewirken, denn man kann eine nicht-materielle, differentielle Vermittlung durch Einführung eines Einbettungsoperators E bewirken, der Zahlwerte auf verschiedene Abbildungsstufen abbildet

E_0: 0 → [0]

E_1: 1 → [1],

in Relationalzahlschreibweise (vgl. Toth 2015a)

E: $n_m \rightarrow n_{m+1}$.

2. Die Anwendung von E auf L, d.h. die Abbildung

e: E → L = [[0, [1]], [[0], 1]], [1, [0]], [[1], 0]]]

ergibt somit nicht-leere Ränder zwischen 0 und 1 und etabliert ferner ontische Orte, d.h. er setzt die Zahlen, die ja sowohl für Objekte als auch für Zeichen stehen können, als ortsfunktionale. Dabei besitzt eine 2-elementige Menge wie L 2 mal 6 sog. Tableaux, welche die Struktur der Ortsfunktionalität von Zahlen angeben

0	1	Ø	Ø	Ø	1	1	Ø	0	Ø	Ø	0
Ø	Ø	0	1	0	Ø	Ø	0	1	Ø	Ø	1

1	0	Ø	Ø	Ø	0	0	Ø	1	Ø	Ø	1
Ø	Ø	1	0	1	Ø	Ø	1	0	Ø	Ø	0.

3. Wie in Toth (2015b) gezeigt, definieren diese 12 zahlentheoretischen Tableaux zugleich die 12 möglichen Ränder von $R[0, 1] \neq R[1, 0]$. Da man

$0 = \Omega$

$1 = Z$

setzen kann, gibt es also genau 12 Ränder zwischen Objekt und Zeichen, die mithilfe von ortsfunktionalen Zahlen differenzierbar sind, und zwar, wie bereits gesagt, ohne die Grundlagen der 2-wertigen aritstotelischen Logik zu verletzen. Subjektive Objekte haben somit andere Ränder als sie objektive Subjekte haben, und da das Zeichen in den beiden möglichen Definitionen

$Z^* = [Z, \Omega]$

$\Omega^* = [\Omega, Z]$

die logische Subjektposition einnimmt, haben wir die beiden Isomorphien

$[Z, \Omega] \cong [\Sigma, \Omega]$

$[\Omega, Z] \cong [\Omega, \Sigma]$.

Subjektive Objekte sind damit genau diejenigen Objekte, welche als Rand die Menge der geordneten Paare $[Z, \Omega]$ haben, und objektive Subjekte sind genau diejenigen Subjekte, welche als Rand die Menge der geordneten Paare $[\Omega, Z]$ haben.

Zur Menge der $[Z, \Omega]$ gehören neben allen wahrgenommenen Objekten auch Objekte, an denen, wie die Umgangssprache sagt, "ein Subjekt hängt" (merkwürdigerweise ist die konverse Relation, daß ein Objekt an einem Subjekt hänge, ungrammatisch, obwohl in einer Logik, in der das Objekt tot, d.h. objektiv, ist, genau die umgekehrte Grammatikalitätsverteilung zu erwarten wäre), wie z.B. der Teddybär des Sohnes, die Puppe der Tochter, die Vuitton-Tasche der Mutter und der Oldtimer des Vaters.

Zur Menge der $[\Omega, Z]$ gehören neben allen zum Zeichen erklärten Objekten, d.h. den benseschen Metaobjekten, diejenigen Subjekte, die von einem Ich-Subjekt aus das Du-Subjekt darstellen, nicht aber Er-Subjekte, denen der gleiche Status wie den (objektiven) Objekten zukommt, nämlich die logische Objektposition. (Eine Unterscheidung zwischen Er-Subjekten und Es-Objekten bedürfte tatsächlich einer mindestens 3-wertigen, nicht-aristotelischen Logik.)

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Objektabhängigkeit und Ortsabhängigkeit von Zahlen

1. Da man n-tupel von Zahlen durch Paare darstellen kann, sind die Peanozahlen, wenn man sie als Paare der Form (n, (n+1)) betrachtet, paarweise 2-seitig objektabhängig (vgl. Toth 2015a), aber sie sind, wie alle Zahlen der klassischen Arithmetik, nicht ortsabhängig. Diese Ortsabhängigkeit gilt jedoch für die von Zahlen gezählten Objekte, da der ontische Satz gilt, daß jedes Objekt ortsfunktional ist, d.h. $\Omega = f(\omega)$ ist (vgl. Toth 2015b). Nun sind Zahlen jedoch formal gesehen Zeichen, d.h. Mengen von Objekten (vgl. Toth 2015c), aber sie bilden keine 2-wertigen logischen Dichotomien, wie dies die Zeichen in den beiden möglichen Formen $Z^* = [Z, \Omega]$ und $\Omega^* = [\Omega, Z]$ tun. Um also die für Objekte und, vermöge ontisch-semiotischer Isomorphie, auch für Zeichen gültige Ortsabhängigkeit neben der Objektabhängigkeit einzuführen, müssen Zählweisen gefunden werden, bei denen Zahl und Gezähltes eine zu Z^* bzw. Ω^* isomorphie 2-wertige Dichotomie bilden. Wie in Toth (2014) gezeigt, kann dies ohne Verletzung des logischen Drittensatzes nicht-substantiell, d.h. nicht durch Einführung eines dritten Wertes, durch den differentiell wirkenden Einbettungsoperator E geschehen. Dadurch ist es möglich, jedes Paar von Peanozahlen auf genau 12 Tableaux abzubilden, die sich in nicht-objektabhängige und objektabhängige unterteilen lassen.

2.1. Nicht-objektabhängige Dichotomien

[0, 1] =			[1, 0] =	
0	1		1	0
Ø	Ø		Ø	Ø
[[0, 1]] =			[[1, 0]]	
Ø	Ø		Ø	Ø
0	1		1	0

[[0], [1]] =	[[1], [0]] =
0 Ø	1 Ø
1 Ø	0 Ø
[[[0], [1]]] =	[[[1], [0]]] =
Ø 0	Ø 1
Ø 1	Ø 0

2.2. Objektabhängige Dichotomien

[[0], 1] =	[[1], 0] =
Ø 1	Ø 0
0 Ø	1 Ø
[0, [1]] =	[1, [0]] =
0 Ø	1 Ø
Ø 1	Ø 0

Literatur

Toth, Alfred, Einbettungsoperatoren. In: Electronic Journal for Mathematical Semiotics, 2014

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Chiastische Zyklen ortsfunktionaler Zahlen

1. Bekanntlich ist die von Gotthard Günther inaugurierte Polykontexturalitätstheorie ein n-wertiges Vermittlungssystem 2-wertiger Logiken, in dem jeder ontologische Ort einer Subjektposition korrespondiert, d.h. Kontexturen sind ausschließlich subjektfunktional, denn nur das Subjekt ist iterierbar, das Objekt in der n-wertigen Günther-Logik ist genauso ein totes, nicht-reflektierbares Objekt wie es dies in der 2-wertigen aristotelischen Logik ist. Ferner gibt es für jedes Subjekt in der ihm abgebildeten 2-wertigen Logik weiterhin keine Vermittlung für die Werte der logischen Basisdichotomie L = [0, 1], denn die güntherschen Rejektionswerte betreffen ja die ganzen Alternativen von L, d.h. bei der Einführung eines neuen Wertes 2 wird sowohl 0 als auch 1 verworfen, wodurch das Ausgeschlossenheitsgesetz der klassischen Logik, das im Falle der 2-wertigen als Tertium non datur erscheint, einfach zu einem Quartum non datur wird, entsprechend geht es weiter bei der Einführung weiterer neuer Werte. In Sonderheit gibt es somit auch keine Vermittlung zwischen der Menge L = [0, 1] und der Menge der Rejektionswerte, d.h. die beiden Mengen von Zahlen haben einen leeren Rand, wie dies im Falle von 0 und 1 in L der Fall ist, die sich aus diesem Grunde wie Spiegelbilder voneinander darstellen. Wenn nun Mitterauer glaubt, daß "the function of self-reference is permanently integrating polyontological and disontological realities, so that we are aware of our personal ego" (2013, S. 514), so folgt daraus, daß diese mysteriöse Selbstreferenz (wovon eigentlich?) zwischen einer Menge von Werten und der Menge ihrer Rejektionswerte vermittelt. Wie wir aber soeben gezeigt haben, ist der Rand zwischen den beiden Mengen leer, d.h. unvermittelt, und somit gibt es dort auch nichts, was selbstreferent sein und vermitteln könnte.

2. Hingegen kann man unter Verzicht auf die Einführung neuer Werte, die wegen der Nicht-Iterierbarkeit des Objekts und der Absenz einer Vermittlung zwischen den Werten von L ohnehin sinnlos ist, jedoch durch Einführung eines nicht-substantiellen und damit nicht-wertigen, sondern rein differentiell fungierenden Einbettungsoperators (vgl. Toth 2014) Zahlen definieren, die nicht nur objektabhängig, sondern auch ortsabhängig sind (vgl. Toth 2015a, b). Wie im folgenden gezeigt wird, lassen sich diese nicht gegen den logischen Drittensatz verstoßenden ortsfunktionalen Zahlen in 3 chiastischen Zyklen

darstellen, bei denen somit paarweise Vermittlung stattfindet und Selbstreferenz möglich ist.

2.1. Chiastischer Zyklus von [0, 1] × [1, 0] und [[0, 1]] × [[1, 0]]

[0, 1] =		[1, 0] =	
0	1	1	0
Ø	Ø	Ø	Ø
[[0, 1]] =		[[1, 0]]	
Ø	Ø	Ø	Ø
0	1	1	0

2.2. Chiastischer Zyklus von [[0], [1]] × [[1], [0]] und [[[0], [1]]] × [[[1], [0]]]

[[0], [1]] =		[[1], [0]] =	
0	Ø	1	Ø
1	Ø	0	Ø
[[[0], [1]]] =		[[[1], [0]]] =	
Ø	0	Ø	1
Ø	1	Ø	0

2.3. Chiastischer Zyklus von [[0], 1] × [1, [0]] und [0, [1]] × [[1], 0]

[[0], 1] =		[1, [0]] =	
Ø	1	1	Ø
0	Ø	Ø	0
[0, [1]] =		[1, [0]] =	
0	Ø	Ø	0
Ø	1	1	Ø

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Reflexivität, Dualität und Selbstreferenz in der Semiotik

1. In der klassischen Semiotik, die auf der 2-wertigen aristotelischen Logik basiert, fallen Reflexivität, Dualität und Selbstreferenz für jedes Subzeichen und damit auch für die aus ihnen konstruierten Zeichenklassen und Realitätsthematiken zusammen, d.h. es gilt

$\times(x.y) = (y.x)$

mit $x, y \in \{1, 2, 3\}$.

2. Führt man jedoch den Einbettungsoperator ein, der differentiell, aber nicht-substantiell operiert, d.h. der die Gültigkeit des logischen Drittensatzes nicht außer Kraft setzt, kann man, wie in Toth (2015a) gezeigt, jedes Subzeichen der Form

$S = \langle x.y \rangle$

auf eine Menge von 12 zahlentheoretischen Tableaux abbilden, die sich in 6 zueinander duale einteilen lassen.

[x, y] =		[y, x] =	
x	y	y	x
Ø	Ø	Ø	Ø
[[x, y]] =		[[y, x]]	
Ø	Ø	Ø	Ø
x	y	y	x
[[x], [y]] =		[[y], [x]] =	
x	Ø	y	Ø
y	Ø	x	Ø

[[[x], [y]]] = [[[y], [x]]] =

Ø x Ø y

Ø y Ø x

[[x], y] = [[y], x] =

Ø y Ø x

x Ø y Ø

[x, [y]] = [y, [x]] =

x Ø y Ø

Ø y Ø x

Diese 6 dualen Paare lassen sich nun nach dem in Toth (2015b) gezeigten Schema in 3 Zyklen chiastischer Relationen darstellen.

2.1. Chiastischer Zyklus von [x, y] × [y, x] und [[x, y]] × [[y, x]]

[x, y] = [y, x] =

x y y x

Ø Ø Ø Ø

[[x, y]] = [[y, x]]

Ø Ø Ø Ø

x y y x

2.2. Chiastischer Zyklus von [[x], [y]] × [[y], [x]] und [[[x], [y]]] × [[[y], [x]]]

[[x], [y]] =		[[y], [x]] =	
x	Ø	y	Ø
y	Ø	x	Ø
[[[x], [y]]] =		[[[y], [x]]] =	
Ø	x	Ø	y
Ø	y	Ø	x

2.3. Chiastischer Zyklus von [[x], y] × [y, [x]] und [x, [y]] × [[y], x]

[[x], y] =		[y, [x]] =	
Ø	y	y	Ø
x	Ø	Ø	x
[x, [y]] =		[y, [x]] =	
x	Ø	Ø	x
Ø	y	y	Ø

Selbstreferenz gibt es somit nur bei den 3 chiastischen Zyklen, welche zwischen nicht-reflexiven Paaren dualer Paare dyadischer semiotischer Relationen vermitteln, d.h. Reflexivität, Dualität und Selbstreferenz koinzidieren bei ortsfunktionalen Zeichen nicht.

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Die Enttrivialisierung der Logik

1. Zwei für unser Thema einschlägige Sätze aus Wittgensteins "Tractatus" (1980) lauten

6.1. Die Sätze der Logik sind Tautologien.

6.1251 Darum kann es in der Logik auch nie Überraschungen geben.

Den Grund, warum das so ist, vermag Wittgenstein, der ganz auf dem Boden der klassischen 2-wertigen aristotelischen Logik steht, freilich nicht zu geben. Der Versuch der polykontexturalen Günther-Logik, in der es nur Subjekt-, aber keine Objektkontexturen und ferner keine Vermittlung zwischen den für jede Kontextur gültigen Werten der klassischen Dichotomie L = [0, 1] gibt, die Logik zu enttrivialisieren, muß nach der Argumentation in Toth (2015a) als fehlgeschlagen betrachtet werden. Hingegen können durch Definition eines Einbettungsoperators, der logische Werte auf ontische Orte (die nicht mit ontologischen Orten zu verwechseln sind) abbildet, differentielle, d.h. nicht-substantielle Vermittlungen zwischen 0 und 1 in L eingeführt werden, ohne gegen den logischen Drittensatz zu verstoßen (vgl. Toth 2014). Auf diese Weise wird die aristotelische Logik nicht außer Kraft gesetzt, aber sie wird zu einem Spezialfall einer ortsfunktionalen Logik und dadurch, wie im folgenden andeutungsweise aufgezeigt wird, enttrivialisiert.

2.1. Negation in einer ortsfunktionalen 2-wertigen Logik

N[0, 1] =			N[1, 0] =	
1	0		0	1
Ø	Ø		Ø	Ø
N[[0, 1]] =			N[[1, 0]]	
Ø	Ø		Ø	Ø
1	0		0	1

N[[0], [1]] =			N[[1], [0]] =	
1	Ø		0	Ø
0	Ø		1	Ø
N[[[0], [1]]] =			N[[[1], [0]]] =	
Ø	1		Ø	0
Ø	0		Ø	1
N[[0], 1] =			N[[1], 0] =	
Ø	0		Ø	1
1	Ø		0	Ø
N[0, [1]] =			N[1, [0]] =	
1	Ø		0	Ø
Ø	0		Ø	1

2.2. Konjunktion und Disjunktion in einer ortsfunktionalen 2-wertigen Logik

Die klassischen aristotelischen Wahrheitsfunktionen für Konkjunktion und Disjunktion sind bekanntlich

p	q	$p \wedge q$
0	0	0
0	1	1
1	0	1
1	1	1

p	q	$p \vee q$
0	0	0
0	1	0
1	0	0
1	1	1

In einer ortsfunktionalen Logik korrespondiert jede der vier Wahrheitswertkombinationen [0, 0], [0, 1], [1, 0] und [1, 1] einem 12-tupel von Wahrheitswertfeldern oder Tableaux, für welche die Resultate für Konjunktion und

Disjunktion nur für diese vier juxtaponierten Fälle, die den beiden Strukturen [0, 1] und [1, 0] entsprechen, nicht aber für die nicht-jutaponierten Fälle

[0, [1]], [[0], 1], [[1], 0], [1, [0]], [[0], [1]], [[1], [0]], [[0, 1]], [[1, 0]], [[[0], [1]]] und [[[1], [0]]]

gültig sind. Zur Bestimmung der Resultate der letzteren müssen die in Toth (2015b) eingeführten Relationalzahlen verwendet werden.

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Der leere Rand zwischen einem Objekt und seiner Reflexion

1. In Wittgensteins "Tractatus" (vgl. Wittgenstein 1980) findet sich im Paragraphen 5.513 die bemerkenswerte Festellung: "Zwei Sätze sind einander entgegengesetzt, wenn sie nichts miteinander gemein haben, und: Jeder Satz hat nur ein Negativ, weil es nur einen Satz gibt, der ganz außerhalb seiner liegt".

2. Ontisch gesehen liegt Wittgensteins korrekte Feststellung daran, daß die Werte 0 und 1 in der logischen Basisdichotomie L = [0, 1] unvermittelt sind, da das logische Grundgesetz des Ausgeschlossenen Dritten die Existenz eines Wertes 2 mit L' = [0, 2, 1] verbietet. In einer 3-wertigen Logik der Form L' gälte also

$$2 = R[0, 1] = R[1, 0],$$

d.h. trotz eines nun nicht mehr leeren Randes blieben die beiden Elemente 0 und 1 der Dichotomie L austauschbar, d.h. es wäre

$$L' = [0, 2, 1] = [1, 2, 0],$$

und somit ändert sich abgesehen von der Nicht-Leerheit des Randes durch die Abbildung l: L → L' überhaupt nichts, denn sowohl in L als auch in L' kann man eine Logik sowohl auf der Position 0 als auch auf der Negation 1 aufbauen, und die beiden daraus resultierenden Logik werden einander isomorph sein.

3. Einführung von Werten, d.h. von Substanz, nützt also nichts, um zu verhindern, daß sich ein Objekt und seine Reflexion nicht mehr austauschen lassen, d.h. daß ein erkenntnistheoretischer Unterschied zwischen einem Objekt und seinem Spiegelbild auf logischer Ebene existiert, der doch auf ontischer Ebene realiter vorhanden ist. Die zahlreichen literarischen und bildnerischen Darstellungen des Aus-dem-Spiegel-Tretens legen davon Zeugnis ab: " 'Laß mir dein Spiegelbild, du innig Geliebter, es soll mein und bei mir bleiben immerdar'. – 'Giulietta', rief Erasmus ganz verwundert, 'was meinst du denn? – Mein Spiegelbild?' [...]. Da rief Erasmus, wahnsinnig vor tötendem Liebesschmerz: 'Muß ich denn fort von dir? – Muß ich fort, so soll mein Spiegelbild dein bleiben auf ewig und immerdar. Keine Macht – der Teufel soll es dir nicht entreißen, bis du mich selbst hast mit Seele und Leib'. Giuliettas Küsse brannten wie Feuer auf

seinem Munde, als er dies gesprochen, dann ließ sie ihn los und streckte sehnsuchtsvoll die Arme aus nach dem Spiegel. Erasmus sah, wie sein Bild unabhängig von seinen Bewegungen hervortrat, wie es in Giuliettas Arme glitt, wie es mit ihr im seltsamen Duft verschwand" (E.T.A. Hoffmann, Die Abenteuer der Silvesternacht).

Hingegen kann man, wie dies in Toth (2014) vorgeschlagen wurde, einen nicht-substantiellen, sondern differentiellen Einbettungsoperator E als Abbildung der Form

$$E: \quad x \rightarrow [x]$$

mit $x \in \{0, 1\}$ definieren. Dadurch wird $L = [0, 1]$ auf 12 mögliche ortsfunktionale Zahlfelder abgebildet (vgl. Toth 2015a)

$$\begin{array}{cc} 0 & 1 \\ \emptyset & \emptyset \end{array} \qquad \begin{array}{cc} \emptyset & \emptyset \\ 0 & 1 \end{array} \qquad \begin{array}{cc} \emptyset & 1 \\ 0 & \emptyset \end{array} \qquad \begin{array}{cc} 1 & \emptyset \\ \emptyset & 0 \end{array} \qquad \begin{array}{cc} 0 & \emptyset \\ 1 & \emptyset \end{array} \qquad \begin{array}{cc} \emptyset & 0 \\ \emptyset & 1 \end{array}$$

$$\begin{array}{cc} 1 & 0 \\ \emptyset & \emptyset \end{array} \qquad \begin{array}{cc} \emptyset & \emptyset \\ 1 & 0 \end{array} \qquad \begin{array}{cc} \emptyset & 0 \\ 1 & \emptyset \end{array} \qquad \begin{array}{cc} 0 & \emptyset \\ \emptyset & 1 \end{array} \qquad \begin{array}{cc} 1 & \emptyset \\ 0 & \emptyset \end{array} \qquad \begin{array}{cc} \emptyset & 1 \\ \emptyset & 0, \end{array}$$

dessen Ränder nun nicht nur nicht-leer sind, sondern erkenntnistheoretisch geschiedene ontische Loci thematisieren (vgl. Toth 2015b)

$$[0, 1] = \begin{array}{cc} 0 & 1 \\ \emptyset & \emptyset \end{array} \qquad [1, 0] = \begin{array}{cc} 1 & 0 \\ \emptyset & \emptyset \end{array}$$

$$R[0, 1] = [[0, 1], [0, \emptyset], [\emptyset, \emptyset], [\emptyset, 1]]$$

$$R[1, 0] = [[1, 0], [\emptyset, 0], [\emptyset, \emptyset], [1, \emptyset]]$$

[[0, 1]] = [[1, 0]]

Ø Ø Ø Ø

0 1 1 0

R[[0, 1]] = [[[0, 1]], [[0, Ø]], [[Ø, Ø]], [[Ø, 1]]]

R[[1, 0]] = [[[1, 0]], [[Ø, 0]], [[Ø, Ø]], [[1, Ø]]]

[[0], [1]] = [[1], [0]] =

0 Ø 1 Ø

1 Ø 0 Ø

R[0, 1] = [[[0], [1]], [[0], [Ø]], [[Ø], [Ø]], [[Ø], [1]]]

R[1, 0] = [[[1], [0]], [[Ø], [0]], [[Ø], [Ø]], [[1], [Ø]]]

[[[0], [1]]] = [[[1], [0]]] =

Ø 0 Ø 1

Ø 1 Ø 0

R[0, 1] = [[[[0], [1]]], [[[0], [Ø]]], [[[Ø], [Ø]]], [[[Ø], [1]]]]

R[1, 0] = [[[[1], [0]]], [[[Ø], [0]]], [[[Ø], [Ø]]], [[[1], [Ø]]]]

[[0], 1] = [[1], 0] =

Ø 1 Ø 0

0 Ø 1 Ø

R[[0], 1] = [[Ø, 1], [Ø, 0], [0, Ø], [Ø, 1]]

R[[1], 0] =[[Ø, 0], [Ø, 1], [1, Ø], [Ø, 0]]

[0, [1]] =		[1, [0]] =	
0	Ø	1	Ø
Ø	1	Ø	0

R[0, [1]] = [[0, Ø], [0, Ø], [Ø, 1], [1, Ø]]

R[1, [0]] = [[1, Ø], [1, Ø], [Ø, 0], [0, Ø]].

In einer Logik, in der vermöge E die Ortsfunktionalität der beiden Werte 0 und 1 in L = [0, 1] gilt, die jedoch in verschiedenen Einbettungsstufen auftreten, stellt also die klassische aristotelische Logik nur vermöge der juxtaponierten Wert-Strukturen eine Teillogik dar. Im Gegensatz zur polykontexturalen Logik, die mehrwertig ist, bleibt eine solche ortsfunktionale Logik jedoch 2-wertig, und ein differentielles statt eines substantiellen Tertiums wird durch das Gesetz des Tertium non datur ja nicht ausgeschlossen. Das bedeutet also, daß in einer solchen Logik, in der die Operatoren nicht über 2 juxtaponierten, sondern über 12 juxtaponierten und nicht-juxtaponierten ontischen Werten operieren, Wittgensteins Feststellung lediglich einen trivialen Sonderfall darstellt, allerdings einen, welcher der ontischen Situation, daß ein Objekt, das gespiegelt wird, nie mit seinem Spiegelbild identisch sein kann und daß es somit auch keine Austauschrelationen à la Giulietta geben kann, widerspricht. Eine solche Logik beschreibt also keineswegs die Welt, geschweige denn ist sie mit ihr identisch, wie dies Wittgenstein verschiedentlich behauptet, sondern eine solche Logik ist eine Kontradiktion der Ontik, und sie beschreibt nichts weniger als die Welt der Objekte und der mit ihr isomorphen Zeichen.

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Logischer und ontischer Ort

1. In Wittgensteins "Tractatus" (vgl. Wittgenstein 1980) liest man:

3.4. Der Satz bestimmt einen Ort im logischen Raum. Die Existenz dieses logischen Ortes ist durch die Existenz der Bestandteile allein verbürgt, durch die Existenz des sinnvollen Satzes.

Danach gibt es also keinen vorgegebenen logischen Ort, auf den die Werte der logischen Basisdichotomie L = [0, 1] abgebildet werden. Dies ist durchaus korrekt, denn es ist ja L = [0, 1] = [1, 0]. Am besten hatte diesen Sachverhalt bereits Günther formuliert: "Beide Werte einer solchen Logik aber sind metaphysisch äquivalent. Das heißt, man kann sie beliebig miteinander vertauschen. Sie verhalten sich zueinander in einer totalen logischen Disjunktion, wie rechts und links. Es gibt keinen theoretischen Grund, welche Seite rechts und welche Seite links von der Zugspitze ist. Die Benennung beruht auf einer willkürlichen Entscheidung, und wenn man seinen Standpunkt wechselt, sind die rechte und die linke Seite miteinander vertauscht (2000, S. 230 f.). In Sonderheit bedeutet dies, daß L auf keinen Fall als Abbildung der Form

f: $[0, 1] \rightarrow [Ø, Ø]$

darstellbar ist, darin Ø eine Leerstelle markiert, die mit 0 oder mit 1 belegbar ist.

2. Dennoch ist es, wie in Toth (2015a) gezeigt, möglich, die 2-wertige aristotelische Logik durch Einführung eines Einbettungsoperators E

E: $x \rightarrow [x]$

mit $x \in L$

auf ein System von 12 Zahlfeldern abzubilden, in denen die Werte von L somit 12 verschiedene ontische Orte einnehmen.

[0, 1] =		[1, 0] =	
0	1	1	0
Ø	Ø	Ø	Ø

$R[0,1] = [[0,1],[0,\emptyset],[\emptyset,\emptyset],[\emptyset,1]]$

$R[1,0] = [[1,0],[\emptyset,0],[\emptyset,\emptyset],[1,\emptyset]]$

$[[0,1]] =$			$[[1,0]]$	
Ø	Ø		Ø	Ø
0	1		1	0

$R[[0,1]] = [[[0,1]],[[0,\emptyset]],[[\emptyset,\emptyset]],[[\emptyset,1]]]$

$R[[1,0]] = [[[1,0]],[[\emptyset,0]],[[\emptyset,\emptyset]],[[1,\emptyset]]]$

$[[0],[1]] =$			$[[1],[0]] =$	
0	Ø		1	Ø
1	Ø		0	Ø

$R[0,1] = [[[0],[1]],[[0],[\emptyset]],[[\emptyset],[\emptyset]],[[\emptyset],[1]]]$

$R[1,0] = [[[1],[0]],[[\emptyset],[0]],[[\emptyset],[\emptyset]],[[1],[\emptyset]]]$

$[[[0],[1]]] =$			$[[[1],[0]]] =$	
Ø	0		Ø	1
Ø	1		Ø	0

$R[0,1] = [[[[0],[1]]],[[[0],[\emptyset]]],[[[\emptyset],[\emptyset]]],[[[\emptyset],[1]]]]$

$R[1,0] = [[[[1],[0]]],[[[\emptyset],[0]]],[[[\emptyset],[\emptyset]]],[[[1],[\emptyset]]]]$

$[[0],1] =$			$[[1],0] =$	
Ø	1		Ø	0
0	Ø		1	Ø

$R[[0],1] = [[\emptyset,1],[\emptyset,0],[0,\emptyset],[\emptyset,1]]$

$R[[1],0] =[[\emptyset,0],[\emptyset,1],[1,\emptyset],[\emptyset,0]]$

[0, [1]] =		[1, [0]] =	
0	Ø	1	Ø
Ø	1	Ø	0

R[0, [1]] = [[0, Ø], [0, Ø], [Ø, 1], [1, Ø]]

R[1, [0]] = [[1, Ø], [1, Ø], [Ø, 0], [0, Ø]].

Wie man erkennt, verletzt eine solche Logik, die durch L* = [L, E] darstellbar ist, die 2-wertige Logik nicht, da E als nicht-substantielles, sondern differentielles "Tertium" wirkt. Die bemerkenswerteste Tatsache besteht aber darin, daß in diesem für eine 2-elementige Menge wie L = [0, 1] minimalen System von 12 Zahlfeldern die Größe des Zahlfeldes variabel ist. Während in der polykontexturalen Logik in einer Kontextur der Länge 2 die beiden Proto-, Deutero- und Tritozahlen [0, 1], [1, 0] und [0, 0] = [1, 1] darstellbar sind, also nur 2 ontische Orte benötigen, benötigen sie in L* 4 ontische Orte, aber sie können auch auf Zahlfelder mit mehr ontischen Orten abgebildet werden, denn sobald eine Leerstelle Ø mit 0 oder 1 belegt wird, wird nicht nur Ø → 0 oder Ø → 1 abgebildet, sondern der Raum erweitert sich gleichzeitig, d.h. es besteht ein Zusammenspiel zwischen Kontraktion und Distraktion der Menge ontischer Orte bei jeder Abbildung, und zwar im Gegensatz zur polykontexturalen Logik unabhängig davon, ob L durch Rejektionswerte erweitert wird, wie z.B. bei

			0	Ø	2
0	2	→	Ø	Ø	Ø
Ø	1		Ø	Ø	1,

oder ob keine Rejektionswerte auftreten, denn gemäß dem in Toth (2015b) formulierten Satz, daß nicht nur kein Zeichen, sondern vermöge ontisch-semiotischer Isomorphie auch kein Objekt allein auftreten kann, gilt z.B. auch

			0	Ø	Ø
0	Ø	→	Ø	Ø	Ø
Ø	1		Ø	Ø	1.

Damit korrespondiert die ontisch überprüfbare Tatsache, daß es in einem 3-dimensionalen Raum immer eine Möglichkeit gibt, ein weiteres Objekt vor, hinter, über oder neben einem Objekt (bzw. zwischen zwei Objekten) zu plazieren.

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Negation und ontischer Ort

1. Wir beginnen mit einem Satz aus Wittgensteins "Tractatus" (vgl. Wittgenstein 1980).

5.44 Wenn man z.B. eine Bejahung durch doppelte Verneinung erzeugen kann, ist dann die Verneinung – in irgendeinem Sinn – in der Bejahung enthalten? Verneint "$\neg\neg$ p" $\neg$p, oder bejaht es p; oder beides?

Man sollte allerdings auch den folgenden Satz hinzuziehen

4.0621 Daß aber die Zeichen "p" und "$\neg$p" das gleiche sagen *können*, ist wichtig. Denn es zeigt, daß dem Zeichen "$\neg$" in der Wirklichkeit nichts entspricht.

2. Der letztere Satz, daß es keine ontische Negation gibt, ist trivial. Interessanter wäre daher, sich mit der Frage zu beschäftigen, warum es denn überhaupt eine logische Negation gibt, da es doch auch keine semiotische Negation gibt. Hinzu kommt, daß die doppelte Verneinung metasemiotisch nicht-gleich der Bejahung ist und daß auch die einfache Negation auf metasemiotischer Ebene sogar als Nicht-Verneinung auftreten kann, vgl.

Ich werde nicht weichen, bis Du nicht bekommen bist.

Wittgensteins ganz auf die Logik beschränktes Problem der Negation sowie der Negation der Negation und ihr Verhältnis zur Position läßt sich jedoch sogleich beseitigen, wenn man den Spezialfall der Juxtaposition der beiden logischen Werte in der logischen Basisdichotomie L = [0, 1] als einer Menge von Relationen von 0 und 1 betrachtet, die auch eingebettet, d.h. in Subposition auftreten können. Wie bereits in Toth (2015a) gezeigt, kann man durch einen nicht gegen die 2-wertige Logik verstoßenden differentiellen Einbettungsoperator E als Abbildung der Form

$$E: \quad x \rightarrow [x]$$

mit $x \in \{0, 1\}$ definieren. Dadurch wird L = [0, 1] auf 12 mögliche ortsfunktionale Zahlfelder abgebildet

0	1	Ø	Ø	Ø	1	1	Ø	0	Ø	Ø	0
Ø	Ø	0	1	0	Ø	Ø	0	1	Ø	Ø	1

1	0	Ø	Ø	Ø	0	0	Ø	1	Ø	Ø	1
Ø	Ø	1	0	1	Ø	Ø	1	0	Ø	Ø	0,

und die Negation in einer solchen 2-wertigen Logik, in welcher E die Zahlwerte auf 12 ontische Orte abbildet, wird damit sogleich enttrivialisiert (vgl. Toth 2015b).

N[0, 1] =		N[1, 0] =	
1	0	0	1
Ø	Ø	Ø	Ø
N[[0, 1]] =		N[[1, 0]]	
Ø	Ø	Ø	Ø
1	0	0	1
N[[0], [1]] =		N[[1], [0]] =	
1	Ø	0	Ø
0	Ø	1	Ø
N[[[0], [1]]] =		N[[[1], [0]]] =	
Ø	1	Ø	0
Ø	0	Ø	1
N[[0], 1] =		N[[1], 0] =	
Ø	0	Ø	1
1	Ø	0	Ø

N[0, [1]] =			N[1, [0]] =	
1	Ø		0	Ø
Ø	0		Ø	1.

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Die chiastischen Relationen ontischer Orte von Zahlen

1. Wie in Toth (2015a) gezeigt, ist es möglich, die 2-wertige aristotelische Logik durch Einführung eines Einbettungsoperators E

E: $x \rightarrow [x]$

mit $x \in (L = [0, 1])$

auf ein System von 12 Zahlfeldern abzubilden, in denen die Werte von L somit 12 verschiedene ontische Orte einnehmen.

2. Im folgenden zeigen wir, daß die in Toth (2015b) bestimmten vier Ränder für jedes der 12 Zahlfelder chiastische Relationen bilden. Diese bestimmen somit die ontischen Orte von Zahlen wie umgekehrte die ontischen Orte von Zahlen die chiastischen Relationen bestimmen.

2.1.

[0, 1] =		[1, 0] =	
0	1	1	0
Ø	Ø	Ø	Ø

R[0, 1] = [[0, 1], [0, Ø], [Ø, Ø], [Ø, 1]]

R[1, 0] = [[1, 0], [Ø, 0], [Ø, Ø], [1, Ø]]

[0, 1], [0, Ø]	[Ø, Ø], [Ø, 1]
×	×
[Ø, 0], [1, 0]	[1, Ø], [Ø, Ø]

2.2.

[[0, 1]] =		[[1, 0]]	
Ø	Ø	Ø	Ø
0	1	1	0

$R[[0, 1]] = [[[0, 1]], [[0, Ø]], [[Ø, Ø]], [[Ø, 1]]]$

$R[[1, 0]] = [[[1, 0]], [[Ø, 0]], [[Ø, Ø]], [[1, Ø]]]$

$[[0, 1]], [[0, Ø]] \qquad [[Ø, Ø]], [[Ø, 1]]$

$\times \qquad \times$

$[[Ø, 0]], [[1, 0]] \qquad [[1, Ø]], [[Ø, Ø]]$

2.3.

$[[0], [1]] = \qquad [[1], [0]] =$

0	Ø	1	Ø
1	Ø	0	Ø

$R[0, 1] = [[[0], [1]], [[0], [Ø]], [[Ø], [Ø]], [[Ø], [1]]]$

$R[1, 0] = [[[1], [0]], [[Ø], [0]], [[Ø], [Ø]], [[1], [Ø]]]$

$[[0], [1]], [[0], [Ø]] \qquad [[Ø], [Ø]], [[Ø], [1]]$

$\times \qquad \times$

$[[Ø], [0]], [[1], [0]] \qquad [[1], [Ø]], [[Ø], [Ø]]$

2.4.

$[[[0], [1]]] = \qquad [[[1], [0]]] =$

Ø	0	Ø	1
Ø	1	Ø	0

$R[0, 1] = [[[[0], [1]]], [[[0], [Ø]]], [[[Ø], [Ø]]], [[[Ø], [1]]]]$

$R[1, 0] = [[[[1], [0]]], [[[Ø], [0]]], [[[Ø], [Ø]]], [[[1], [Ø]]]]$

[[[0], [1]]], [[[0], [Ø]]]		[[[Ø], [Ø]]], [[[Ø], [1]]]

×		×

[[[Ø], [0]]], [[[1], [0]]]		[[[1], [Ø]]], [[[Ø], [Ø]]]

2.5.

[[0], 1] =		[[1], 0] =

Ø	1		Ø	0

0	Ø		1	Ø

R[[0], 1] = [[Ø, 1], [Ø, 0], [0, Ø], [Ø, 1]]

R[[1], 0] =[[Ø, 0], [Ø, 1], [1, Ø], [Ø, 0]]

[Ø, 1], [Ø, 0]		[0, Ø], [Ø, 1]

×		×

[Ø, 0], [Ø, 1]		[1, Ø], [Ø, 0]

2.6.

[0, [1]] =		[1, [0]] =

0	Ø		1	Ø

Ø	1		Ø	0

R[0, [1]] = [[0, Ø], [0, Ø], [Ø, 1], [1, Ø]]

R[1, [0]] = [[1, Ø], [1, Ø], [Ø, 0], [0, Ø]]

[0, Ø], [0, Ø]		[Ø, 1], [1, Ø]

×		×

[Ø, 0], [0, Ø]		[1, Ø], [1, Ø]

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Identität und Identifizierbarkeit

1. "Die konstituierte Identität von Zeichenklasse und Realitätsthematik des 'ästhetischen Zustandes' besagt selbstverständlich auch, daß er nur als 'Repräsentamen' und nicht als 'Präsentamen' identifiziert werden kann; d.h. das 'Repräsentierte' ist das 'Präsentierte' (der 'Schein' die 'Realität'). Was so nur durch sich selbst identifiziert werden kann, ist nur durch sich selbst gegeben, ein Original und nur ideeierend (...) übertragbar" (Bense 1979, S. 116).

2. Bense bezieht sich hier (wie später hauptsächlich in seinem letzten Buch von 1992) auf die Dualidentität von Zeichen- und Realitätsthematik des semiotischen Dualsystems

$DS = (3.1, 2.2, 1.3) \times (3.1, 2.2, 1.3)$,

in dem also

$\times(3.1, 2.2, 1.3) = (3.1, 2.2, 1.3)$

gilt. Diese Identität kann es jedoch nur in einer solchen 2-wertigen Logik geben, welche an der beliebigen Austauschbarkeit der Werte in $L = [0, 1]$ festhält. Bereits Günther hatte festgestellt: "Beide Werte einer solchen Logik aber sind metaphysisch äquivalent. Das heißt, man kann sie beliebig miteinander vertauschen. Sie verhalten sich zueinander in einer totalen logischen Disjunktion, wie rechts und links. Es gibt keinen theoretischen Grund, welche Seite rechts und welche Seite links von der Zugspitze ist. Die Benennung beruht auf einer willkürlichen Entscheidung, und wenn man seinen Standpunkt wechselt, sind die rechte und die linke Seite miteinander vertauscht (2000, S. 230 f.).

3. Führt man hingegen, wie dies in Toth (2014) und nachfolgenden Arbeiten (v.a. Toth 2015) getan wurde, einen Einbettungsoperator ein, der die Juxtaposition der Werte zwar als Spezialfall gelten läßt, sie jedoch vermöge

$$\begin{array}{cccccccccc} 0 & 1 & & Ø & Ø & & 0 & Ø & & Ø & 0 \\ Ø & Ø & & 0 & 1 & & 1 & Ø & & Ø & 1 \end{array}$$

1	0	Ø	Ø	1	Ø	Ø	1
Ø	Ø	1	0	0	Ø	Ø	0

mehrdeutig werden läßt und als Gesamtsystem für L = [0, 1] 12 ontische Orte einführt

0	1	Ø	Ø	Ø	1	1	Ø	0	Ø	Ø	0
Ø	Ø	0	1	0	Ø	Ø	0	1	Ø	Ø	1

1	0	Ø	Ø	Ø	0	0	Ø	1	Ø	Ø	1
Ø	Ø	1	0	1	Ø	Ø	1	0	Ø	Ø	0,

deren Teilmenge somit die vier Juxtapositionen sind, so fällt die Identität einer solchen Logik, die man einbettungstheoretisch nennen könnte, für alle nicht-juxtapositiven Strukturen, d.h. für

S = [0, [1]]   S = [[1], 0]

S = [[0], 1]   S = [1, [0]]

dahin, denn es gilt

$\times$[0, [1]] $\neq$ [[1], 0]

$\times$[[0], 1] $\neq$ [1, [0]].

3. Was hingegen die Identifizierbarkeit betrifft, so steht sie entgegen Benses Angaben in überhaupt keinem Zusammenhang mit der logischen oder semiotischen Identität, sondern mit der Selbstgegebenheit der von Zeichen bezeichneten Objekte, und diese tritt bekanntlich nur in der Form von Selbst-identität auf. Für das eigenreale Dualsystem könnte dies also nur bedeuten, daß Zeichenthematik und Realitätsthematik einander gleich sind. Wären sie nämlich identisch, wären sich folglich gar nicht unterscheidbar, aber das sind sie für Bense, der ja seine gesamte Argumentation auf diesem Unterschied aufbaut.

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Zur Arithmetik ontischer Einbettung

1. Wie bekannt (vgl. Toth 2015a, b), beruht die klassische Peano-Arithmetik (wie natürlich die gesamte quantitative Mathematik) auf dem Grundschema der zweiwertigen aristotelischen Logik, $L = [0, 1]$, darin die Werte nicht nur vermöge des Satzes vom Ausgeschlossenen Dritten unvermittelt sein müssen, sondern worin sie aus diesem Grunde auch in keinem Abhängigkeitsverhältnis zueinander bzw. voneinander stehen dürfen. 0 und 1 sind somit juxtaponiert und können daher beliebig ausgetauscht werden (vgl. Günther 2000, S. 230 f.), d.h. sie sind spiegelbildlich. Man kann nun allerdings, ohne einen materialen dritten Wert einzuführen, mittels der Einführung eines Einbettungsoperators E die beiden Werte in ein Abhängigkeitsverhältnis setzen, d.h. es gilt entweder $0 = f(1)$ oder $1 = f(0)$. Da E iterierbar ist, kann arithmetische Einbettung mehrstufig sein, d.h. eine Zahl kann n-fach eingebettet sein mit $n \geqq 0$. In einer solchen Arithmetik stellen also die Peanozahlen den Spezialfall für $n = 0$ dar.

2. Grundtypen einer 3-stufigen Arithmetik

2.1. 0-stufige Einbettung

Die Folge der Peanozahlen ist linear, d.h. sie ist einem "Gänsemarsch" vergleichbar (vgl. Kronthaler 1990).

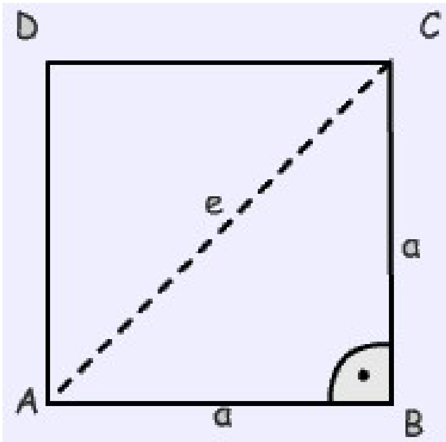
Für $n = 0$ gibt es somit nur eine Ordnung O.

2.1.1. $O = 1$

$S = [0, 1, 2]$

2.2. 1-stufige Einbettung

Für $n = 1$ wechselt die 1-dimensionale zu einer 2-dimensionalen Zählung. Diese betrifft allerdings nicht nur die Horizontale und die Vertikale, sondern auch die Diagonale, d.h. wir haben nun im Gegensatz zur einen und einzigen Peanozählweise bereits drei Zählarten.



2.2.1. O = (0, 1)

S = [[0], 1, 2] S = [2, 1, [0]]

S = [0, [1], 2] S = [2, [1], 0]

S = [0, 1, [2]] S = [[2], 1, 0]

S = [[0, 1], 2] S = [2, [1, 0]]

S = [0, [1, 2]] S = [[2, 1], 0]

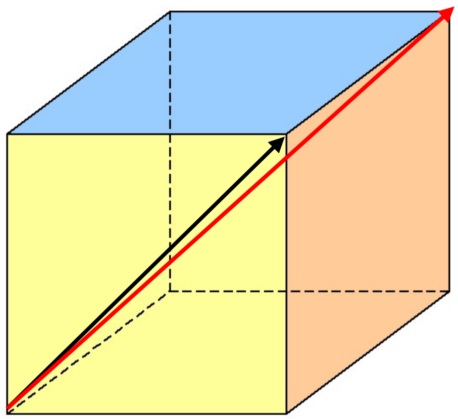
S = [[0], 1, [2]] S = [[2], 1, [0]]

2.2.2. O = 1

S = [[0, 1, 2]] S = [[2, 1, 0]]

2.3. 2-stufige Einbettung

Für n = 2 entstehen aus den 2-dimensionalen nun 3-dimensionale Zählarten. Man beachte, daß jede Seite des Kubus natürlich den Fall n = 1 repräsentiert, d.h. die drei 2-dimensionalen Zählarten sind in den 3-dimensionalen enthalten. Während der schwarze Pfeil eine 1-stufige Einbettung beinhaltet, beinhaltet der rote Pfeil eine 2-stufige Einbettung.



2.3.1. O = 2

S = [[[0, 1, 2]]]

2.3.2. O = (2, 0)

S = [[[0]], 1, 2] S = [2, 1, [[0]]]

S = [0, [[1]], 2] S = [2, [[1]], 0]

S = [0, 1, [[2]]] S = [[[2]], 1, 0]

S = [[[0, 1]], 2] S = [2, [[1, 0]]]

S = [0, [[1, 2]]] S = [[[2, 1]], 0]

S = [[[0]], 1, [[2]]] S = [[[2]], 1, [[0]]]

2.3.3. O = (2, 1)

S = [[[0]], [1], [2]] S = [[2], [1], [[0]]]

S = [[[1]], [0], [2]] S = [[2], [0], [[1]]]

S = [[[2], [0], [1]] S = [[1], [0], [[2]]]

S = [[[0, 1]], [2]] S = [[2], [[1, 0]]]

S = [[[0, 2]], [1]] S = [[1], [[2, 0]]]

S = [[[1, 2]], [0]]   S = [[0], [[2, 1]]]

2.3.4. O = (2, 1, 0)

S = [[[0]], [1], 2]   S = [2, [1], [[0]]]

S = [[[1]], [0], 2]   S = [2, [0], [[1]]]

S = [[[2]], [0], 1]   S = [1, [0], [[2]]]

2.3.5. O = (2, 2, 0)

S = [[[0]], [[1]], 2]   S = [2, [[1]], [[0]]]

S = [[[1]], [[0]], 2]   S = [2, [[0]], [[1]]]

S = [[[2]], [[0]], 1]   S = [1, [[0]], [[2]]]

2.3.6. O = (2, 2, 1)

S = [[[0]], [[1]], [2]]   S = [[2], [[1]], [[0]]]

S = [[[1]], [[0]], [2]]   S = [[2], [[0]], [[1]]]

S = [[[2]], [[0]], [1]]   S = [[1], [[0]], [[2]]]

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Einbettungstheoretische Semiotik I

1. Im Anschluß an Toth (2015a-c) definieren wir im folgenden eine Semiotik, welche im Gegensatz zu derjenigen von Peirce und Bense über eingebettete Zeichenzahlen verfügt (vgl. Toth 2014). Sie enthält die peirce-bensesche Semiotik, geht aber weit über sie hinaus.

2.1. Definition der Zeichenrelation

$Z = [(1, 2, 3), E]$,

darin E der Einbettungsoperator ist mit

$E(1) = [1]$

$E(2) = [2]$

$E(3) = [3]$

2.2. Jede dyadische Subrelation der Form $S = [x.y]$ (mit, $x, y \in \{1, 2, 3\}$) läßt sich durch Anwendung von E in vier möglichen Formen notieren

$[x, [y]] \qquad [[y], x]$

$[[x], y] \qquad [y, [x]]$.

Damit steht der paarweisen Ungleichung nicht-eingebetteter Subrelationen

$[x.y] \neq [y.x]$

das Quadrupel von Ungleichungen

$$\begin{array}{ccc} [x, [y]] & \neq & [[y], x] \\ \neq & & \neq \\ [[x], y] & \neq & [y, [x]] \end{array}$$

gegenüber.

2.3. Das vollständige System semiotischer Subrelationen

[1, [1]]	[[1], 1]	[1, [2]]	[[2], 1]	[1, [3]]	[[3], 1]
[[1], 1]	[1, [1]]	[[2], 1]	[1, [2]]	[[3], 1]	[1, [3]]
[2, [1]]	[[2], 1]	[2, [2]]	[[2], 2]	[2, [3]]	[[3], 2]
[[2], 1]	[1, [2]]	[[2], 2]	[2, [2]]	[[3], 2]	[2, [3]]
[3, [1]]	[[1], 3]	[3, [2]]	[[2], 3]	[3, [3]]	[[3], 3]
[[3], 1]	[1, [3]]	[[2], 3]	[3, [2]]	[[3], 3]	[3, [3]]

3. Die triadische Relation der Form Z = [x, y, z] kann auf 4 Einbettungsstufen erscheinen.

3.1. 0 Einbettungsstufen

Z = [1, 2, 3]	Z = [2, 1, 3]	Z = [3, 1, 2]
Z = [1, 3, 2]	Z = [2, 3, 1]	Z = [3, 2, 1]

3.2. 1 Einbettungsstufe

Z = [1, 2, [3]]	Z = [2, 1, [3]]	Z = [3, 1, [2]]
Z = [1, 3, [2]]	Z = [2, 3, [1]]	Z = [3, 2, [1]]
Z = [1, [2], 3]	Z = [2, [1], 3]	Z = [3, [1], 2]
Z = [1, [3], 2]	Z = [2, [3], 1]	Z = [3, [2], 1]
Z = [[1], 2, 3]	Z = [[2], 1, 3]	Z = [[3], 1, 2]
Z = [[1], 3, 2]	Z = [[2], 3, 1]	Z = [[3], 2, 1]

Z = [[1, 2, 3]]	Z = [[2, 1, 3]]	Z = [[3, 1, 2]]
Z = [[1, 3, 2]]	Z = [[2, 3, 1]]	Z = [[3, 2, 1]]

3.3. 2 Einbettungsstufen

Z = [1, [2, 3]]	Z = [2, [1, 3]]	Z = [3, [1, 2]]
Z = [1, [3, 2]]	Z = [2, [3, 1]]	Z = [3, [2, 1]]

Z = [[1], 2, [3]]	Z = [[2], 1, [3]]	Z = [[3], 1, [2]]
Z = [[1], 3, [2]]	Z = [[2], 3, [1]]	Z = [[3], 2, [1]]

Z = [[1, 2], 3]	Z = [[2, 1], 3]	Z = [[3, 1], 2]
Z = [[1, 3], 2]	Z = [[2, 3], 1]	Z = [[3, 2], 1]

3.4. 3 Einbettungsstufen

Z = [1, [2, [3]]]	Z = [2, [1, [3]]]	Z = [3, [1, [2]]]
Z = [1, [3, [2]]]	Z = [2, [3, [1]]]	Z = [3, [2, [1]]]

Z = [[[1], 2], 3]	Z = [[[2], 1], 3]	Z = [[[3], 1], 2]
Z = [[[1], 3], 2]	Z = [[[2], 3], 1]	Z = [[[3], 2], 1]

Weitere Einbettungsstufen setzen die Iteration von $E \rightarrow \{E^2, ..., E^n\}$ voraus, dann erhält man eingebettete Zeichenrelation wie z.B. Z = [[[1], 2], 3], [1, [[2], [[3]]], [[[2], [3]], 1], usw. Ferner kann man iterierte Einbettungsstufen natürlich miteinander kombinieren. Dadurch wird die einbettungstheoretische Semiotik zu einem System von kaum vorstellbarer Komplexität.

4. Operatoren

Für die nicht-einbettungstheoretische Semiotik gelten seit Beckmann (1976) die verbandstheoretische Vereinigung $\sqcup$ und der Durchschnitt $\sqcap$ anstelle der Peano-Addition und –Subtraktion. Sie gilt selbstverständlich darüber hinaus für alle Zeichenrelationen der gleichen Einbettungsstufe.

4.1. Addition von Subrelationen

[1, [2] + [1, [3]] = [1, [3]]

[[2], 1] + [[3], 1] = [[3], 1]

[[1], 2] + [[2], 1] = [[2], 1]]

[2, [1]] + [3, [1]] = [3, [1]]

4.2. Subtraktion von Subrelationen

[1, [3] – [1, [2]] = [1, [2]]

[[3], 1] – [[2], 1] = [[2], 1]

[[1], 3] – [[1], 2] = [[1], 2]]

[3, [1]] – [2, [1]] = [2, [1]]

4.3. Dagegen können Subrelationen ungleicher Einbettungsstufen nicht addiert bzw. subtrahiert werden.

[1, [2]] + [[2], 1] = [(1 + [2]), ([2] + 1)]

[1, [2]] + [[1], 2] = [(1 + [1]), ([2] + 2)]

[1, [2]] + [2, [1]] = [(1 + 2), ([2] + [1])], usw.

[1, [2]] – [[2], 1] = [(1 – [2]), ([2] – 1)]

[1, [2]] – [[1], 2] = [(1 – [1]), ([2] – 2)]

[1, [2]] – [2, [1]] = [(1 – 2), ([2] – [1])], usw., d.h. statt Addition (Vereinigung) bzw. Subtraktion (Durchschnitt) tritt hier eine neue Operation ein, die wir Determination nennen können, insofern jedes Paar der Form [[x.y], E] durch die beiden möglichen Fälle

$[[x.y], E] \pm [[z.w], E] = [[[x.y], E] \leftarrow [[z.w], E]]$

$[[x.y], E] \pm [[z.w], E] = [[[x.y], E] \rightarrow [[z.w], E]]$

darstellbar ist.

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Einbettungstheoretische Semiotik II

1. Die Einführung eines Einbettungsoperators E, der jede Dichotomie der Form L = [0, 1] auf ein Quadrupel der Form

L^4 = [[0, [1]], [[1], 0], [[0], 1], [1, [0]]]

abbildet, bedeutet, ein differentielles Tertium in die 2-wertige Logik einzuführen, d.h. ein solches, das zwar nicht durch die Einführung eines dritten Wertes die aristotelische Logik sprengt, aber indem die beiden in L spiegelbildlichen Werte in ein vierfach mögliches Abhängigkeitsverhältnis gesetzt werden. Da die Semiotik, wie im übrigen natürlich sämtliche Wissenschaften, auf der 2-wertigen Logik beruht, bedeutet die Abbildung

E: $L \rightarrow L^4$

keine Aufhebung von L, sondern dessen Einbettung in eine sehr viel komplexere Struktur, von der L lediglich einen trivialen Fall – nämlich den beliebigen Austausch der Werte von L – darstellt. Mit E geht somit auch eine Relativierung sowohl der horizontalen Linearität als auch des Nachfolgeprinzips der Peanozahlen einher, denn E bewirkt eine Abbildung eines 1-dimensionalen Zahlenstrahles auf ein 2-dimensionales Zahlenfeld, indem es somit neben horizontaler auch vertikale und zwei diagonale Zählweisen gibt. Im folgenden werden die Grundtypen einer dermaßen zu konzipierenden einbettungstheoretischen Semiotik angegeben.

2. Grundtypen einer 3-stufigen Semiotik

2.1. 0-stufige Einbettung

Für n = 0 gibt es somit nur eine Ordnung O.

2.1.1. O = 1

S = [M, O, I]

2.2. 1-stufige Einbettung

2.2.1. O = (0, 1)

S = [[M], O, I]	S = [I, O, [M]]
S = [M, [O], I]	S = [I, [O], M]
S = [M, O, [I]]	S = [[I], O, M]

S = [[M, O], I]	S = [I, [O, M]]
S = [M, [O, I]]	S = [[I, O], M]
S = [[M], O, [I]]	S = [[I], O, [M]]

2.2.2. O = 1

S = [[M, O, I]]	S = [[I, O, M]]

2.3. 2-stufige Einbettung

2.3.1. O = 2

S = [[[M, O, I]]]

2.3.2. O = (2, 0)

S = [[[M]], O, I]	S = [I, O, [[M]]]
S = [M, [[O]], I]	S = [I, [[O]], M]
S = [M, O, [[I]]]	S = [[[I]], O, M]

S = [[[M, O]], I]	S = [I, [[O, M]]]
S = [M, [[O, I]]]	S = [[[I, O]], M]
S = [[[M]], O, [[I]]]	S = [[[I]], O, [[M]]]

2.3.3. O = (2, 1)

S = [[[M]], [O], [I]]		S = [[I], [O], [[M]]]

S = [[[O]], [M], [I]]		S = [[I], [M], [[O]]]

S = [[[I], [M], [O]]		S = [[O], [M], [[I]]]

S = [[[M, O]], [I]]		S = [[I], [[O, M]]]

S = [[[M, I]], [O]]		S = [[O], [[I, M]]]

S = [[[O, I]], [M]]		S = [[M], [[I, O]]]

2.3.4. O = (2, 1, 0)

S = [[[M]], [O], I]		S = [I, [O], [[M]]]

S = [[[O]], [M], I]		S = [I, [M], [[O]]]

S = [[[I]], [M], O]		S = [O, [M], [[I]]]

2.3.5. O = (2, 2, 0)

S = [[[M]], [[O]], I]		S = [I, [[O]], [[M]]]

S = [[[O]], [[M]], I]		S = [I, [[M]], [[O]]]

S = [[[I]], [[M]], O]		S = [O, [[M]], [[I]]]

2.3.6. O = (2, 2, 1)

S = [[[M]], [[O]], [I]]		S = [[I], [[O]], [[M]]]

S = [[[O]], [[M]], [I]]		S = [[I], [[M]], [[O]]]

S = [[[I]], [[M]], [O]]		S = [[O], [[M]], [[I]]]

Man beachte, daß die Semiotik, wie sie von Bense auf der kategorietheoretischen Zeichendefinition

$$Z = (M \rightarrow ((M \rightarrow O) \rightarrow (M \rightarrow O \rightarrow I)))$$

begründet worden war (vgl. Bense 1979, S. 53 u. 67), somit der Einbettungsordnung O = (2, 1, 0) korrespondiert. Damit setzt die Semiotik also alle drei Einbettungsstufen einer dreistufigen Semiotik voraus. Sie widerspricht damit also nicht nur dem Fundierungsaxiom der klassischen Mengentheorie, sondern auch der aristotelischen Logik, und man darf daher sogar soweit gehen zu behaupten, daß die hier präsentierte einbettungstheoretische Semiotik lediglich eine "Auffaltung" des bereits in Benses Zeichendefinition enthaltenen Strukturreichtums darstellt.

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Einführung der peirce-benseschen Semiotik mit Hilfe von Relationalzahlen

1. Im folgenden werden die Grundlagen der peirce-benseschen Semiotik mit Hilfe der in Toth (2015a, b) skizzierten Theorie der Relationalzahlen neu eingeführt.

2.1. Primzeichen

Die Menge der von Bense (1981, S. 17 ff.) eingeführten, Primzeichen genannten, Zeichenzahlen P = (1, 2, 3) teilt sich in eine Teilmenge der triadischen

$P_{td} = <x.>$

und in eine Teilmenge der trichotomischen

$P_{tt} = <.y>$

Zeichenzahlen, mit x, y ∈ P. Diese unterscheiden sich also lediglich durch ihren Einbettungsgrad, d.h. für die zugehörigen Relationalzahlen R gilt

$R(P_{td}) \supset R(P_{tt})$.

2.2. Subzeichen

Subzeichen werden als kartesische Produkte der Form

$S = <x.y>$

mit $<x.y> \subset P \times P$

definiert. Damit können wir die $3^2 = 9$ Subzeichen in der Form der folgenden Matrixdarstellung über P mit Hilfe von Relationalzahlen wie folgt definieren

$(1_m, 1_n)$ $(1_m, 2_{n+1})$ $(1_m, 3_{n+2})$

$(2_{m+1}, 1_n)$ $(2_{m+1}, 2_{n+1})$ $(2_{m+1}, 3_{n+2})$

$(3_{m+2}, 1_n)$ $(3_{m+2}, 2_{n+1})$ $(3_{m+2}, 3_{n+2})$.

2.3. Zeichenklassen und Realitätsthematiken

Semiotische Dualysteme, bestehend aus Zeichenklassen und Realitästhematiken, werden nach einem Vorschlag Walthers (1979, S. 79) als Konkatenationen von Paaren von Dyaden von Subzeichen gebildet. Damit erhalten wir sogleich

$((3_{m+2}, 1_n), (2_{m+1}, 1_n), (1_m, 1_n)) \quad \times ((1_m, 1_n), (1_m, 2_{n+1}), (1_m, 3_{n+2}))$

$((3_{m+2}, 1_n), (2_{m+1}, 1_n), (1_m, 2_{n+1})) \quad \times ((2_{m+1}, 1_n), (1_m, 2_{n+1}), (1_m, 3_{n+2}))$

$((3_{m+2}, 1_n), (2_{m+1}, 1_n), (1_m, 3_{n+2})) \quad \times ((3_{m+2}, 1_n), (1_m, 2_{n+1}), (1_m, 3_{n+2}))$

$((3_{m+2}, 1_n), (2_{m+1}, 2_{n+1}), (1_m, 2_{n+1})) \quad \times ((2_{m+1}, 1_n), (2_{m+1}, 2_{n+1}), (1_m, 3_{n+2}))$

$((3_{m+2}, 1_n), (2_{m+1}, 2_{n+1}), (1_m, 3_{n+2})) \quad \times ((3_{m+2}, 1_n), (2_{m+1}, 2_{n+1}), (1_m, 3_{n+2}))$

$((3_{m+2}, 1_n), (2_{m+1}, 3_{n+2}), (1_m, 3_{n+2})) \quad \times ((3_{m+2}, 1_n), (3_{m+2}, 2_{n+1}), (1_m, 3_{n+2}))$

$((3_{m+2}, 2_{n+1}), (2_{m+1}, 2_{n+1}), (1_m, 2_{n+1})) \times ((2_{m+1}, 1_n), (2_{m+1}, 2_{n+1}), (2_{m+1}, 3_{n+2}))$

$((3_{m+2}, 2_{n+1}), (2_{m+1}, 2_{n+1}), (1_m, 3_{n+2})) \times ((3_{m+2}, 1_n), (2_{m+1}, 2_{n+1}), (2_{m+1}, 3_{n+2}))$

$((3_{m+2}, 2_{n+1}), (2_{m+1}, 3_{n+2}), (1_m, 3_{n+2})) \times ((3_{m+2}, 1_n), (3_{m+2}, 2_{n+1}), (2_{m+1}, 3_{n+2}))$

$((3_{m+2}, 3_{n+2}), (2_{m+1}, 3_{n+2}), (1_m, 3_{n+2})) \times ((3_{m+2}, 1_n), (3_{m+2}, 2_{n+1}), (3_{m+2}, 3_{n+2}))$

Man beachte, daß die Dualisierung nur die Peanozahlenanteile der Relationalzahlen, nicht aber deren Einbettungsgrad betreffen, d.h. der qualitativ, nämlich als differentielles (nicht-materiales) "Tertium" wirkende Einbettungsoperator hebt die Identität zwischen dualen Subzeichen der nicht-eingebetteten Form

$\times\langle x.y\rangle = \langle y.x\rangle$

$\times\times\langle x.y\rangle = \langle x.y\rangle$

auf. Daraus folgt die Ungültigkeit der Eigenrealität, deren Spiegelbildlichkeit (vgl. Bense 1992) sich als nur scheinbar entpuppt

$((3_{m+2}, 1_n), (2_{m+1}, 2_{n+1}), (1_m, 3_{n+2})) \quad \times ((3_{m+2}, 1_n), (2_{m+1}, 2_{n+1}), (1_m, 3_{n+2})),$

denn wir haben

$\times(3_{m+2}, 1_n) \neq (1_m, 3_{n+2})$

$\times(2_{m+1}, 2_{n+1}) \neq (2_{m+1}, 2_{n+1})$

$\times(1_m, 3_{n+2}) \neq \times(3_{m+2}, 1_n)$.

Dasselbe gilt für die Kategorienrealität, da auch die Dualisation der eingebetteten genuinen Subzeichen, d.h. der automorphen Semiosen, nicht-identitiv ist, wie man sich leicht selbst überzeugt.

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Das qualitative 2-dimensionale Inklusionsschema der Semiotik

1. Wie in Toth (2015a) ausgeführt, bedeutet die Einführung eines Einbettungsoperators E für die ortsfunktionale Arithmetik, durch den also die logische Dichotomie L = (0, 1) sowie alle ihr isomorphen Dichotomien auf Quadrupel der Form

$$
\begin{array}{ccc}
[0, [1]] & \times & [[1], 0] \\
 & \times & \\
[[0], 1] & \times & [1, [0]]
\end{array}
$$

abgebildet werden, insofern eine Qualifizierung der ontischen Arithmetik, wie sie auch für die Ontik und die Semiotik gültig ist, als die Auflösung der Spiegelbildlichkeit und der daraus folgenden Austauschbarkeit der beiden Werte in L ein zwar nicht materiales, dafür aber ein differentielles "Tertium" darstellen, das von der 2-wertigen aristotelischen Logik explizit verboten wird.

2. Für die Semiotik ist ein solcher Einbettungsoperator zwar nie zuvor eingeführt worden (vgl. Toth 2015b), aber er liegt implizit vor in der selbsteinbettenden Zeichendefinition, die Bense (1979, S. 53 u. 67) gegeben hatte

$$Z = (1 \rightarrow ((1 \rightarrow 2) \rightarrow (1 \rightarrow 2 \rightarrow 3))),$$

darin sich nicht nur das Zeichen selbst im triadischen Interpretantenbezug enthält, sondern in dem die Erstheit sowohl in der Zweitheit als auch in der Drittheit und die Zweitheit in der Drittheit eingeschlossen sind, d.h. es liegt der sog. Droste-Effekt vor.

Genau dieses Prinzip wird von Z aus auf die Z konstituierenden Teilrelationen von Z, die sog. Subzeichen, übertragen, indem die von Bense (1975, S. 37) eingeführte semiotische Matrix auf die folgende einbettungstheoretische Matrix abgebildet wird

$$
\begin{array}{ccccc}
(1_m, 1_n) & \subset & (1_m, 2_{n+1}) & \subset & (1_m, 3_{n+2}) \\
\cap & & \cap & & \cap \\
(2_{m+1}, 1_n) & \subset & (2_{m+1}, 2_{n+1}) & \subset & (2_{m+1}, 3_{n+2}) \\
\cap & & \cap & & \cap \\
(3_{m+2}, 1_n) & \subset & (3_{m+2}, 2_{n+1}) & \subset & (3_{m+2}, 3_{n+2}).
\end{array}
$$

Würde es sich hier um rein quantitative Zahlen der Form

$$
\begin{array}{ccc}
1 & 2 & 3 \\
2 & 3 & 5 \\
3 & 5 & 8
\end{array}
$$

handeln, würde sowohl für die Triaden als auch für die Trichotomien bzw. für die Zeilen und für die Spalten der Matrix natürlich die Additvität gelten, d.h. wir hätten in beiden Dimensionen dieses 2-dimensionalen quantitativen Inklusionsschemas

$1 + 2 = 3$

$2 + 3 = 5$

$3 + 5 = 8.$

Dieses gilt jedoch nicht für die Semiotik, denn für die von Bense (1981, S. 17 ff.) eingeführten und von ihm Primzeichen genannten Zeichenzahlen, d.h. also qualitativen Zahlen, gilt selbstverständlich die Hyperaddivität, insofern für die Trichotomien

$(1.1) + (1.2) < (1.3)$

$(2.1) + (2.2) < (2.3)$

$(3.1) + (3.2) < (3.3)$

und für die Triaden

$(1.1) + (2.1) < (3.1)$

$(1.2) + (2.2) < (3.2)$

$(1.3) + (2.3) < (3.3)$

gilt. Was der Einbettungsoperator also leistet, besteht darin, die Begründung für diese qualitative Hyperaddivität zu liefern, und zwar in der Form der Abbildung der semiotischen Teilrelationen auf ontische Orte.

Literatur

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Toth, Alfred, Einführung der peirce-benseschen Semiotik mit Hilfe von Relationalzahlen. In: Electronic Journal for Mathematical Semiotics, 2015b

Elementares System einer vollständigen qualitativen Arithmetik

1. Wie bereits in Toth (2015a) dargelegt, führt die Einführung eines Einbettungsoperators E und seine Anwendung auf das Zahlenpaar P = (0, 1), das als arithmetische Struktur der 2-wertigen aristotelischen Logik sowie aller ihr isomorphen rein quantitativen Systeme dient, zu 2-dimensionalen Zahlenfeldern, in denen zwar 1-dimensionale Zahlenfolgen nicht aufgehoben, aber stark marginalisiert sind und in denen es statt einer drei Zählweisen gibt, die wir als adjazente, subjazente und transjazente bezeichnet hatten. Benutzt man nun die in Toth (2015b) definierten Relationalzahlen zur Definition dieser Raumfelder, ergibt sich das im folgenden präsentierte elementare System einer vollständigen qualitativen Arithmetik.

2.1. Arithmetische Adjazenz

2.1.1. Definition

R(adj) = (x_m, y_n) mit $x \neq y$ und $m = n$

2.1.2. Zahlenfelder

0_i	1_j		1_i	0_j		1_j	0_i		0_j	1_i
$Ø_i$	$Ø_j$		$Ø_i$	$Ø_j$		$Ø_j$	$Ø_i$		$Ø_j$	$Ø_i$
		×			×			×		
$Ø_i$	$Ø_j$		$Ø_i$	$Ø_j$		$Ø_j$	$Ø_i$		$Ø_j$	$Ø_i$
0_i	1_j		1_i	0_j		1_j	0_i		0_j	1_i

2.1.3. Relationalzahlen

(0, 1)	(1, 0)	$(0_1, 1_1)$	$(1_1, 0_1)$
$(0_{-1}, 1_{-1})$	$(1_{-1}, 0_{-1})$	(0, 1)	(1, 0)

2.2. Arithmetische Subjazenz

2.2.1. Definition

$R(subj) = (x_m, y_n)$ mit $x = y$ und $m \neq n$

2.2.2. Zahlenfelder

$$\begin{array}{ccccccccccc}
0_i & \varnothing_j & & \varnothing_i & 0_j & & \varnothing_j & 0_i & & 0_j & \varnothing_i \\
1_i & \varnothing_j & & \varnothing_i & 1_j & & \varnothing_j & 1_i & & 1_j & \varnothing_i \\
 & & \times & & & \times & & & \times & & \\
1_i & \varnothing_j & & \varnothing_i & 1_j & & \varnothing_j & 1_i & & 1_j & \varnothing_i \\
0_i & \varnothing_j & & \varnothing_i & 0_j & & \varnothing_j & 0_i & & 0_j & \varnothing_i
\end{array}$$

2.2.3. Relationalzahlen

$(0 \leftarrow 1_{-1}) \qquad (1_{-1} \rightarrow 0)$

$(0_{-1} \leftarrow 1) \qquad (1 \rightarrow 0_{-1})$

2.3. Arithmetische Transjazenz

2.3.1. Definition

$R(transj) = (x_n, y_m)$ mit $x \neq y$ und $m \neq n$

2.3.2. Zahlenfelder

$$\begin{array}{ccccccccccc}
0_i & \varnothing_j & & \varnothing_i & 0_j & & \varnothing_j & 0_i & & 0_j & \varnothing_i \\
\varnothing_i & 1_j & & 1_i & \varnothing_j & & 1_j & \varnothing_i & & \varnothing_j & 1_i \\
 & & \times & & & \times & & & \times & & \\
\varnothing_i & 1_j & & 1_i & \varnothing_j & & 1_j & \varnothing_i & & \varnothing_j & 1_i \\
0_i & \varnothing_j & & \varnothing_i & 0_j & & \varnothing_j & 0_i & & 0_j & \varnothing_i
\end{array}$$

2.3.3. Relationalzahlen

$(0, 1_{-1})$ $(1_{-1}, 0)$

$(0_{-1}, 1)$ $(1, 0_{-1})$

2.4. Als einzige nicht-verwandte Definition verbleibt somit

$R = (x_n, y_m)$ mit $x = y$ und $m = n$,

d.h. es handelt sich um Zahlenfolgen der Form

(1), (1, 1), (1, 1, 1), ...,

die überhaupt keine Einbettungsgrade – außer der trivialen mit $E = 0$ – kennen.

Literatur

Toth, Alfred, Peanozahlen und ihre ontischen Orte I-II. In: Electronic Journal for Mathematical Semiotics, 2015a

Toth, Alfred, Zur Arithmetik der Relationalzahlen I-II. In: Electronic Journal for Mathematical Semiotics, 2015b

Konvexe und nichtkonvexe Relationalzahlen

1. Gegeben sei die Menge von Peanozahlen P = (0, 1) und der in Toth (2015a) eingeführte Einbettungsoperator E. Dann erhält man statt der linearen Peanofolge mit einer einzigen, horizontalen, Zählweise, eine Menge von Quadrupeln von Zahlenfeldern und drei Zählweisen, neben der horizontalen noch die vertikale und die beiden diagonalen.

1.1. Adjazente Zählweise

0	1		1	0		1	0		0	1
Ø	Ø		Ø	Ø		Ø	Ø		Ø	Ø
		×			×			×		
Ø	Ø		Ø	Ø		Ø	Ø		Ø	Ø
0	1		1	0		1	0		0	1

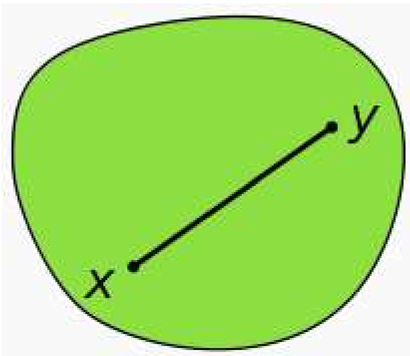
1.2. Subjazente Zählweise

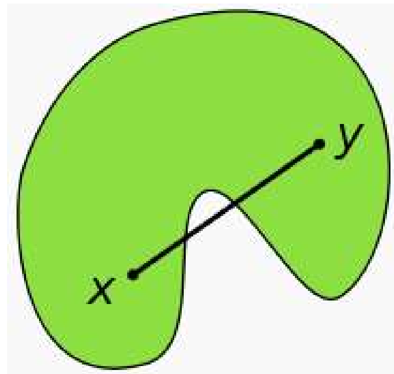
0	Ø		Ø	0		Ø	0		0	Ø
1	Ø		Ø	1		Ø	1		1	Ø
		×			×			×		
1	Ø		Ø	1		Ø	1		1	Ø
0	Ø		Ø	0		Ø	0		0	Ø

1.3. Transjazente Zählweise

0	Ø		Ø	0		Ø	0		0	Ø
Ø	1		1	Ø		1	Ø		Ø	1
		×			×			×		
Ø	1		1	Ø		1	Ø		Ø	1
0	Ø		Ø	0		Ø	0		0	Ø

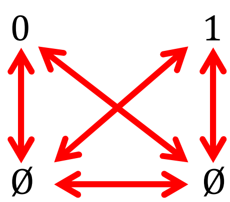
2. Eine Menge heißt konvex gdw. wenn für 2 Punkte der Menge auch die Verbindungsstrecke der beiden Punkte zur Menge gehört (vgl. dazu Toth 2015b, c), vgl. die beiden folgenden Bilder aus der Wikipedia.



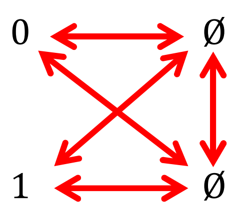


Ein Relationalzahlenfeld ist somit nichtkonvex gdw. es ein Teilfeld enthält, daß eine Nullstelle (Ø) enthält. In den folgenden Beispielen sind nichtkonvexe Verbindungsstrecken mit roten Doppelpfeilen eingezeichnet.

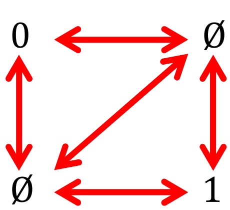
2.1. Beispiel für adjazente Nichtkonvexität

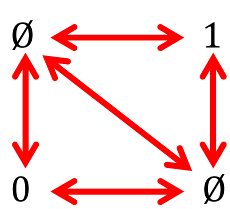


2.2. Beispiel für subjazente Nichtkonvexität



2.3. Beispiel für transjazente Nichtkonvexität





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Toth, Alfred, Nichtkonvexe Systeme. In: Electronic Journal for Mathematical Semiotics, 2015b

Toth, Alfred, Nichtkonvexe Umgebungen. In: Electronic Journal for Mathematical Semiotics, 2015c

Ortsfunktionale Mengentheorie

1. Wie bekannt, läßt sich die logische Basisrelation L = [0, 1], auf der auch die quantitative Mathematik beruht, vermöge Toth (2015a) auf ein Quadrupel der Form

$$L = [0, 1] \quad \rightarrow \left(\begin{array}{ll} L_1 = [0, [1]] & L_2 = [[1], 0] \\ L_3 = [[0], 1] & L_4 = [1, [0]] \end{array} \right)$$

abbilden. Einzige Voraussetzung ist ein Einbettungsoperator E, der als differentielles Tertium in L wirkt, der also die beiden Werte auf verschiedene Einbettungsstufen abbildet und sie somit aus ihrer juxtapositiven Linearität befreit. Wie in Toth (2015b) gezeigt wurde, führt E zu Zahlenfeldern, in denen statt der einen Peano-Zählweise drei ortsfunktionale Zählweisen unterschieden werden müssen.

1.1. Adjazente Zählweise

1.1.1. Zahlenfelder

$$\begin{array}{cccccccccccc}
0 & 1 & & 1 & 0 & & 1 & 0 & & 0 & 1 \\
\emptyset & \emptyset & & \emptyset & \emptyset & & \emptyset & \emptyset & & \emptyset & \emptyset \\
 & & \times & & & \times & & & \times & & \\
\emptyset & \emptyset & & \emptyset & \emptyset & & \emptyset & \emptyset & & \emptyset & \emptyset \\
0 & 1 & & 1 & 0 & & 1 & 0 & & 0 & 1
\end{array}$$

1.1.2. Relationalzahlen

$R = (0_{\pm n}, 1_{\pm m})$

1.2. Subjazente Zählweise

1.2.1. Zahlenfelder

$$\begin{array}{ccccccccccc}
0 & Ø & & Ø & 0 & & Ø & 0 & & 0 & Ø \\
1 & Ø & & Ø & 1 & & Ø & 1 & & 1 & Ø \\
 & & \times & & & \times & & & \times & & \\
1 & Ø & & Ø & 1 & & Ø & 1 & & 1 & Ø \\
0 & Ø & & Ø & 0 & & Ø & 0 & & 0 & Ø
\end{array}$$

1.2.2. Relationalzahlen

$R = (0_{\pm n}, 1_{\pm m})$

1.3. Transjazente Zählweise

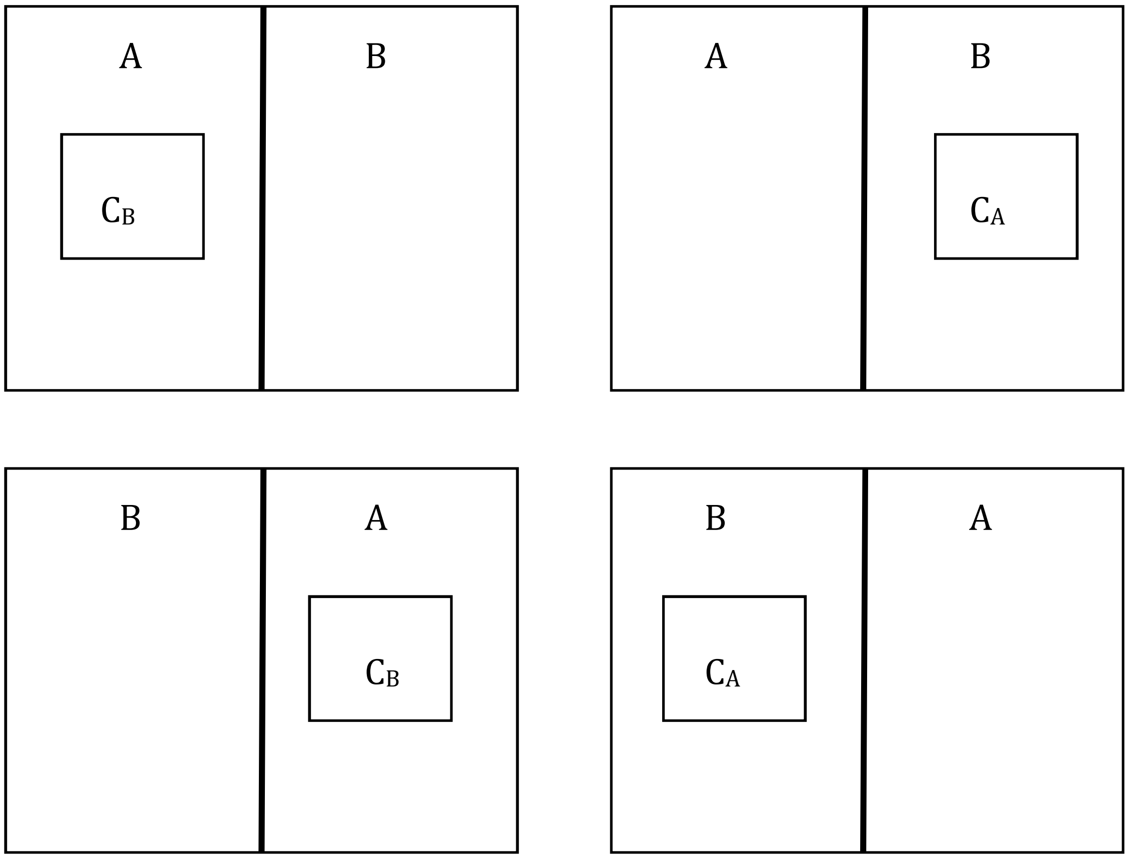
1.3.1. Zahlenfelder

$$\begin{array}{ccccccccccc}
0 & Ø & & Ø & 0 & & Ø & 0 & & 0 & Ø \\
Ø & 1 & & 1 & Ø & & 1 & Ø & & Ø & 1 \\
 & & \times & & & \times & & & \times & & \\
Ø & 1 & & 1 & Ø & & 1 & Ø & & Ø & 1 \\
0 & Ø & & Ø & 0 & & Ø & 0 & & 0 & Ø
\end{array}$$

1.3.2. Relationalzahlen

$R = ((0_{\pm n}, 1_{\pm n}), (0_{\pm n}, 1_{\pm m}))$

2. Man kann nun, wie bereits in Toth (2015c) angedeutet, einen entscheidenden Schritt von der ortsfunktionalen Arithmetik zu einer ortsfunktionalen Mengentheorie gehen. Gegeben seien zwei Mengen A und B und eine Teilmenge C, die entweder zu A oder zu B gehören kann. Dann erhält man wiederum ein Quadrupel der Form



mit den zugehörigen mengentheoretischen Relationen

$R_1 = [[C_B \subset A], B]$

$R_2 = [A, [C_A \subset B]]$

$R_3 = [B, [C_B \subset A]$

$R_4 = [[C_A \subset B, A]$.

Vermöge Kapitel 1 gilt somit ebenfalls

$R_1 = [[x_1 \subset 0], B]$

$R_2 = [0, [x_0 \subset 1]]$

$R_3 = [1, [x_1 \subset 0]]$

$R_4 = [[x_0 \subset 1], 0]$,

sofern 0 und 1 als Mengen eingeführt werden. Man erhält also für die Durchschnittsoperation

$R_1 = [[x_1 \subset 0], 1] \cap R_2 = [0, [x_0 \subset 1]] = [[[x_1 \subset 0], 1], [0, [x_0 \subset 1]]]$

$R_1 = [[x_1 \subset 0], 1] \cap R_3 = [1, [x_1 \subset 0]] = [[[x_1 \subset 0], 1, [x_1 \subset 0]]]$

$R_1 = [[x_1 \subset 0], 1] \cap R_4 = [[x_0 \subset 1], 0] = [[[x_1 \subset 0], 1], [[x_0 \subset 1], 0]]$

$R_2 = [0, [x_0 \subset 1]] \cap R_3 = [1, [x_1 \subset 0]] = [[0, [x_0 \subset 1]], [1, [x_1 \subset 0]]]$

$R_2 = [0, [x_0 \subset 1]] \cap R_4 = [[x_0 \subset 1], 0] = [[0, [x_0 \subset 1], 0]]$

$R_3 = [1, [x_1 \subset 0]] \cap R_4 = [[x_0 \subset 1], 0] = [[1, [x_1 \subset 0]], [[x_0 \subset 1], 0]]$

und für die Vereinigungsoperation

$R_1 = [[x_1 \subset 0], 1] \cup R_2 = [0, [x_0 \subset 1]] = [[[x_1 \subset 0], 1], [0, [x_0 \subset 1]]]$

$R_1 = [[x_1 \subset 0], 1] \cup R_3 = [1, [x_1 \subset 0]] = [[[x_1 \subset 0], 1], [1, [x_1 \subset 0]]]$

$R_1 = [[x_1 \subset 0], 1] \cup R_4 = [[x_0 \subset 1], 0] = [[[x_1 \subset 0], 1], [[x_0 \subset 1], 0]]$

$R_2 = [0, [x_0 \subset 1]] \cup R_3 = [1, [x_1 \subset 0]] = [[0, [x_0 \subset 1]], [1, [x_1 \subset 0]]]$

$R_2 = [0, [x_0 \subset 1]] \cup R_4 = [[x_0 \subset 1], 0] = [[0, [x_0 \subset 1]], [[x_0 \subset 1], 0]]$

$R_3 = [1, [x_1 \subset 0]] \cup R_4 = [[x_0 \subset 1], 0] = [[1, [x_1 \subset 0]], [[x_0 \subset 1], 0]]$.

Literatur

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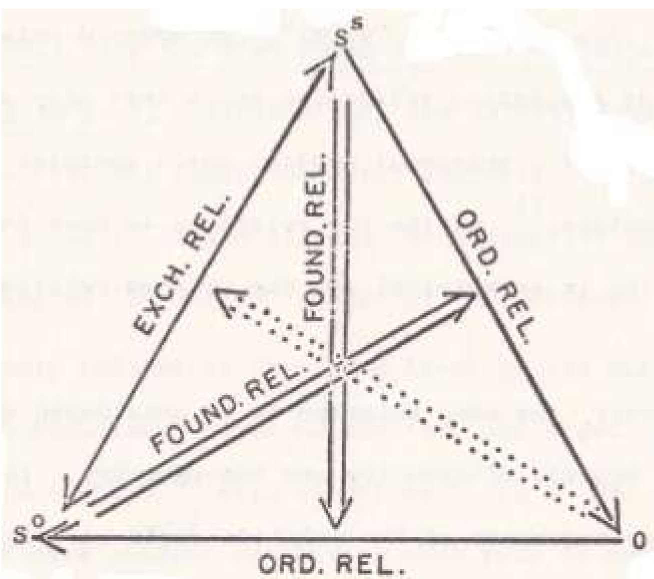
Toth, Alfred, Zur Arithmetik der Relationalzahlen I-II. In: Electronic Journal for Mathematical Semiotics, 2015b

Toth, Alfred, Konvexität und Nichtkonvexität von Enklaven und Exklaven. In: Electronic Journal for Mathematical Semiotics, 2015c

Wie qualitativ ist die Mathematik der Qualitäten?

1. Die von Engelbert Kronthaler geschaffene "Mathematik der Qualitäten" (Kronthaler 1973/86) gehört ohne Zweifel zu den großen mathematischen Leistungen. Übrigens dürfte der Begriff der "Mathematik der Qualitäten" auf Natorp (1903, S. 419) zurückgehen, der ihn im Zusammenhang mit der platonischen Ideenlehre eingeführt hatte. Allerdings basiert die MdQ auf der polykontexturalen Logik Gotthard Günthers (1976-1980), und diese ist vor dem Hintergrund der in Toth (2015a) eingeführten qualitativen Arithmetik der Relationalzahlen, wie im folgenden gezeigt werden soll, in drei kapitalen Punkten angreifbar.

2.1. Aus dem folgenden Schema der Subjekt- und Objektfunktionen, das Günther (1976, S. 337) aufgestellt hatte,



geht hervor, daß es in der Polykontexturalitätstheorie lediglich die folgenden drei Abbildungen gibt (im folgenden verwenden wir oO für objektives Objekt, oS für objektives Subjekt und sS für subjektives Subjekt)

2.1.1. oO → sS

2.1.2. sS → oO

2.1.3. oS ⇄ sS

In Sonderheit fehlt also die Funktion des subjektives Objektes, die vermöge der folgenden Tabelle allein aus kombinatorischen Gründen existieren muß

	O	S
O	oO	oS
S	sO	sS.

Nun bedeutet, wie zuletzt in Toth (2015b) dargelegt, sO das (von einem Subjekt) wahrgenommene Objekt. Der Grund dafür, daß es fehlt, beruht darin, daß in der Polykontexturalitätstheorie das Objekt weiterhin als "totes" Objekt betrachtet wird, denn die polykontexturale Logik ist nichts anderes als ein Verbundsystem von zweiwertigen aristotelischen Logiken, die lediglich über mehr als ein Subjekt distribuiert sind.

2.2. Da die Dichotomien E = [Objekt, Subjekt] und L = [Position, Negation] isomorph sind, folgt aus 2.1., daß nur das Subjekt iterierbar ist. Da das Subjekt die Negativität darstellt, konstruiert Günther (1980, S. 286) in Hamiltonkreisen angeordnete "Negationszyklen" (aus denen G.G. Thomas später seine auch in der Mathematik bekannt gewordenen "Permutographen" konstruieren sollte). Beispielsweise umfaßt der Negationszyklus für eine 4-wertige Logik 4! = 24 Wertfunktionen.

P	N	1	2	3	2	3	2	1	2	1	2	3	2	3	2	1	2	1	2	3	2	3	2	1	2	P
1		2	3	4	4	3	2	1	1	2	3	4	4	3	2	1	1	2	3	4	4	3	2	1	1	
2		1	1	1	1	1	1	2	3	3	2	2	3	4	4	4	4	4	4	3	2	2	3	3	2	
3		3	2	2	3	4	4	4	4	4	4	3	2	2	3	3	2	1	1	1	1	1	1	2	3	
4		4	4	3	2	2	3	3	2	1	1	1	1	1	1	2	3	3	2	2	3	4	4	4	4	

2.3. Das Objekt bleibt somit nicht-iterierbar. Man könnte also sagen: In der polykontexturalen Logik bekommt jedes Subjekt seine eigene 2-wertige Logik. In dieser Trivialität besteht im Grunde der einzige Unterschied zwischen der polykontexturalen güntherschen und der monokontexturalen aristotelischen Logik. Diese Annahme ist aber, wie bereits in 2.1. gezeigt, deswegen falsch, weil eine Reduktion der vier möglichen Objekt-Subjekt-Funktionen auf nur drei strukturell unterdeterminiert ist. In Sonderheit bildet das bei Günther fehlende subjektive Objekt das Domänenelement der thetischen Einführung von Zeichen

$$\mu: \quad \Omega = f(\Sigma) \rightarrow \Sigma = f(\Omega),$$

denn es ist ja

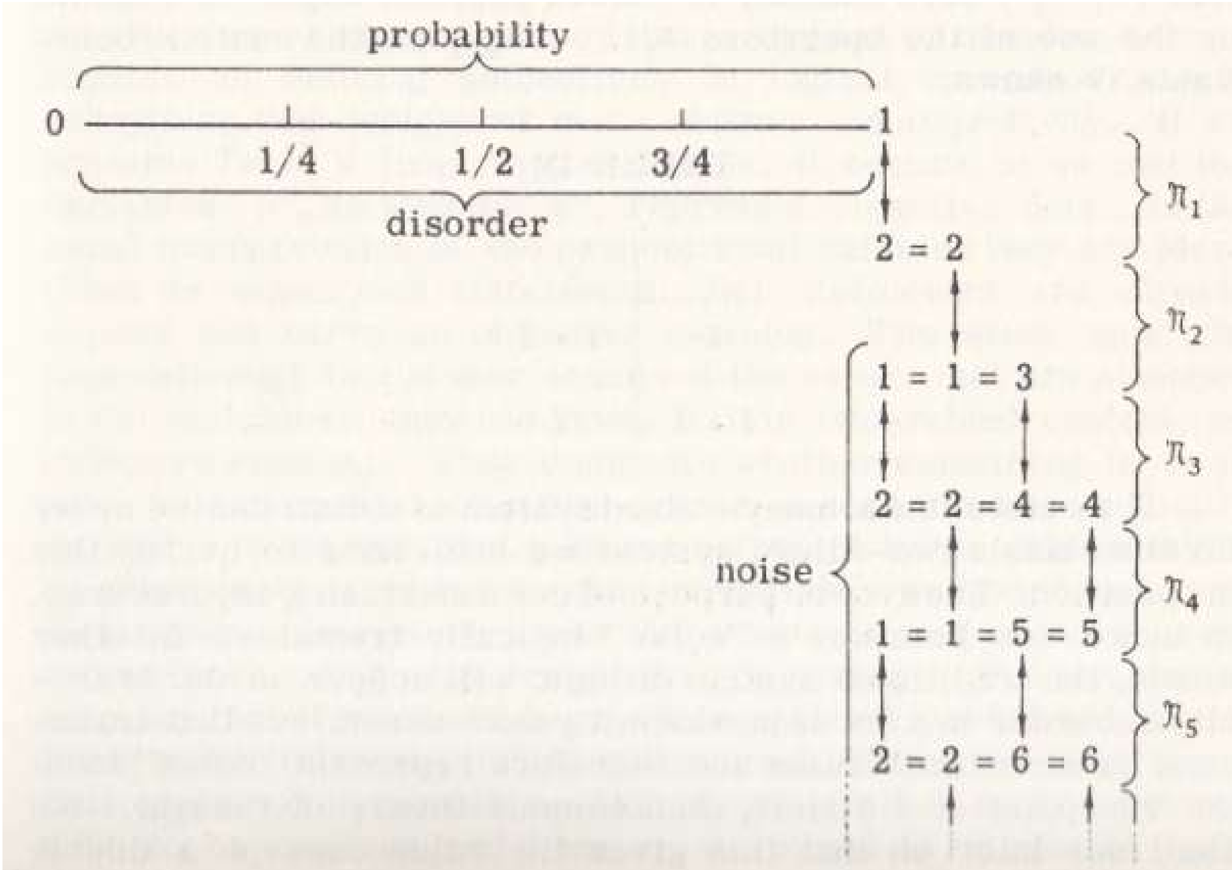
$sO = (\Omega = f(\Sigma))$. Eine Semiotik kann es somit innerhalb der Polykontexturalitätstheorie, welche nach Günther explizit nicht nur eine Logik, sondern auch eine Ontologie enthält und die auf erkenntnistheoretischen Funktionen basiert, paradoxerweise nicht geben, da als einzige nicht-subjektiven Objekte die absoluten, d.h. objektiven Objekte der klassischen Logik übernommen werden. Für Günther gilt somit weiterhin die klassische logische Dichotomie der Form

$$L = [0, 1]$$

mit

$$R[0, 1] = R[1, 0] = \emptyset,$$

d.h. es gibt keine Vermittlung zwischen den Werten der für jede Kontxtur gültigen L. Auch das Subjekt ist innerhalb jeder Kontextur ein subjektives Subjekt, da die Iteration der Negativität einzig und allein dazu dient, subjektale Deixis in die klassische Logik einzuführen, also zwischen Ich-, Du-, Er- usw. Subjekten zu unterscheiden. Beim Übergang von der 2-wertigen aristotelischen zur n-wertigen polykontexturalen Logik wird also das Tertium non datur nicht aufgehoben, sondern lediglich mit wachsender Anzahl von Subjekten in ein Quartum Quintum, Sextum ... non datur verschoben. Weil Günther an der nachweislich falschen Vorstellung absoluter Objekte und Subjekte festhält (vgl. Toth 2015c), kann er die Idee der Vermittlung der Werte in $L = [0, 1]$ lediglich im Rahmen der reichenbachschen Quantenlogik sehen, wie aus dem folgenden, aus Günther (1976, S. 343) reproduzierten Schema in eindeutiger Weise hervorgeht.



Günther kommt in Sonderheit nicht auf die Idee, daß es neben der Einführung dritter, vierter, fünfter ... Werte zwischen den Werten von L = [0, 1] noch die Möglichkeit gibt, das Tertiumgesetz nicht substantiell, sondern differentiell aufzuheben, indem L wie folgt auf ein Quadrupel von Einbettungsrelationen abgebildet wird, in denen also ein Einbettungsoperator E ein differentielles Tertium erzeugt

$$L = [0, 1] \quad \rightarrow \begin{pmatrix} L_1 = [0, [1]] & L_2 = [[1], 0] \\ L_3 = [[0]\,, 1] & L_4 = [1, [0]] \end{pmatrix} \quad .$$

In diesem Quadrupel sind nun 0 und 1 bzw. Objekt und Subjekt vermöge wechselseitiger Abhängigkeit vermittelt, d.h. es gibt nicht nur objektive Subjekte, sondern auch subjektive Objekte – und ohne daß dafür Wahrscheinlichkeitswerte eingeführt werden müßten. Wegen der Möglichkeit perspektivischer Reflexion kann man nun echte qualitative Zahlen, d.h. solche, welche sowohl subjektive Objekte als Domänen- und objektive Subjekte als Codomänen der thetischen Einführung von Zeichen enthalten, auf Octupel abbilden. Da solche qualitativen Zahlen in 2-dimensionalen Zahlenfeldern gezählt werden müssen, kann ferner zwischen horizontaler, vertikaler und zwei diagonalen Zählweisen unterschieden werden, die wir als adjazente, subjazente und transjazente Zählweisen bezeichnet hatten.

2.3.1. Adjazente Zählweise

0_i	1_j		1_i	0_j		1_j	0_i		0_j	1_i
$\emptyset_i$	$\emptyset_j$		$\emptyset_i$	$\emptyset_j$		$\emptyset_j$	$\emptyset_i$		$\emptyset_j$	$\emptyset_i$
		$\times$			$\times$			$\times$		
$\emptyset_i$	$\emptyset_j$		$\emptyset_i$	$\emptyset_j$		$\emptyset_j$	$\emptyset_i$		$\emptyset_j$	$\emptyset_i$
0_i	1_j		1_i	0_j		1_j	0_i		0_j	1_i

2.3.2. Subjazente Zählweise

0_i	$\emptyset_j$		$\emptyset_i$	0_j		$\emptyset_j$	0_i		0_j	$\emptyset_i$
1_i	$\emptyset_j$		$\emptyset_i$	1_j		$\emptyset_j$	1_i		1_j	$\emptyset_i$
		$\times$			$\times$			$\times$		
1_i	$\emptyset_j$		$\emptyset_i$	1_j		$\emptyset_j$	1_i		1_j	$\emptyset_i$
0_i	$\emptyset_j$		$\emptyset_i$	0_j		$\emptyset_j$	0_i		0_j	$\emptyset_i$

2.3.3. Transjazente Zählweise

0_i	$\emptyset_j$		$\emptyset_i$	0_j		$\emptyset_j$	0_i		0_j	$\emptyset_i$
$\emptyset_i$	1_j		1_i	$\emptyset_j$		1_j	$\emptyset_i$		$\emptyset_j$	1_i
		$\times$			$\times$			$\times$		
$\emptyset_i$	1_j		1_i	$\emptyset_j$		1_j	$\emptyset_i$		$\emptyset_j$	1_i
0_i	$\emptyset_j$		$\emptyset_i$	0_j		$\emptyset_j$	0_i		0_j	$\emptyset_i$

Wie man anhand der verwendeten Indizes erkennt, kann man also die Positionen von subjektiven Objekten (0) und objektiven Subjekten (1) ohne Probleme im Sinne Günthers kontexturieren, d.h. subjektdeiktisch differenzieren, d.h. die qualitative Arithmetik dieser Relationalzahlen enthält die MdQ, aber das Umgekehrte gilt selbstverständlich nicht. Vor allem aber lassen sich bei den Relationalzahlen nun auch die Objekte kontexturieren, denn wegen des für subjektive Objekte und objektive Subjekte bestehenden Austauschs von Subjekt- und Objektanteilen verändert die Wahrnehmung eines Objektes durch verschiedene Subjekte auch die Objekte, insofern diese von verschiedenen Subjekten in verschiedener Weise wahrgenommen werden können. In der Arithmetik der Relationalzahlen gibt es somit nicht nur Hamiltonkreise für subjektale Negativität, sondern auch für objektale Positivität, und nur in diesem Sinne sollte von einer Mathematik der Qualitäten gesprochen werden.

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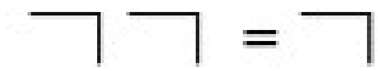
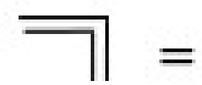
Die beiden Grundaxiome der Laws of Form

1. Spencer Browns Kalkül der Form beruht, wie allgemein bekannt ist, auf der Operation des Unterscheidens (vgl. Spencer Brown 1969). Diese selbst ist jedoch undefiniert: "Draw a distinction". Die beiden Grundaxiome lauten

1.1. Gesetz der Kondensation



1.2. Gesetz der Suspension



.

2. Beide Gesetze haben gemein, daß Iteration einer Operation nichts Neues bringt, d.h. sie sind rein quantitativ. Quantität beruht aber, wie zuletzt in Toth (2015a) anhand der "merkwürdig rapportierenden epischen Satzverkettungen" (Bense 1957, S. 518) in Gertrude Steins Texten gezeigt wurde, darauf, daß die den ortslosen Peanozahlen zugrunde liegende 2-wertige logische Dichotomie

$L = [0, 1]$

keine Vermittlung der beiden Werte 0 und 1 kennt, d.h. Strukturen wie

$[[0], 1], [1, [0]], [0, [1]], [[1], 0]$

sind genauso verboten wie Strukturen der Form $L = [2, 0, 1]$, $L = [0, 2, 1]$, $L = [0, 1, 2]$, nämlich durch das gleiche Grundgesetz des Tertium non datur. In den letzteren Fällen verletzt ein substantieller Wert, 2, in den ersteren Fällen verletzt ein differentieller Einbettungsoperator der Form

E: $x \rightarrow [x]$

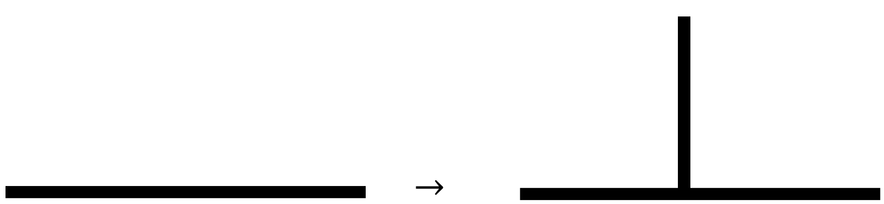
mit $x \in \{0, 1\}$ das Gesetz des Ausgeschlossenen Dritten.

3. Wie Spencer-Browns Basisaxiome allerdings zeigen, führt die Einführung von Differenz noch nicht zur Qualifizierung einer durch und durch quantitativen Logik, deren erkenntnistheoretische Werte das absolute, d.h. objektive, Objekt und das absolute, d.h. subjektive, Subjekt sind. Eine Logik, die stattdessen auf subjektiven, d.h. wahrgenommenen Objekten und objektiven, d.h. wahrnehmenden, Subjekten beruht, gibt es bis heute nicht, denn auch die sog. polykontexturale Logik Gotthard Günthers ist lediglich ein Verbundsystem von Kontexturen, innerhalb derer die 2-wertige Logik weiterhin unangetastet gilt (vgl. Toth 2015b).

3.1. Gehen wir aus von der folgenden Linie, wie man sie etwa als Ordnungsstruktur für die Peanozahlen betrachten kann

Ohne Subjektpräsenz ist diese Linie noch kein Differendum, sie ist überhaupt nichts, und der kategorische Imperativ des Mach-einen-Unterschied ist ebenso sinnlos, da er eine Entität voraussetzt, an der kein Unterschied eingebracht werden kann.

3.2. Die Linie bedarf also zunächst einer thetischen Setzung, und zwar einer ontischen und keiner semiotischen thetischen Setzung (vgl. Bense 1967, S. 9), und diese kann selbstverständlich nur durch ein Subjekt erfolgen. Erst dann wird die Linie zum Differendum – nämlich für das Subjekt, das sie ontisch thetisch gesetzt hat. Diese Linie ist somit notwendig ein subjektives und kein objektives Objekt mehr, und erst jetzt ist der kategorische Imperativ Spencer-Browns sinnvoll, d.h. man bekommt die Transformation

τ: 

3.3. Nun hat die Linie einen Unterschied bekommen, aber wir stehen, wie man sogleich sieht, vor einer qualitativen Gleichung der Form

$1_i + 1_j = 3$,

darin 1_i für die Linie und 1_j für den Unterschied (oder umgekehrt) steht, denn der Unterschied U teilt die Linie L in zwei Teile, und wir haben somit eine Tripelrelation der Form

$T = [L_i , U, L_j]$.

Das bedeutet aber, daß keine der logischen Dichotomie L = [0, 1] isomorphe Relation mehr vorliegt, sondern eine solche, die über nicht-leere Ränder verfügt, denn wie leicht einzusehen ist, gilt für T

$R[L_i , U] \neq R[U, L_j]$

und ferner natürlich

$R[L_i , L_j] \neq \varnothing$

und daher

$R[L_i , U] \neq R[U, L_j] \neq \varnothing$.

In anderen Worten: U fungiert, obwohl er kein Einbettungsoperator wie E ist, auf die gleiche Weise: Er erzeugt nichtleere Ränder und vermittelt daher zwischen Kategorien und erzeugt also differentielle Tertia durch "Anbettung", genauso wie es E durch Einbettung tut.

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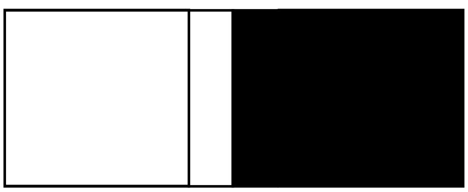
Der Jäger Gracchus und die Vermittlung von Diesseits und Jenseits

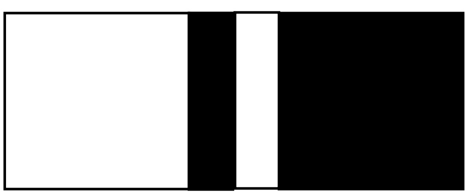
1. Die Vorstellung, daß Diesseits (D) und Jenseits (J) nicht durch eine Grenze G der Form

mit

$G \in D \cup J$,

sondern durch einen Rand der Formen





mit

$R[D, J] \neq R[J, D] \neq \emptyset$

getrennt sind, ist in der Mythologie seit sehr langer Zeit bekannt. Aus neuerer Zeit bekannt ist die folgende Stelle aus Kafkas "Jäger Gracchus".

> Der Jäger nickte und zog die Zungenspitze zwischen den Lippen durch: »Ja, die Tauben fliegen vor mir her. Glauben Sie aber, Herr Bürgermeister, daß ich in Riva bleiben soll?«
>
> »Das kann ich noch nicht sagen«, antwortete der Bürgermeister. »Sind Sie tot?«
>
> »Ja«, sagte der Jäger, »wie Sie sehen. – Vor vielen Jahren, es müssen aber ungemein viel Jahre sein, stürzte ich im Schwarzwald – das ist in Deutschland – von einem Felsen, als ich eine Gemse verfolgte. Seitdem bin ich tot.«

»Aber Sie leben doch auch«, sagte der Bürgermeister.

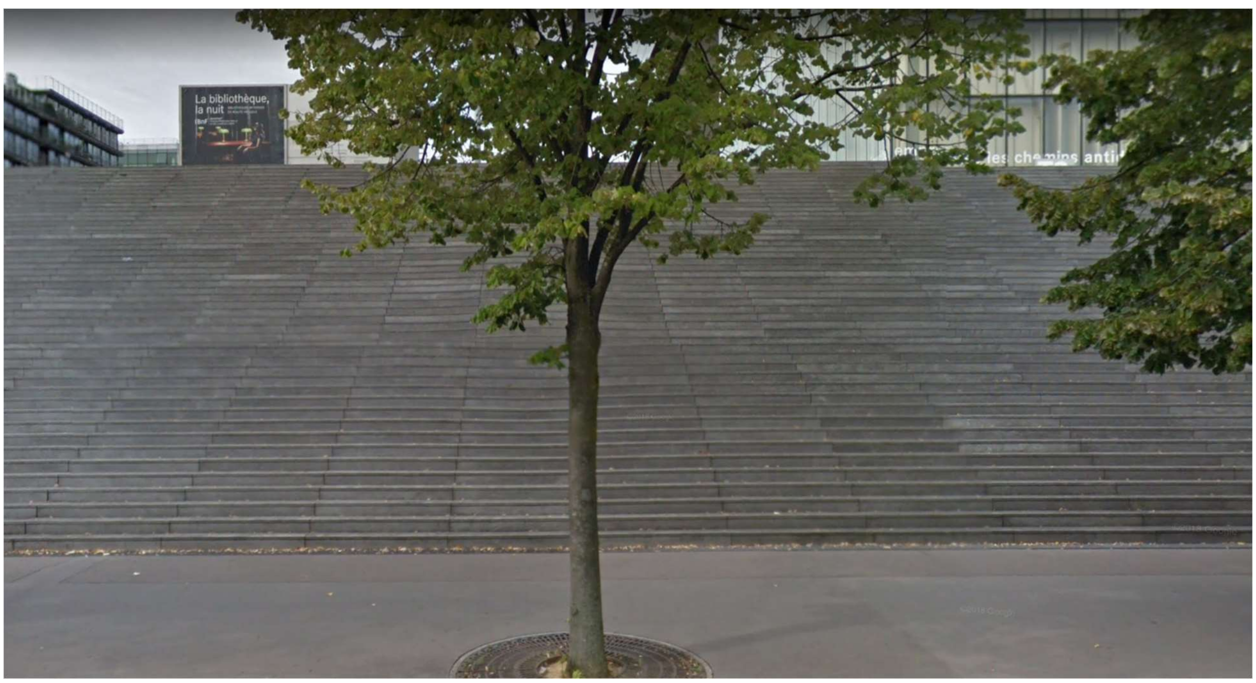
»Gewissermaßen«, sagte der Jäger, »gewissermaßen lebe ich auch. Mein Todeskahn verfehlte die Fahrt, eine falsche Drehung des Steuers, ein Augenblick der Unaufmerksamkeit des Führers, eine Ablenkung durch meine wunderschöne Heimat, ich weiß nicht, was es war, nur das weiß ich, daß ich auf der Erde blieb und daß mein Kahn seither die irdischen Gewässer befährt. So reise ich, der nur in seinen Bergen leben wollte, nach meinem Tode durch alle Länder der Erde.«

»Und Sie haben keinen Teil am Jenseits?« fragte der Bürgermeister mit gerunzelter Stirne.

»Ich bin«, antwortete der Jäger, »immer auf der großen Treppe, die hinaufführt. Auf dieser unendlich weiten Freitreppe treibe ich mich herum, bald oben, bald unten, bald rechts, bald links, immer in Bewegung.

(Franz Kafka, Der Jäger Gracchus)

Als approximatives ontisches Modell könnte das folgende dienen



Quai François Mauriac, Paris.

3. Eine sowohl D als auch J gemeinsame Grenze der Form $G \in D \cup J$ kann nur eine Linie sein. Ein Streifen setzt jedoch mit $R[D, J] \neq R[J, D] \neq \emptyset$ die Nicht-Vertauschbarkeit von D und J voraus, d.h. es muß gelten

$$L = [D, J] \neq L^{-1} = [J, D].$$

Da R gemäß den obigen Diagrammen kein von D und J verschiedener dritter Wert darstellt, d.h. nicht substantiell von D und J verschieden ist, muß er differentiell verschieden sein. Wenn man berücksichtigt, daß man in den Diagrammen noch die ontischen Orte von Weiß und Schwarz bzw. die Abbildungen von D und J auf die entsprechend eingefärbten Kästchen vertauschen kann, bekommen wir die folgenden 4 möglichen Strukturen

$L_1 = [D, [J]]$ $L_1^{-1} = [[J], D]$

$L_2 = [[D], J]$ $L_1^{-1} = [J, [D]]$,

d.h. es kann sowohl das Jenseits auf zwei perspektivisch geschiedene Arten Teil des Diesseits sein als auch das Diesseits auf zwei perspektivisch geschiedene Arten Teil des Jenseits sein. Anders gesagt: Es gibt nicht nur das im Sein nichtende Nichts, sondern auch das im Nichts wesende Sein, und dies auf zweimal zwei qualitativ-arithmetisch differenzierbare Arten. Alles, was dazu benötigt wird, sind zwei Peanozahlen 0 und 1 und ein Einbettungsoperator E, der

$0 \rightarrow [0]$

$1 \rightarrow [1]$

einbettet. Dabei kann natürlich wiederum sowohl 0 als auch 1 sowohl auf D als auch auf J abgebildet werden.

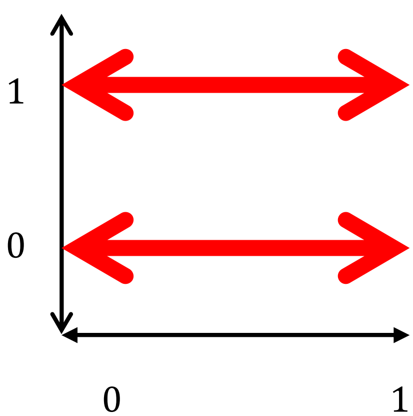
Wie in Toth (2015a-c) gezeigt, erhält man wegen E keine Zahlenlinien, sondern Zahlenfelder, in denen es nicht nur eine horizontale, sondern zusätzlich eine vertikale und zwei diagonale Zählweisen gibt, die wir mit adjazenter, subjazenter und transjazenter Zählweise bezeichnet hatten.

3.1. Adjazente Zählweise

3.1.1. Zahlenfelder

x_i	y_j		y_i	x_j		y_j	x_i		x_j	y_i
$\varnothing_i$	$\varnothing_j$		$\varnothing_i$	$\varnothing_j$		$\varnothing_j$	$\varnothing_i$		$\varnothing_j$	$\varnothing_i$
		$\times$			$\times$			$\times$		
$\varnothing_i$	$\varnothing_j$		$\varnothing_i$	$\varnothing_j$		$\varnothing_j$	$\varnothing_i$		$\varnothing_j$	$\varnothing_i$
x_i	y_j		y_i	x_j		y_j	x_i		x_j	y_i

3.1.2. Zahlenschema

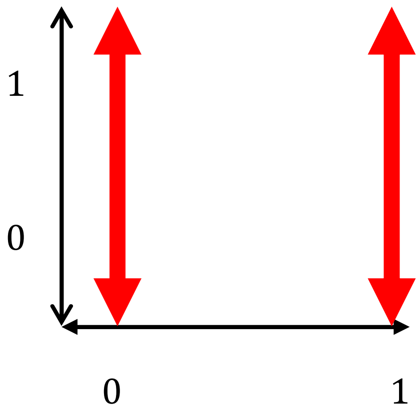


3.2. Subjazente Zählweise

3.2.1. Zahlenfelder

x_i	$\varnothing_j$		$\varnothing_i$	x_j		$\varnothing_j$	x_i		x_j	$\varnothing_i$
y_i	$\varnothing_j$		$\varnothing_i$	y_j		$\varnothing_j$	y_i		y_j	$\varnothing_i$
		$\times$			$\times$			$\times$		
y_i	$\varnothing_j$		$\varnothing_i$	y_j		$\varnothing_j$	y_i		y_j	$\varnothing_i$
x_i	$\varnothing_j$		$\varnothing_i$	x_j		$\varnothing_j$	x_i		x_j	$\varnothing_i$

3.2.2. Zahlenschema

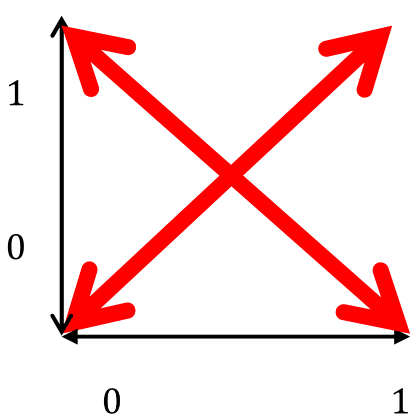


3.3. Transjazente Zählweise

3.3.1. Zahlenfelder

x_i	$Ø_j$		$Ø_i$	x_j		$Ø_j$	x_i		x_j	$Ø_i$
$Ø_i$	y_j		y_i	$Ø_j$		y_j	$Ø_i$		$Ø_j$	y_i
		$\times$			$\times$			$\times$		
$Ø_i$	y_j		y_i	$Ø_j$		y_j	$Ø_i$		$Ø_j$	y_i
x_i	$Ø_j$		$Ø_i$	x_j		$Ø_j$	x_i		x_j	$Ø_i$

3.3.2. Zahlenschema



Da in diesem Zahlenfeldern einer einbettungstheoretischen Arithmetik nicht nur

$0 = f(1)$,

sondern auch

$1 = f(0)$

gilt, kann also hier im Gegensatz zur polykontexturalen Logik G. Günthers NICHT NUR DIE SUBJEKT-, SONDERN AUCH DIE OBJEKTPOSITION ITERIERT WERDEN. Statt der 2-wertigen aristotelischen Dichotomie unvermittelter und daher reflexionssymmetrischer Werte (vgl. Günther 2000, S. 230 f.), die für jede Einzelkontextur auch innerhalb des polykontexturalen Verbundssystems weiterbesteht, geht die der ortsfunktionalen Arithmetik zugehörige Logik also nicht von einer Unvermitteltheitsrelation zwischen objektivem Objekt und subjektivem Subjekt, sondern von einer qua E bewerkstelligten Vermitteltheitsrelation zwischen subjektivem Objekt und objektivem Subjekt aus. Diese beiden vermittelten logischen und erkenntnistheoretischen Kategorien entsprechen aber genau dem Dualverhältnis von wahrgenommenem Objekt und dem Zeichen, das von Bense (1967, S. 9) als Metaobjekt definiert worden war, so zwar, daß das subjektive Objekt als Domäne und das objektive Subjekt als Codomäne des metaobjektiven Prozesses der thetischen Setzung von Zeichen fungiert.

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Die Logik des Jägers Gracchus

1. Für die 2-wertige aristotelische Logik gilt

$L = [0, 1] = L^{-1} = [1, 0]$,

denn das Gesetz vom Ausgeschlossenen Dritten verbietet die Annahme eines vermittelnden Wertes

$0 \vee \neg 0$

$1 \vee \neg 1$.

2. Allerdings gibt es neben der Möglichkeit substantieller dritter Werte die Erzeugung eines differentiellen Tertiums. Dafür benötigen wir einen Einbettungsoperator E (vgl. Toth 2014).

$E \rightarrow L = [0, 1] =$

$$\left(\begin{array}{ll} L_1 = [0, [1]] & L_1^{-1} = [[1], 0] \\ L_2 = [[0], 1] & L_2^{-1} = [1, [0]] \end{array}\right)$$

Anstelle von 0 und 1 bekommen wir somit in diesem minimalen Fall

0, [0]

1, [1],

d.h. für jedes L_i gilt

$0 = f(1)$

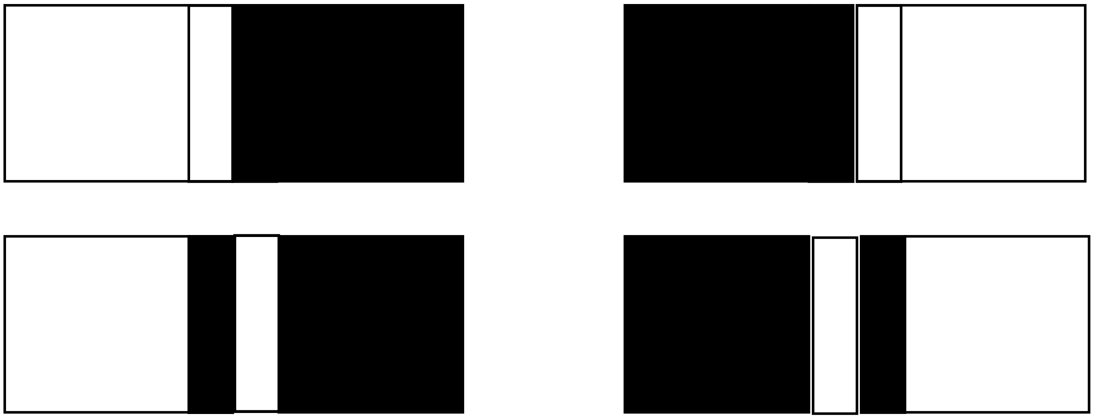
$1 = f(0)$,

und somit ist

$(x \in 0) \subset 1$

$(y \in 1) \subset 0$,

d.h. 0 hat 1-Anteile, und 1 hat 0-Anteile. Man kann dies schematisch wie folgt darstellen (vgl. Toth 2015a).



Die Werte in einer solchen Logik sind also vermöge eines differentiellen Tertiums vermittelt. In Sonderheit gilt also für den Rand R

$R[0, 1] \neq R[1, 0] \neq Ø$,

während für $L = [0, 1]$ natürlich gilt

$R[0, 1] = R[1, 0] = Ø$,

vgl. dazu die folgenden äußerst treffenden Feststellungen: "Beide Werte einer solchen Logik aber sind metaphysisch äquivalent. Das heißt, man kann sie beliebig miteinander vertauschen. Sie verhalten sich zueinander in einer totalen logischen Disjunktion, wie rechts und links. Es gibt keinen theoretischen Grund, welche Seite rechts und welche Seite links von der Zugspitze ist. Die Benennung beruht auf einer willkürlichen Entscheidung, und wenn man seinen Standpunkt wechselt, sind die rechte und die linke Seite miteinander vertauscht (Günther 2000, S. 230 f.).

3. Da entweder 0 oder 1 die logische Objekt- oder Subjektpositionen einnehmen, bedeutet die funktionelle Abhängigkeit beider Werte voneinander, daß das stillschweigend vorausgesetzte Axiom der 2-wertigen Logik, die, wie übrigens auch die polykonxexturale Logik Günthers, auf objektiven Objekten und subjektiven Subjekten basiert, suspendiert wird. Stattdessen sind subjektive Objekte und objekte Subjekte die neuen logischen Basiskategorien.

$\Omega = f(\Sigma)$	subjektives Objekt	Objekt
$\Sigma = f(\Omega)$	objektives Subjekt	Zeichen

Wie man erkennt, ist also das wahrgenommene subjektive Objekt gerade das Domänen- und das objektive Subjekt als dessen "Metaobjekt" (vgl. Bense 1967, S. 9) gerade das Codomänenelement der thetischen Einführung von Zeichen, d.h. die neue logische Basis ist gleichzeitig das vollständige Abbildungsschema der Zeichensetzung. Damit stehen Objekt und Zeichen in einer Dualrelation

$\Omega = f(\Sigma) \times \Sigma = f(\Omega)$,

und diese besagt, daß das Objekt – vermöge seiner Wahrnehmung, die selbstverständlich nur durch ein Subjekt erfolgen kann – Subjektanteile besitzt und daß das Subjekt – vermöge seiner Objektwahrnehmung – Objektanteile besitzt. Daraus folgt aber nicht mehr und nicht weniger, als daß es eine Brücke zwischen dem Diesseits des Subjektes bzw. Objektes und dem Jenseits des Objektes bzw. Subjektes gibt. Subjektanteile und Objektanteile werden also bei der Wahrnehmung vermöge einer Menge von Transformationen ausgetauscht

$[\Sigma = f(\Omega)] \rightleftarrows [\Omega = f(\Sigma)]$ subjektives Objekt $\rightleftarrows$ objektives Subjekt,

die als Partizipationsrelationen definierbar sind. Es nichtet nicht nur das Nichts im Sein des Seienden, sondern es west auch das Sein des Seienden im Nichts.

4. Da die Werte 0 und 1 auch als eingebettete in der Form [0] und [1] auftreten können, bedeutet dies, daß eine Linie zur Darstellung der Peanozahlen nicht mehr ausreicht. Die eingebetteten Zahlen können auch unter- oder oberhalb dieser Linie aufscheinen, d.h. sie bekommen erstens eine Menge von ontischen Orten und nicht nur einen "Stellenwert" (bzw. eine "Wertstelle") zugewiesen, und zweitens wird statt einer Zahlenlinie ein Zahlenfeld vorausgesetzt. In diesem gibt es somit nicht nur die horizontale, sondern auch eine vertikale und eine horizontale Zählweise, die wir in Toth (2015b-d) mit adjazenter, subjazenter und transjazenter Zählweise bezeichnet hatten. In den folgenden vollständigen Zahlenfeldern für die 2-elementige Menge $P = (0, 1)$ sind nun alle partizipativen Austauschrelationen zwischen subjektiven Objekten und objektiven Subjekten qua $0 = f(1)$ und $1 = f(0)$ durch Doppelpfeile eingezeichnet.

4.1. Adjazente Zählweise

$$
\begin{array}{ccccccccccc}
x_i & y_j & & y_i & x_j & & y_j & x_i & & x_j & y_i \\
\emptyset_i & \emptyset_j & \rightleftarrows & \emptyset_i & \emptyset_j & \rightleftarrows & \emptyset_j & \emptyset_i & \rightleftarrows & \emptyset_j & \emptyset_i \\
\uparrow\downarrow & & \times & \uparrow\downarrow & & \times & \uparrow\downarrow & & \times & \uparrow\downarrow & \\
\emptyset_i & \emptyset_j & & \emptyset_i & \emptyset_j & & \emptyset_j & \emptyset_i & & \emptyset_j & \emptyset_i \\
x_i & y_j & \rightleftarrows & y_i & x_j & \rightleftarrows & y_j & x_i & \rightleftarrows & x_j & y_i
\end{array}
$$

3.2. Subjazente Zählweise

$$
\begin{array}{ccccccccccc}
x_i & \emptyset_j & & \emptyset_i & x_j & & \emptyset_j & x_i & & x_j & \emptyset_i \\
y_i & \emptyset_j & \rightleftarrows & \emptyset_i & y_j & \rightleftarrows & \emptyset_j & y_i & \rightleftarrows & y_j & \emptyset_i \\
\uparrow\downarrow & & \times & \uparrow\downarrow & & \times & \uparrow\downarrow & & \times & \uparrow\downarrow & \\
y_i & \emptyset_j & & \emptyset_i & y_j & & \emptyset_j & y_i & & y_j & \emptyset_i \\
x_i & \emptyset_j & \rightleftarrows & \emptyset_i & x_j & \rightleftarrows & \emptyset_j & x_i & \rightleftarrows & x_j & \emptyset_i
\end{array}
$$

4.3. Transjazente Zählweise

$$
\begin{array}{ccccccccccc}
x_i & \emptyset_j & & \emptyset_i & x_j & & \emptyset_j & x_i & & x_j & \emptyset_i \\
\emptyset_i & y_j & \rightleftarrows & y_i & \emptyset_j & \rightleftarrows & y_j & \emptyset_i & \rightleftarrows & \emptyset_j & y_i \\
\uparrow\downarrow & & \times & \uparrow\downarrow & & \times & \uparrow\downarrow & & \times & \uparrow\downarrow & \\
\emptyset_i & y_j & & y_i & \emptyset_j & & y_j & \emptyset_i & & \emptyset_j & y_i \\
x_i & \emptyset_j & \rightleftarrows & \emptyset_i & x_j & \rightleftarrows & \emptyset_j & x_i & \rightleftarrows & x_j & \emptyset_i.
\end{array}
$$

Da, wie bereits angedeutet, in der polykontexturalen Logik G. Günthers und der auf ihr beruhenden Mathematik der Qualitäten E. Kronthalers die 2-wertige aristotelische Logik L = (0, 1) für jede Einzelkontextur unangetastet bleibt und sich die Poly-Kontexturalität also lediglich der Iterierbarkeit des Subjektes verdankt, dieses aber weiterhin ein subjektives Subjekt ist, kann in dieser polykontexturalen Logik, Mathematik und Ontologie keine Rede davon sein, daß man Äpfel und Birnen addieren könne, wie dies ständig behauptet wird (vgl. z.B. Kronthaler 1990). 1 Apfel + 1 Birne ergeben bekanntlich 2 Früchte. Interessant an dieser qualitativen Gleichung ist aber nicht nur der angeblich Qualitätsverlust in der Summe, sondern die Tatsache, daß nur deswegen überhaupt eine Summe gebildet werden kann, weil Apfel und Birne ein vermittelndes Drittes gemeinsam haben, denn die weitere qualitative Gleichung 1 Apfel + 1 Stein hat beispielsweise keine angebbare Summe. Wenn es aber ein vermittelndes Drittes gibt, bedeutet dies natürlich wiederum, daß die Schnittmenge der Merkmalsmengen von Apfel und Birne nicht leer sein kann, und damit sind die Zahlen, welche Apfel und Birne vertreten, also 0 und 1 oder 1 und 0, natürlich vermittelt, d.h. folgend der oben skizzierten qualitativen ortsfunktionalen Arithmetik mit ihren drei 2-dimensionalen Zählweisen. Eine qualitative Mathematik, welche diesen Namen verdient, setzt also zwei fundamentale Änderungen der polykontexturallogischen Basis voraus:

1. die Ersetzung der logischen Basiskategorien des objektiven Objektes und des subjektiven Subjektes durch die vermittelten Kategorien des subjektiven Objektes und des objektiven Subjektes.

2. die daraus resultierende Möglichkeit, nicht nur das Subjekt, sondern auch das Objekt iterieren zu lassen. Damit ergeben sich ungeheuer komplexere "Permutogramme" (G.G. Thomas) bzw. Hamiltonkreise (G. Günther) als diejenigen, welche innerhalb der polykontexturalen Logik benutzt werden.

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"Im uferlosen Meer der Dinge zwischen Sein und Nichts"

1. Der Titel dieses Aufsatzes stammt aus einem späten Gedicht Max Benses (Bense 1985, S. 29). Dieser Satz ist daher mehr als nur erstaunlich, denn gemäß Benses Metaphysik, die natürlich in Sonderheit seiner Semiotik zugrunde liegt, gilt, wie Hausdorff in der von Bense veranstalteten Neuedition eines philosophischen Frühwerkes sagt, "daß es derlei vermittelnde Gebiete nicht gibt, daß vom Empirischen zum Absoluten keine Brücke herüber und hinüber führt" (Hausdorf 1976, S. 27), in anderen Worten, sowohl Hausdorff als auch sein Schüler Bense halten am "unbedingten Dualismus zwischen Erscheinung und Ding an sich" fest (ibd., S. 25).

2. Das Problem liegt, wie von uns zuletzt in Toth (2015a, b) dargestellt, daran, daß die klassische 2-wertige aristotelische Logik von einer dichotomischen Relation

$L = [0, 1]$

ausgeht, in welcher 0 oder 1 für das objektive Objekt und 1 oder 0 für das subjektive Subjekt stehen. Das Grundgesetz des Tertium non datur verbietet mit jeglicher Vermittlung zwischen den beiden dergestalt reflexiven Werten nicht nur einen dritten substantiellen Wert, sondern auch eine differentielle Vermittlung, d.h. die Relation L ist rein koordinativ, und die vier möglichen subordinativen und superordinativen Relationstypen

$L_1 = [0, [1]$

$L_2 = [[1], 0]$

$L_3 = [[0], 1]$

$L_4 = [1, [0]]$

sind daher ausgeschlossen. Geht man jedoch von diesem Quadrupel aus, so enthält jedes Objekt Subjektanteile und jedes Subjekt Objektanteile, bzw., in erkenntnistheoretischer Terminologie gesprochen, das Ding an sich wird durch das von einem Subjekt wahrgenommene und daher subjektive Objekt und die

Erscheinung wird durch das ein Objekt wahrnehemende, d.h. objektive Subjekt ersetzt

$\Omega = f(\Sigma)$ subjektives Objekt Objekt

$\Sigma = f(\Omega)$ objektives Subjekt Zeichen.

3. Das "uferlose Meer der Dinge zwischen Sein qua Objekt und Nichts qua Zeichen bzw. zwischen Objekt und Subjekt kann dann als eine besondere Form einer von Neumann-Hierarchie wie folgt dargestellt werden

$\Omega(\Sigma) \times \Sigma(\Omega)$

$\Sigma(\Omega(\Sigma)) \times \Sigma(\Omega(\Sigma))$

$\Omega(\Sigma(\Omega(\Sigma))) \times \Omega(\Sigma(\Omega(\Sigma)))$

$\Sigma(\Omega(\Sigma(\Omega(\Sigma)))) \times \Sigma(\Omega(\Sigma(\Omega(\Sigma))))$

$\Omega(\Sigma(\Omega(\Sigma(\Omega(\Sigma)))) \times \Omega(\Sigma(\Omega(\Sigma(\Omega(\Sigma)))))$

$\Sigma(\Omega(\Sigma(\Omega(\Sigma(\Omega(\Sigma))))) \times \Sigma(\Omega(\Sigma(\Omega(\Sigma(\Omega(\Sigma)))))), ...,$

in der jede (n+1)-te Dualrelation gegenüber jeder n-ten Dualrelation entweder um einen Objekt- oder einen Subjektanteil ansteigt bzw., rückwärts durchlaufen, absteigt. Da diese Hierarchie prinzipiell ad infinitum fortsetzbar ist, ergibt sich eine enorme Komplexität von sog. Partizipationsrelationen zwischen subjektiven Objekten und objektiven Subjekten, deren Basisschema durch

$[\Sigma = f(\Omega)] \rightleftarrows [\Omega = f(\Sigma)]$

bestimmt worden war.

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"Denn was man erkannt hat, wird uns nicht erkennen"

1. Dieser hier als Titel verwendete Satz stammt aus einem späten Gedicht Max Benses (Bense 1985, S. 25). Obwohl Benses Metaphysik klassisch-aristotelisch ist, setzt dieser Satz, wie nachfolgend gezeigt wird, eine besondere Form von Vermittlungslogik voraus, in welcher die 2-wertige koordinative Dichotomie

$L = [0, 1]$

durch das Quadrupel von sub- und superordinativen Relationen

$L_1 = [0, [1]$

$L_2 = [[1], 0]$

$L_3 = [[0], 1]$

$L_4 = [1, [0]]$

ersetzt werden muß (vgl. Toth 2015).

2. Die Relation des Erkennens ist notwendig eine Relation zwischen einem erkennenden Subjekt und einem erkannten Objekt, d.h. in der folgenden Matrix logisch-erkenntnistheoretischer Funktionen, wie sie dem Quadrupel L_1 bis L_4 zugrunde liegt

	Ω	Σ
Ω	ΩΩ	ΩΣ
Σ	ΣΩ	ΣΣ,

kommen nur die beiden Funktionen

$\Omega\Sigma =: \Omega = f(\Sigma)$

und

$\Sigma\Omega =: \Sigma = f(\Omega)$

in Frage.

Benses Satz behauptet nun, daß die Erkenntnis eines Objektes die Erkenntnis des Erkennenden absorbiert bzw. auslöscht. Da hierfür auch Subjekte in Frage kommen, da jedes von einem Subjekt A erkannte Subjekt B relativ zu A als Objekt (et vice versa) erscheint, können wir die beiden oben definierten vermittelten logisch-erkenntnistheoretischen Funktionen als partiell unbesetzte Funktions-Formen dergestalt definieren, daß jede Funktion auf ein Paar von Funktions-Formen abgebildet wird

$$\Omega\Sigma \begin{array}{ll} \nearrow & \langle —, \Sigma \rangle \\ \searrow & \langle \Omega, — \rangle \end{array}$$

$$\Sigma\Omega \begin{array}{ll} \nearrow & \langle —, \Omega \rangle \\ \searrow & \langle \Sigma, — \rangle. \end{array}$$

Dies ist ein in der Tat neues und hoch interessantes Ergebnis, denn, wie man sieht, koinzidiert keine der vier Funktions-Formen mit irgend einer anderen, d.h. Erkanntes und Erkennendes unterscheiden sich nicht nur im Fehlen des jeweils im Rahmen der Objekt-Subjekt-Dichotomie L fehlenden Gliedes, sondern zusätzlich im ontischen Ort sowohl des fehlenden als auch des nichtfehlenden Gliedes.

Literatur

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Toth, Alfred, Im uferlosen Meer der Dinge zwischen Sein und Nichts. In: Electronic Journal for Mathematical Semiotics, 2015

"Verkleinerung der Entfernung der Worte vom Gegenstand"

1. Der Titel dieses Aufsatzes, der an Toth (2015) anschließt, ist ein Satz aus Max Benses letztem Werk "Poetische Abstraktionen", das wenige Tage nach seinem Tode im April 1990 in limitierter Auflage erschienen ist (vgl. Bense 1990). In einem ein gutes Jahrzehnt zuvor publizierten Band mit Gedichten Benses steht der ähnliche Satz: "Wörter so niedrig hängen,/wie es nur geht./Sie sollen die Dinge berühren,/eh sie verschwinden" (Bense 1981, S. 39).

2. Die Besonderheit dieser Sätze besteht darin, daß sie die Abbildung des dichotomischen Basisschemas der 2-wertigen aristotelischen Logik

$L = [0, 1]$,

darin die Ränder zwischen den Werten leer sind, d.h.

$R[0, 1] = R[1, 0] = \emptyset$

gilt, auf ein Quadrupel von Dichotomien der Form

$L_1 = [0, [1]]$

$L_2 = [[1], 0]$

$L_3 = [[0], 1]$

$L_4 = [1, [0]]$

(mit $L_2 = L_1^{-1}$ und $L_4 = L_3^{-1}$) mit nicht-leeren Rändern, d.h.

$R[0, 1] \neq R\ [1, 0] \neq \emptyset$

voraussetzen. Man beachte, daß diese Nichtleerheit der Ränder durch einen Einbettungsoperator E

$E: \quad x \rightarrow [x]$,

der also differentiell und nicht-substantiell fungiert, bewirkt wird. Anders gesagt: Man braucht keinen dritten logischen Wert, um in $L = [0, 1]$ nicht-leere Ränder zwischen den Werten zu erzeugen.

2. Ferner geht aus den Sätzen Benses hervor, daß offenbar eine asymptotische Funktion zwischen bezeichnendem Zeichen (Wort) und bezeichnetem Objekt (Gegenstand) besteht. Da die fundamentale Dichotomie des Quadrupels L_1 bis L_4 nach Toth (2015) der folgenden von Neumann-Hierarchie korrespondiert

$\Omega(\Sigma) \quad \times \quad \Sigma(\Omega)$

$\Sigma(\Omega(\Sigma)) \quad \times \quad \Sigma(\Omega(\Sigma))$

$\Omega(\Sigma(\Omega(\Sigma))) \quad \times \quad \Omega(\Sigma(\Omega(\Sigma)))$

$\Sigma(\Omega(\Sigma(\Omega(\Sigma)))) \quad \times \quad \Sigma(\Omega(\Sigma(\Omega(\Sigma))))$

$\Omega(\Sigma(\Omega(\Sigma(\Omega(\Sigma)))) \quad \times \quad \Omega(\Sigma(\Omega(\Sigma(\Omega(\Sigma)))))$

$\Sigma(\Omega(\Sigma(\Omega(\Sigma(\Omega(\Sigma))))) \quad \times \quad \Sigma(\Omega(\Sigma(\Omega(\Sigma(\Omega(\Sigma)))))), ...,$

beschreibt diese Hierarchie, von "unten", d.h. von der einfach vermittelten Dualität zwischen subjektivem Objekt und objektivem Subjekt, angefangen, in einer immer feineren Annäherung der Subjektanteile von Objekten und, dual, der Objektanteile von Subjekten, präzise die bensesche "Verkleinerung der Entfernung der Worte vom Gegenstand" (Bense 1990, S. 23). Obwohl dieser Prozess natürlich ad infinitum fortsetzbar ist, tritt im Rahmen des Basisschemas $L = [0, 1]$ der aristotelischen Logik niemals der Fall ein, daß $0 \equiv 1$ bzw. $1 \equiv 0$ wird, d.h. obwohl die Werte in L austauschbar sind in dem Sinne, daß eine auf 0 aufgebaute Logik einer auf 1 aufgebauten Logik notwendig isomorph sein muß, bleibt die Kontexturgrenze zwischen 0 und 1 bzw. 1 und 0 trotz einer immer präziseren Annäherung von Worten an Gegenstände bzw. Gegenständen an Worte stets bestehen. Dies gilt übrigens auch dann, wenn man, statt eines differentiellen Tertiums durch E einzuführen, einen substantiellen dritten Wert einführt, denn in diesem Fall wird lediglich das Tertium non datur zu einem Quartum, Quintum, Sextum, ... non datur verschoben, i.a.W., das 2-wertige aristotelische Schema $L = [0, 1]$ bleibt auch in der von Gotthard Günther begründeten polykontexturalen Logik bestehen. Daraus folgt allerdings, daß die Kontexturgrenze zwischen Objekt und Subjekt bzw. Objekt und Zeichen niemals aufhebbar ist, ganz egal, ob man das logische Basisschema $L = [0, 1]$ durch E auf ein ortsfunktionales Quadrupel abbildet, oder ob man L in ein "polykontexturales" Verbundsystem einbettet. Nicht-beantwortbar ist allerdings die Frage, OB DIESE TRANSZENDENZ ZWISCHEN OBJEKT UND ZEICHEN DURCH DIE THETISCHE SETZUNG DER ZEICHEN ERZEUGT WIRD, ODER OB SIE ES IST, WELCHE ERST DIE TRANSZENDENZ ERZEUGT. Vermutlich liegt hier einer der qualitativen logischen Fälle vor, für welche das bekannte wittgensteinische Verbot, daß eine Funktion nicht als ihr eigenes Argument auftreten darf, suspendiert ist.

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Die Ränder der Wörter

1. Der Titel dieses Aufsatzes ist ein Zitat aus Bense (1985, S. 37). Es ist allerdings doppeldeutig. Wörter sind bekanntlich Zeichen, also können die Ränder zwischen Zeichen gemeint sein. Andererseits bezeichnen Zeichen Objekte, denn Zeichen wurden von Bense ausdrücklich als "Metaobjekte" eignеführt (vgl. Bense 1967, S. 9), d.h. die Ränder von Wörtern können auch Partizipationsrelationen zwischen Objekten und Zeichen sein. Wir haben somit die beiden folgenden Alternativen

$R[Z_i, Z_j]$

$R[Z, \Omega]$.

2. Diese beiden Randtypen sind jedoch nicht-symmetrisch, denn während

$R[Z_i, Z_j] = R[Z_j, Z_i]$

gilt, denn es ist z.B.

$R[[3.1, 2.1, 1.3], [3.1, 2.2, 1.3]] = R[[3.1, 2.2, 1.3], [3.1, 2.1, 1.3]] = [3.1, 1.3]$,

gilt für Zeichen und Objekte die Ungleichung

$R[Z, \Omega] \neq R[\Omega, Z]$,

denn anders als zwischen Zeichen und Zeichen verläuft zwischen Zeichen und Objekten eine Kontexturgrenze, d.h. es gibt die folgenden vier Möglichkeiten

$R_1[Z, [\Omega]]$ $\quad$ $R_2[[\Omega], Z]$

$R_3[[Z], \Omega]$ $\quad$ $R_4[\Omega, [Z]]$,

so daß für alle Paare des Quadrupels der Rand nichtnull ist. Dies trifft für Zeichen nicht zu, denn trotz der Tatsache, daß die eigenreale Zeichenklasse mit jeder anderen Zeichenklasse und Realitätsthematik in mindestens einem Subzeichen zusammenhängen muß (vgl. Walther 1982), gibt es Paare von Zeichenklassen und Realitätsthematiken mit leeren Rändern, z.B.

$R[[3.1, 2.1, 1.1], [3.2, 2.2, 1.2]] = Ø$.

3. Während dies für Zeichen jedoch kein Problem darstellt, stellt es für Ränder zwischen Zeichen und Objekten ein beinahe unüberwindliches Problem dar, denn das Quadrupel von Relationen ist das Ergebnis der Anwendung eines Einbettungsoperators E

E: $x \rightarrow [x]$,

d.h. E erzeugt ein differentielles (nicht-substantielles) Tertium und widerspricht somit der 2-wertigen aristotelischen Logik.

Ein noch größeres Problem stellt, wie bereits in Toth (2015) angedeutet, die Tatsache dar, daß die von Bense eingeführten semiotischen Funktionen, d.h. die Bezeichnungsfunktion ($M \rightarrow O$), die Bedeutungsfunktion ($O \rightarrow I$) und die Gebrauchsfunktion ($I \rightarrow M$), nicht-bijektiv auf die ihnen isomorphen ontischen Funktionen abbildbar sind

Semiotik	Ontik
$M \rightarrow O$	$\Omega \rightarrow (M = Z)$
$O \rightarrow I$	$\Omega \rightarrow \Sigma$
$I \rightarrow M$	$\Omega \rightarrow \Sigma$
	$(M = Z) \rightarrow \Sigma$.

Wie man sogleich erkennt, tritt die Abbildung ($\Omega \rightarrow \Sigma$) nicht nur bei der Bedeutungsfunktion, sondern auch bei der Gebrauchsfunktion auf. Indessen korrespondiert die zyklische semiotische Transformation

t: $(M \rightarrow O) \rightarrow (O \rightarrow I) = (I \rightarrow M)$,

die Bense (1971, S. 81) als Kreisgraphen dargestellt hatte, der ebenfalls zyklischen ontischen Transformation

u: $\Omega \rightarrow (M = Z) \rightarrow \Omega \rightarrow \Sigma \rightarrow (M = Z) \rightarrow \Sigma$,

in der das Zeichen zwischen dem bezeichneten Objekt und dem es bezeichnenden Subjekt vermittelt.

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"Die Unterschiede wären, wenn sie wären, alles oder leer"

1. Das vollständige Zitat, das hier verkürzt als Titel verwendet wird, lautet: "Die grauen Unterschiede,/weder Ding noch Schatten,/wären, wenn sie wären,/alles oder leer (Bense 1983, S. 21). Diese Aussage nimmt natürlich Bezug auf das dichotomische Basisschema der 2-wertigen aristotelischen Logik

$L = [0, 1]$,

darin die Ränder zwischen den Werten leer sind, d.h. es gilt

$R[0, 1] = R[1, 0] = \emptyset$.

Ferner gilt

$L = [0, 1] = L^{-1} = [1, 0]$,

wozu es eine unübertreffliche Kommentierung gibt: "Beide Werte einer solchen Logik aber sind metaphysisch äquivalent. Das heißt, man kann sie beliebig miteinander vertauschen. Sie verhalten sich zueinander in einer totalen logischen Disjunktion, wie rechts und links. Es gibt keinen theoretischen Grund, welche Seite rechts und welche Seite links von der Zugspitze ist. Die Benennung beruht auf einer willkürlichen Entscheidung, und wenn man seinen Standpunkt wechselt, sind die rechte und die linke Seite miteinander vertauscht (Günther 2000, S. 230 f.).

2. Die Lösung der polykontexturalen Logik Günthers besteht nun darin, $L = [0, 1]$ in ein Verbundsystem theoretisch unendlich vieler Logiken einzubetten, zwischen denen Transoperatoren vermitteln. Für jede durch L definierte Kontextur gilt aber weiterhin die 2-wertige aristotelische Logik, d.h. wir haben hier die von Bense angedeutete Alternative zur Leerheit des Randes von L, nämlich die Allesheit von L. Genau genommen determinieren sich Leerheit und Allesheit gegenseitig, denn auch in der polykontexturalen Logik gibt es keine Vermittlung zwischen den Werten 0 und 1 bzw. 1 und 0 in jedem eine Monokontextur definierenden L innerhalb des gesamten polykontexturalen Systems. Transoperatoren gibt es also nur durch Verwerfung ganzer L's, nicht aber durch Übergänge zwischen 0 und 1, wie sie in Toth (2015) durch Einführung eines Einbettungsoperators E

$E: x \rightarrow [x]$

vorgeschlagen wurden, der $L = [0, 1]$ auf das Quadrupel

$L_1 = [0, [1]]$

$L_2 = [[1], 0]$

$L_3 = [[0], 1]$

$L_4 = [1, [0]]$

(mit $L_2 = L_1^{-1}$ und $L_4 = L_3^{-1}$) abbildet. Erst durch Sub- bzw. Superordination, d.h. durch die Aufbrechung der Koordination der spiegelbildlichen Werte in $L = [0, 1]$ kann es eine Vermittlung zwischen 0 und 1 bzw. 1 und 0 geben, die nicht durch einen dritten, vierten, fünften, ... logischen Wert vonstatten geht, der lediglich zur Folge hätte, daß das 2-wertige Tertium non datur zu einem 3-wertigen Quartum non datur, einem 4-wertigen Quintum non datur, usw. verschoben würde. Für dieses Quadrupel gibt es also eine dritte Alternative neben der Leerheit und der Allesheit, nämlich vier Formen von gegenseitiger Abhängigkeit von 0 und 1, die durch Einbettung bewirkt wird und dadurch differentiell, d.h. nicht-substantiell, nichtleere Ränder erzeugt. Hier handelt es sich somit um echte Differenzen, denn es gilt natürlich z.B.

$[0, [1]] \neq [0, 1] \neq [[0], 1]$,

d.h. ein Wert, der in einen anderen eingebettet ist, bekommt durch diese Einbettung Anteile dieses anderen Wertes. Beispielsweise bekommt also ein Objekt Subjektanteile, indem es von einem Subjekt wahrgenommen wird, und umgekehrt bekommt ein Subjekt Objektanteile, indem es ein Objekt wahrnimmt. Man kann, wenn man will, hier die heideggersche Jemeinigkeit des Etwas erkennen, nur ist sie insofern unvollständig, da es konvers dazu auch eine Jeetwasigkeit der Meinigkeit geben muß.

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Toth, Alfred, Der Jäger Gracchus und die Vermittlung von Diesseits und Jenseits. In: Electronic Journal for Mathematical Semiotics, 2015

Ein allgemeines Modell für Colinearität

1. Das in Toth (2015a) zur Verfeinerung der von Bense skizzierten Raumsemiotik (vgl. Bense/Walther 1973, S. 80) vorgeschlagene 3-stufige Abbildungsmodell

$$\left.\begin{array}{l} (2.1) = f(2.1)\\ (2.1) = f(2.2)\\ (2.1) = f(2.3)\\ (2.2) = f(2.2)\\ (2.2) = f(2.3)\\ (2.3) = f(2.3) \end{array}\right\} \qquad \left.\begin{array}{l} \text{Adjazenz}\\ \text{Subjazenz}\\ \text{Transjazenz} \end{array}\right\} \qquad \begin{array}{l} \text{Exessivität}\\ \text{Adessivität}\\ \text{Inessivität,} \end{array}$$

das wir durch die Formel

L = [(2.x), (n, E), (R(S, U)]

abkürzen, können, darin $x \in P = \{1, 2, 3\}$ die von Bense (1981, S. 17 ff.) eingeführten Primzeichen sind, darin n eine Peanozahl und E der Einbettungsoperator E: $x \to [x]$ ist und darin R(S, U) die drei möglichen ontischen Lagerelationen eines Systems, Teilsystems oder Objektes relativ zu seiner Umgebung angeben, können wir leicht als allgemeines Modell für Colinearität verwenden.

2. Colinearität, wie sie v.a. in Toth (2015b-e) sowie einer langen Reihe von Einzelstudien ein- und weitergeführt wurde, setzt zunächst eine ontische Struktur der Form

C = [L, Abb, L]

voraus, darin Abb für die von Bense eingeführten raumsemiotischen Abbildungen stehen, also z.B. Straßen, Gassen oder Wege, aber auch Brücken, Stege und verwandte Objekte. Da L per definionem nicht nur iconisch sein kann, wie es z.B. bei 2-reihigen Häuserzeilen der Fall ist, ergeben sich (mit S = System und Rep = Repertoire) für C die folgenden 4 homogenen Möglichkeiten

C = [S, Abb, S]		C = [Abb, S, Abb]

C = [S, Rep, S]		C = [Rep, S, Rep]

und die folgenden 6 heterogenen Möglichkeiten

C = [S, Abb, Rep]		C = [Abb, S, Rep]		C = [Rep, S, Abb]

C = [S, Rep, Abb]		C = [Abb, Rep, S]		C = [Rep, Abb, S].

Dazu kommen die wohl auf die reine Theorie restringierten 3 weiteren homogen-undifferenzierten Möglichkeiten

C = [S, S, S]

C = [Abb, Abb, Abb]

C = [Rep, Rep, Rep].

Während der Fall C = [S, S, S] deshalb unwahrscheinlich ist, weil die Ränder zwischen Systemen Umgebungen implizieren, scheidet dieser Fall de facto sogar aus. Im Falle von C = [Abb, Abb, Abb] und C = [Rep, Rep, Rep] gilt natürlich die Gleichheitsrelation mit C = Abb und C = Rep, d.h. es liegt überhaupt keine Colinearität vor.

3. Für L = [(2.x), (n, E), (R(S, U)] gelten folgende semiotischen, ontischen und arithmetischen Sätze.

3.1. (2.x) ⊂ L

1. Jedes Icon teilt den semiotischen Raum des Repertoires in zwei Bereiche (z.B. in Übereinstimmungsmerkmale und Nichtübereinstimmungsmerkmale bzw. inhärente oder nichtinhärente Prädikate u. dgl.).

2. Jeder Index stellt die Verknüpfung zweier beliebiger Elemente des semiotischen Raums des Repertoires dar (ein Weg als Index, bezeichnet durch den Wegweiser, verknüpft stets zwei Örter).

3. Jedes Symbol ist eine Darstellung des semiotischen Raumes als pures Repertoire (Bense/Walther 1973, S. 80).

3.2. $P = (n, E)$

Ortsfunktionale Peanozahlen, d.h. Peanozahlen, die auf ontische Orte abgebildet werden, stellen Abbildungen der Form

$L = [0, 1] \rightarrow$

$L = [0, [1]]$ $\qquad$ $L = [[1], 0]$

$L = [[0], 1]$ $\qquad$ $L = [1, [0]]$,

d.h. E ersetzt die koordinative Relation von L durch eine Relation von Sub- und Superordination unter nicht-perspektivischem, d.h. subjektunabhängigem Wechsel der ontischen Orte der Werte von L, mit $L = [0, 1]$ als Trivialfall.

3.3. R(S, U)

Ein System, Teilsystem oder Objekt kann sich gemäß Toth (2012) in drei fundamentalen Lagerelationen (d.h. solchen, die u.U. kombiniert werden können) befinden. S ist exessiv gdw.

$exS = S \subset U$

gilt. S ist adessiv gdw.

$adS = S \cap U \neq \emptyset$,

gilt, und S ist inessiv gdw. gilt

$inS = S \cap U = \emptyset$,

d.h. befindet sich z.B. ein Tisch in einer Nische, so ist er relativ zu seiner Umgebung exessiv, ist er an eine Wand angelehnt, so ist er relativ zu seiner Umgebung adessiv, und steht er mitten in einem Zimmer, so ist er relativ zu seiner Umgebung inessiv.

4. Die erweiterte Raumsemiotik geht also von semiotischen Kategorien aus, die zuerst qualitativ-arithmetisch und hernach ontisch-lagetheoretisch subkategorisiert werden. Sie gilt selbstverständlich zunächst für den linearen Fall, also etwa eine Häuserzeile. Bett man diese in eine der 10 folgenden Strukturen ein

C = [S, Abb, S]	C = [Abb, S, Abb]	
C = [S, Rep, S]	C = [Rep, S, Rep]	
C = [S, Abb, Rep]	C = [Abb, S, Rep]	C = [Rep, S, Abb]
C = [S, Rep, Abb]	C = [Abb, Rep, S]	C = [Rep, Abb, S],

so stellt der Fall einer Straße mit zwei durch eine ontische Abbildung getrennten (reihigen) Häuserzeilen den Spezialfall C = [S, Abb, S] dar. Colinearität ist somit eine ontische Eigenschaft, welche sämtlichen raumsemiotischen Kombinationen und sämtliche qualitativ-arithmetischen sowie ontisch-lagetheoretischen Subkombinationen zu beschreiben im Stande ist, und zwar, wie bereits angedeutet, zunächst völlig subjektunabhängig (also etwa unter Vernachläßigung der Tatsache, daß ein Subjekt eine Straße von ihrer Domäne zur Codomäne oder in umgekehrter Richtung durchlaufen kann). Will man dennoch neben den den obigen Definitionen inhärierenden nicht-subjektabhängigen perspektivischen Relationen subjektabhängige betrachten, d.h. die kybernetische Beobachterposition (etwa im Falle einer automatentheoretischen Definition der Colinearität) einbauen, so genügt es, die 10 C-Strukturen in Subjektfunktion zu setzen. Daraus folgt natürlich weiter, daß Colinearität keinesfalls auf horizontale Relationen beschränkt ist, sondern daß in einem 3-dimensionalen Raum jede einzelne Seite sowie jedes Paar von Seiten mit Hilfe der colinearen Modells formal maximal exakt bestimmbar ist.

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Definition der qualitativen Zahl

1. Peanozahlen basieren auf der logischen unvermittelten und linearen Dichotomie

$L = [0, 1]$,

darin die Werte austauschbar sind, d.h. es gilt $L = L^{-1} = [1, 0]$, und damit spielen auch ontische Orte keine Rolle. In Sonderheit verbietet das Tertiumgesetz daher nicht nur die Einführung eines dritten Wertes, sondern auch die funktionelle Abhängigkeit der beiden Werte voneinander, d.h.

$0 = f(1)$

$1 = f(0)$.

Dasselbe gilt natürlich in Sonderheit für $0 = f(0)$ und $1 = f(1)$, denn eine Funktion darf nicht als ihr eigenes Argument fungieren (vgl. Wittgenstein, Tractatus 5.251, vgl. auch 3.333).

2. Im Anschluß an Toth (2015a-d) kann man allerdings das Verbot der funktionellen Abhängigkeit durch Einführung eines Einbettungsoperators

$E: \quad x \rightarrow [x]$

umgehen, und wir bekommen zunächst

$[0] = f(0)$

$[1] = f(1)$.

Damit werden nun auch ontische Orte, an denen Zahlen stehen, relevant

$0 \rightarrow [0] \quad \Rightarrow \quad [\omega_1, \omega_2]$

$1 \rightarrow [1] \quad \Rightarrow \quad [\omega_1, \omega_2]$

und weiter

$[0] \rightarrow [[0]] \quad \Rightarrow \quad [\omega_1, \omega_2, \omega_3, \omega_4]$

$[1] \rightarrow [[1]] \quad \Rightarrow \quad [\omega_1, \omega_2, \omega_3, \omega_4]$

und weiter

$[[0]] \rightarrow [[[0]]] \qquad \Rightarrow \qquad [\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8]$

$[[1]] \rightarrow [[[1]]] \qquad \Rightarrow \qquad [\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8]$,

usw.

3. Jede gleiche Zahl kann somit, sofern sie nicht auf der gleichen Einbettungsstufe steht, von jedem anderen Zahlwert funktionell abhängig sein und einen ontischen Ort entsprechend den obigen Schemata einnehmen, z.B.

$0 = f(1) \qquad 1 = f(0)$

$0 = f([0]) \qquad 1 = f([1])$

$0 = f([[0]]) \qquad 1 = f([[1]])$, usw.

$[0] = f(0) \qquad [1] = f(1)$

$[0] = f([[0]]) \qquad [1] = f([[1]])$

$[[0]] = f([0]) \qquad [[1]] = f([1])$, usw.

Graphisch können wir also zwischen der 0-stufige Peanozählweise und den n-stufigen Nicht-Peanozählweisen (n > 0) unterscheiden.

0-stufige Zählweise

0 1 1 0

--------- ----------

1-stufige Zählweise

0 1

--------- ---------

1 0

2-stufige Zählweise

0 0

--------- ---------

0 0

----------------- -----------------

1 1, usw.

Die qualitative Zahl läßt sich somit definieren als Tripel

$Z = [(x \in \mathbb{N}), E, \omega]$.

Literatur

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Grundlegung einer qualitativen Semiotik

> Nur Gründe haben Folgen. Aber Gründe haben keine grundlose Tiefe; echte Gründe sind letzte Gründe. Tiefere Gründe gibt es nicht. Man erkennt die letzten Gründe an der reflexiven Autoreproduktion ihrer selbst.
>
> Max Bense

1. Die in Toth (2015a) definierte qualitative Zahl

$Z = [(x \in \mathbb{N}), E, \omega]$,

darin x eine natürliche Zahl, z.B. eine Peanozahl, ist, E den Einbettungsoperator

$E: x \rightarrow [x]$

und ω den ontischen Ort bezeichnet, hat den außerordentlichen Vorteil, zugleich als (wohl abstraktest mögliche) Definition des Zeichens zu dienen. Man bedenke dabei, daß bereits Bense (1992) das invariante semiotische Dualsystem

$DS = (3.1, 2.2, 1.3) \times (3.1, 2.2, 1.3)$

gleichzeititg als Repräsentationsschema des Zeichens und der Zahl (sowie des, mathematisch meßbaren) ästhetischen Zustandes bestimmt hatte.

2. Damit können Zeichen genauso wie Zahlen vermöge Toth (2015b-e) in 2-dimensionalen Zeichenfeldern adjazent, subjazent und transjazent dargestellt werden.

2.1. Adjazente Zeichenfelder

x_i	y_j		y_i	x_j		y_j	x_i		x_j	y_i
$Ø_i$	$Ø_j$		$Ø_i$	$Ø_j$		$Ø_j$	$Ø_i$		$Ø_j$	$Ø_i$
		×			×			×		

$$\begin{array}{ccccccccccc}
\emptyset_i & \emptyset_j & & \emptyset_i & \emptyset_j & & \emptyset_j & \emptyset_i & & \emptyset_j & \emptyset_i \\
x_i & y_j & & y_i & x_j & & y_j & x_i & & x_j & y_i
\end{array}$$

2.2. Subjazente Zeichenfelder

$$\begin{array}{ccccccccccc}
x_i & \emptyset_j & & \emptyset_i & x_j & & \emptyset_j & x_i & & x_j & \emptyset_i \\
y_i & \emptyset_j & & \emptyset_i & y_j & & \emptyset_j & y_i & & y_j & \emptyset_i \\
 & & \times & & & \times & & & \times & & \\
y_i & \emptyset_j & & \emptyset_i & y_j & & \emptyset_j & y_i & & y_j & \emptyset_i \\
x_i & \emptyset_j & & \emptyset_i & x_j & & \emptyset_j & x_i & & x_j & \emptyset_i
\end{array}$$

2.3. Transjazente Zeichenfelder

$$\begin{array}{ccccccccccc}
x_i & \emptyset_j & & \emptyset_i & x_j & & \emptyset_j & x_i & & x_j & \emptyset_i \\
\emptyset_i & y_j & & y_i & \emptyset_j & & y_j & \emptyset_i & & \emptyset_j & y_i \\
 & & \times & & & \times & & & \times & & \\
\emptyset_i & y_j & & y_i & \emptyset_j & & y_j & \emptyset_i & & \emptyset_j & y_i \\
x_i & \emptyset_j & & \emptyset_i & x_j & & \emptyset_j & x_i & & x_j & \emptyset_i
\end{array}$$

Man beachte, daß jedes der 3 mal 8 Zeichenfelder verschieden ist. Die insgesamt 24 Zeichenfelder geben also für die erkenntnistheoretische (ontisch-semiotische) Basisdichotomie $E = [\Omega, Z]$, d.h. für ein Objekt, das durch ein Zeichen bezeichnet wird und ein Zeichen, das ein Objekt bezeichnet, alle theoretisch möglichen Fälle einschließlich der wechselnden Subjektperspektiven des die thetische Einführung bzw. Metaobjektivation (vgl. Bense 1967, S. 9) vollziehenden Subjektes (anhand der Indizes i und j) an. Ob dabei $x = \Omega$ oder $x = Z$ bzw. $y = Z$ oder $y = \Omega$ gesetzt wird, ist natürlich völlig belanglos, da die ortsfunktionale Differenzierung auf dem dyadischen logischen Schema $L = [0, 1]$ basiert, dessen Werte bekanntlich austauschbar sind (vgl. dazu Günther 2000, S. 203 f.).

3. An dieser Stelle ist ein Wort zu der von Bense (1975, S. 37) eingeführten semiotischen Matrix nötig. Bekanntlich ist die peircesche Zeichenrelation eine triadische Relation über einer monadischen (.1.), einer dyadischen (.2.) und einer triadischen (.3.) Relation, so daß sich das Zeichen in seiner Drittheit also selbst enthält (vgl. Bense 1979, S. 53 u. 67). Vom rein quantitativen Standpunkt des nicht durch E vermittelten logischen Schemas $L = [0, 1]$ aus betrachtet kommen also die durch kartesische Produktbildung erzeugten Subzeichen

1.1., 2.2, 3.3

deswegen nicht in Frage, weil sie Funktionen der Form

$1 = f(1)$

$2 = f(2)$

$3 = f(3)$

sind (vgl. Wittgenstein, Tractatus 5.251 u. 3.333). Ebenfalls ausgeschlossen sein müssten Subzeichen der Form $S = \langle x.y \rangle$ mit $y > x$, d.h. die Funktionen

$1 = f(2)$

$1 = f(3)$

$2 = f(3)$,

da eine n-stellige Relation von ihrer Valenz her keine m-stellige Relation mit $m > n$ binden kann. Solche Funktionen sind hingegen typisch für qualitative Systeme, in denen Funktionen als ihre eigenen Argumente auftreten können. Man bedenke auch, daß die semiotische Matrix im Gegensatz zum doppelt positiven Quadranten der komplexen Zahlen anordbar ist, d.h. es handelt sich bei den Subzeichen um nicht-komplexe, aber 2-dimensionale Zahlen, denn je nach Anordnung der Matrix sind die Trichotomien adjazent, die Triaden subjazent und die beiden Diagonalen transjazent. Wie es also aussieht, ist unsere ortsfunktionale qualitative Begründung einer mathematischen Semiotik durchaus mit der ursprünglichen Intention einer mathematischen Begründung der Semiotik vereinbar, auch wenn dies bislang völlig unerkannt geblieben ist.

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Qualitative semiotische Funktionen

1. Gehen wir aus von der von Bense (1975, S. 37) eingeführten semiotischen Matrix. In dieser Anordnung stellen die Trichotomien die adjazente, die Triaden die subjazente und die beiden Diagonalen die transjazente semiotische Zählweise dar

	.1	.2	.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3.

Der wesentliche Unterschied zwischen dem doppelt positiven Quadranten des komplexen Zahlenfeldes und der benseschen Matrix besteht ja darin, daß die Zeichenzahlen der letzteren im Gegensatz zu den komplexen Zahlen anordbar sind, da stets zwischen Hauptwert der Form $T_{td} = <x.>$ und Stellenwert der Form $T_{tt} = <.y>$ unterschieden wird. Das bedeutet aber, daß die aus den Peanozahlen durch kartesische Produktbildung erzeugten Subzeichen qualitative Zeichen der Form

$Z = [(x \in \mathbb{N}), E, \omega]$

sind, wie sie in Toth (2015) definiert wurden und worin E den Einbettungsoperator E: $x \rightarrow [x]$ und ω den ontischen Ort bezeichnen. Auch wenn jedes Subzeichen relativ zu E und ω bijektiv ist, hat doch auch jedes seinen festen ontischen Ort und seine feste Einbettungsstufe. Die semiotische Matrix macht somit den Eindruck eines pseudo-quantitativen Systems vermöge "erstarrter" Qualität. Denn geht man von Z aus, so kann jedes Subzeichen 1. auf jeder Einbettungsstufe aufscheinen und 2. kann es an jedem ontischen Ort erscheinen, und d.h. Funktion jedes anderen Subzeichens sein.

2. Die für quantitative Systeme verbotene Relation einer Funktion zu seinem eigenen Argument (vgl. Wittgenstein Tractatus 5.251 u. 3.333) ist also für qualitative Systeme geradezu charakteristisch. Damit erhalten wir für die triadisch-trichotomische Zeichenrelation, mit einem entsprechenden Mini-

mum von drei Einbettungsstufen, die folgenden qualitativen semiotischen Funktionen.

2.1. n = f(n)

1 = f(1)	2 = f(2)	3 = f(3)

2.2. n = f(n-1)

1 = f([1])	2 = f([2])	3 = f([3])

2.3. (n-1) = f(n)

[1] = f(1)	[2] = f(2)	[3] = f(3)

2.4. n = f(n-2)

1 = f([[1]])	2 = f([[2]])	3 = f([[3]])

2.5. (n-2) = f(n)

[[1]] = f(1)	[[2]] = f(2)	[[3]] = f(3)

2.6. (n-1) = f(n-2)

[1] = f([[1]])	[2] = f([[2]])	[3] = f([[3]])

2.7. (n-2) = f(n-1)

[[1]] = f([1])	[[2]] = f([2])	[[3]] = f([3])

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Isomorphie ontischer Orte von Quaternionen und qualitativen Zahlen

1. Der folgende Beitrag weist auf ein ganz erstaunliches Ergebnis hin. Wie man seit Toth (2015a, b) weiß, haben die qualitativen Zahlen, wie sie in der ortsfunktionalen Arithmetik, die Ontik und Semiotik zugrunde liegt, benutzt werden, die 2-Dimensionalität von Zahlenfeldern mit den komplexen Zahlen gemeinsam (und stehen somit beide der Peano-Linearität gegenüber). Das ist aber auch schon alles, denn im Gegensatz zu den komplexen Zahlen sind bereits die Subzeichen-Zahlen, die Bense (1975, S. 37) in der Form der sog. semiotischen Matrix eingeführt hatte, anordbar

$$\begin{array}{ccccc}
(1.1) & < & (1.2) & < & (1.3) \\
\wedge & & \wedge & & \wedge \\
(2.1) & < & (2.2) & < & (2.3) \\
\wedge & & \wedge & & \wedge \\
(3.1) & < & (3.2) & < & (3.3),
\end{array}$$

während die komplexen Zahlen bekanntlich nicht-anordbar sind.

2. Ein Quaternion bzw. eine Hamilton-Zahl kann man durch das Quadrupel (in einer bewußt gewählten unüblichen Form)

$R(Quat) = (\pm 1, \pm i, \pm j, \pm k)$

definieren. Diese sog. hyperkomplexe Zahl besteht also aus dem reellen Zahlenanteil 1 und drei differenzierbaren imaginären Zahlenanteilen i, j und k, wobei alle vier Glieder sowohl positiv als auch negativ auftreten können. Der Zusammenhang zwischen den Relata von R regeln die sog. Hamilton-Regeln:

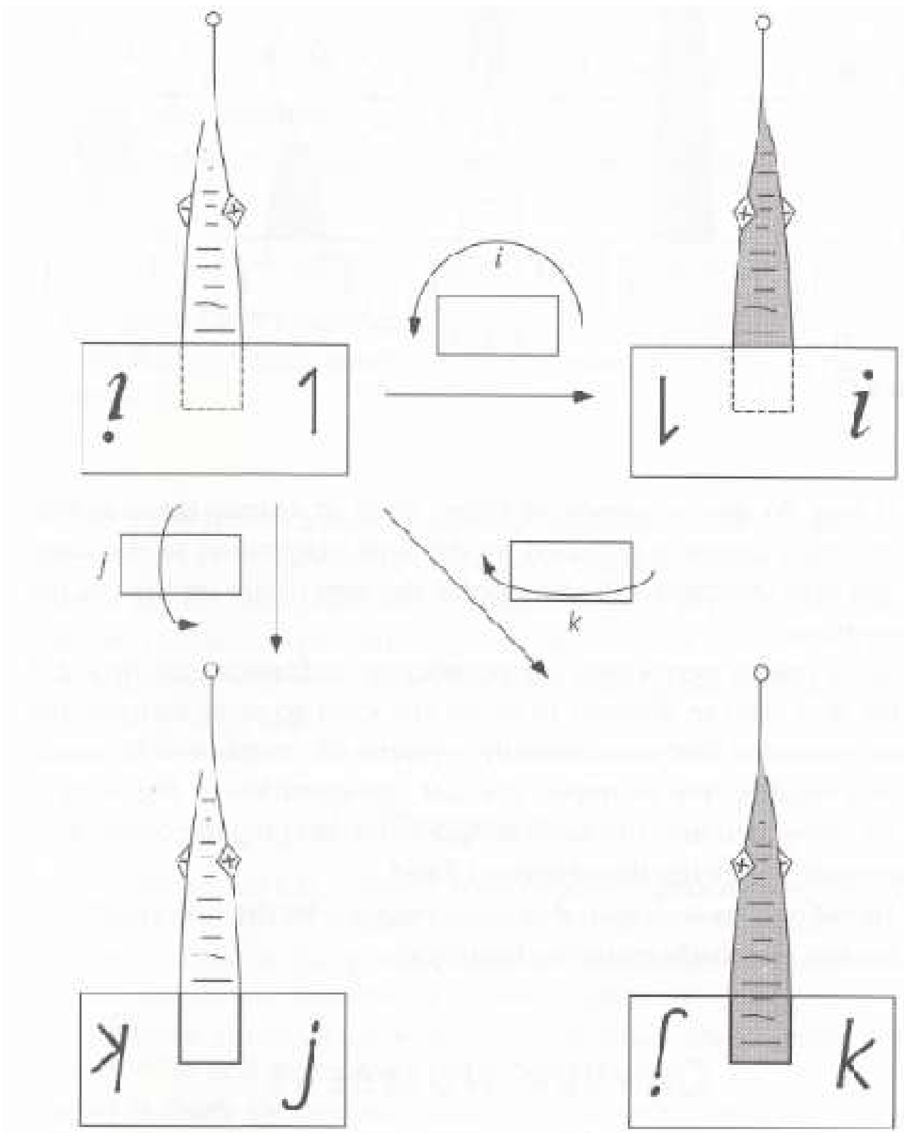
$$\begin{array}{l}
i^2 = j^2 = k^2 = -1 \\
ij = +k, \quad jk = +i, \quad ki = +j \\
ji = -k, \quad kj = -i, \quad ik = -j
\end{array}$$

.

Ausführlich orientiert über alle 8 × 8 möglichen paarweisen reell-imaginären Produkte die folgende vollständige Tabelle.

	−1	**−i**	**−j**	**−k**	**1**	**i**	**j**	**k**
−1	1	i	j	k	−1	−i	−j	−k
−i	i	−1	k	−j	−i	1	−k	j
−j	j	−k	−1	i	−j	k	1	−i
−k	k	j	−i	−1	−k	−j	i	1
1	−1	−i	−j	−k	1	i	j	k
i	−i	1	−k	j	i	−1	k	−j
j	−j	k	1	−i	j	−k	−1	i
k	−k	−j	i	1	k	j	−i	−1

Wie Conway und Guy (1995, S. 233) gezeigt haben, kann man sowohl Paare von reellen und imaginären als auch von imaginären Zahlenanteilen von Quaternionen durch sog. "Quaternionenmaschinen" darstellen. Wie man aus der folgenden, äußerst suggestiven Darstellung Conways leicht ersieht, werden dabei die ontischen Orte der Relata der Quaternionen einerseits in der Oben-Unten-Relation und andererseits in der Links-Rechts-Relation vertauscht.



Vier Positionen der conwayschen Quaternionenmaschinen (Conway/Guy 1995, S. 233)

3. Dagegen wurde die qualitative ortsfunktionale Zahl in Toth (2015a, b) wie folgt definiert

$R(Qual) = ((n \in \mathbb{N}), E, \omega)$,

darin n wie üblich eine natürliche Zahl ist, E den Einbettungsoperator

E: $n \rightarrow [n]$

und ω den ontischen Ort einer Zahl angibt. Qualitative Zahlen sind also insofern "komplex", als sie sowohl ontisch als auch ordinativ (d.h. koordinativ, subordinativ oder superordinativ) verankerte Peanozahlen sind. Während also für Peanozahlen die strikte unvermittelte Linearität

$L = [0, 1]$

der logischen Basisdichotomie gilt, ergeben sich durch Anwendung von E die folgenden 4 möglichen Zahlenstrukturen

$L_1 = [0, [1]]$ $\quad$ $L_2 = [[1], 0]$

$L_3 = [[0], 1]$ $\quad$ $L_4 = [1, [0]]$,

d.h. wir haben neben der koordinativen Zahlenstruktur L nun außerdem die subordinativ/superordinativen Zahlenstrukturen L_1 bis L_4, für die natürlich außerdem $L_2 = L_1^{-1}$ und $L_4 = L_3^{-1}$ gilt. Das bedeutet also nicht anderes, als daß in L die beiden Zahlenwerte 0 und 1 funktionell unabhängig voneinander und daher beliebig austauschbar sind, während für L_1 bis L_4 gilt

$0 = f(1)$

$1 = f(0)$.

Da aber E auch die ontischen Orte ω vertauscht, gilt nun außerdem

$0 = f(1) \quad \neq \quad f(1) = 0$

$1 = f(0) \quad \neq \quad f(0) = 1$.

Daraus folgt, daß es für alle drei möglichen Zählweisen in 2-dimensionalen Zahlenfeldern, d.h. für die horizontale, die vertikale und die beiden diagonalen Zählweisen, jeweils genau 4 mögliche Positionen gibt. In Toth (2015c-e), wo die qualitative Arithmetik eingeführt worden war, waren hierfür die "geometriefreien" Begriffe der adjazenten, subjazenten und transjazenten Zählweisen eingeführt worden. Ferner können, da die Werte 0 und 1 logisch durch die Objekt- und Subjektposition besetzt werden können, diese 4 Positionen verdoppelt aufscheinen, nämlich zusätzlich in perspektivischem Wechsel von einem kybernetischen Subjektstandpunkt aus betrachtet.

Die zweimal 4 möglichen Positionen für die adjazente Zählweise

$$
\begin{array}{ccccccccccc}
x_i & y_j & & y_i & x_j & & y_j & x_i & & x_j & y_i \\
\varnothing_i & \varnothing_j & & \varnothing_i & \varnothing_j & & \varnothing_j & \varnothing_i & & \varnothing_j & \varnothing_i \\
 & & \times & & & \times & & & \times & & \\
\varnothing_i & \varnothing_j & & \varnothing_i & \varnothing_j & & \varnothing_j & \varnothing_i & & \varnothing_j & \varnothing_i \\
x_i & y_j & & y_i & x_j & & y_j & x_i & & x_j & y_i
\end{array}
$$

Die zweimal 4 möglichen Positionen für die subjazente Zählweise

$$
\begin{array}{ccccccccccc}
x_i & \varnothing_j & & \varnothing_i & x_j & & \varnothing_j & x_i & & x_j & \varnothing_i \\
y_i & \varnothing_j & & \varnothing_i & y_j & & \varnothing_j & y_i & & y_j & \varnothing_i \\
 & & \times & & & \times & & & \times & & \\
y_i & \varnothing_j & & \varnothing_i & y_j & & \varnothing_j & y_i & & y_j & \varnothing_i \\
x_i & \varnothing_j & & \varnothing_i & x_j & & \varnothing_j & x_i & & x_j & \varnothing_i
\end{array}
$$

Die zweimal 4 möglichen Positionen für die transjazente Zählweise

$$
\begin{array}{ccccccccccc}
x_i & \varnothing_j & & \varnothing_i & x_j & & \varnothing_j & x_i & & x_j & \varnothing_i \\
\varnothing_i & y_j & & y_i & \varnothing_j & & y_j & \varnothing_i & & \varnothing_j & y_i \\
 & & \times & & & \times & & & \times & & \\
\varnothing_i & y_j & & y_i & \varnothing_j & & y_j & \varnothing_i & & \varnothing_j & y_i \\
x_i & \varnothing_j & & \varnothing_i & x_j & & \varnothing_j & x_i & & x_j & \varnothing_i
\end{array}
$$

Sowohl bei den Quaternionen $R = R(\text{Quat}) = (\pm 1, \pm i, \pm j, \pm k)$ als auch bei den qualitativen Zahlen $R(\text{Qual}) = ((n \in \mathbb{N}), E, \omega)$ werden also die Oben-Unten-Relationen und die Links-Rechts-Relationen gleichzeitig umgekehrt, d.h. die ontischen Orte von Quaternionen und von qualitativen Zahlen sind isomorph.

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Qualitative logische Zweiwertigkeit

1. In Toth (2015a, b) wurde die qualitative Zahl wie folgt definiert

$Z(\text{Qual}) = ((n \in \mathbb{N}), E, \omega)$,

darin n wie üblich eine natürliche Zahl ist, E den Einbettungsoperator

$E: \quad n \rightarrow [n]$

und ω den ontischen Ort einer Zahl angibt. Qualitative Zahlen sind also insofern "komplex", als sie sowohl ontisch als auch ordinativ (d.h. koordinativ, subordinativ oder superordinativ) verankerte Peanozahlen sind. Während also für Peanozahlen die strikte unvermittelte Linearität

$L = [0, 1]$

der logischen Basisdichotomie gilt, ergeben sich durch Anwendung von E die folgenden 4 möglichen Zahlenstrukturen

$L_1 = [0, [1]] \qquad L_2 = [[1], 0]$

$L_3 = [[0], 1] \qquad L_4 = [1, [0]]$,

d.h. wir haben neben der koordinativen Zahlenstruktur L nun außerdem die subordinativ/superordinativen Zahlenstrukturen L_1 bis L_4, für die natürlich außerdem $L_2 = L_1^{-1}$ und $L_4 = L_3^{-1}$ gilt. Das bedeutet also nicht anderes, als daß in L die beiden Zahlenwerte 0 und 1 funktionell unabhängig voneinander und daher beliebig austauschbar sind, während für L_1 bis L_4 gilt

$0 = f(1)$

$1 = f(0)$.

Da aber E auch die ontischen Orte ω vertauscht, gilt außerdem

$0 = f(1) \qquad \neq \qquad f(1) = 0$

$1 = f(0) \qquad \neq \qquad f(0) = 1$.

Daraus folgt, daß es für alle drei möglichen Zählweisen in 2-dimensionalen Zahlenfeldern, d.h. für die horizontale, die vertikale und die beiden diagonalen Zählweisen, jeweils genau 4 mögliche Positionen gibt. In Toth (2015c-e), wo die qualitative Arithmetik eingeführt worden war, waren hierfür die "geometriefreien" Begriffe der adjazenten, subjazenten und transjazenten Zählweisen eingeführt worden. Ferner können, da die Werte 0 und 1 logisch durch die Objekt- und Subjektposition besetzt werden können, diese 4 Positionen verdoppelt aufscheinen, nämlich zusätzlich in perspektivischem Wechsel von einem kybernetischen Subjektstandpunkt aus betrachtet.

Die 2 mal 4 möglichen Positionen für die adjazente Zählweise

$$\begin{array}{ccccccccccc}
x_i & y_j & & y_i & x_j & & y_j & x_i & & x_j & y_i \\
\emptyset_i & \emptyset_j & & \emptyset_i & \emptyset_j & & \emptyset_j & \emptyset_i & & \emptyset_j & \emptyset_i \\
 & & \times & & & \times & & & \times & & \\
\emptyset_i & \emptyset_j & & \emptyset_i & \emptyset_j & & \emptyset_j & \emptyset_i & & \emptyset_j & \emptyset_i \\
x_i & y_j & & y_i & x_j & & y_j & x_i & & x_j & y_i
\end{array}$$

Die 2 mal 4 möglichen Positionen für die subjazente Zählweise

$$\begin{array}{ccccccccccc}
x_i & \emptyset_j & & \emptyset_i & x_j & & \emptyset_j & x_i & & x_j & \emptyset_i \\
y_i & \emptyset_j & & \emptyset_i & y_j & & \emptyset_j & y_i & & y_j & \emptyset_i \\
 & & \times & & & \times & & & \times & & \\
y_i & \emptyset_j & & \emptyset_i & y_j & & \emptyset_j & y_i & & y_j & \emptyset_i \\
x_i & \emptyset_j & & \emptyset_i & x_j & & \emptyset_j & x_i & & x_j & \emptyset_i
\end{array}$$

Die 2 mal 4 möglichen Positionen für die transjazente Zählweise

$$\begin{array}{ccccccccccc}
x_i & \emptyset_j & & \emptyset_i & x_j & & \emptyset_j & x_i & & x_j & \emptyset_i \\
\emptyset_i & y_j & & y_i & \emptyset_j & & y_j & \emptyset_i & & \emptyset_j & y_i \\
 & & \times & & & \times & & & \times & &
\end{array}$$

$Ø_i$	y_j	y_i	$Ø_j$	y_j	$Ø_i$	$Ø_j$	y_i
x_i	$Ø_j$	$Ø_i$	x_j	$Ø_j$	x_i	x_j	$Ø_i$

2. Die wesentliche Neuerung von L_1 bis L_4 gegenüber L beruht somit darauf, daß die absoluten Kategorien des objektiven Objektes und des subjektiven Subjektes, wie sie der aristotelischen Logik, welche die reine Quantität der Mathematik garantiert, zugrundeliegen, nur noch als koordinativer Spezialfall existieren. Die beiden möglichen Funktionsabhängigkeiten

$0 = f(1)$

$1 = f(0)$

mit

$0 = f(1) \quad \neq \quad f(1) = 0$

$1 = f(0) \quad \neq \quad f(0) = 1$

bedeuten also nichts anderes, als daß das Objekt Subjektanteile und das Subjekt Objektanteile bekommen kann. Nimmt ein Subjekt ein Objekt wahr, dann handelt es sich beim Objekt um ein wahrgenommenes und somit subjektives Objekt und beim Subjekt um ein wahrnehmendes und somit objektives Subjekt. Diese beiden Fälle werden also je nach Subjektabhängigkeit von Objekten oder Objektabhängigkeit von Subjekten durch die vier subordinativen und superordinativen Fälle abgedeckt, d.h. der Einbettungsoperator fungiert als differentielles Tertium, das jedoch, da es eben nicht material ist, nicht gegen die logische Zweiwertigkeit verstößt. Somit bringt der Einbettungsoperator Qualität in die Quantität.

Im Gegensatz zur polykontexturalen Logik Günthers (vgl. Günther 1976-80) kann hier allerdings nicht nur die logische Subjektposition iteriert werden, während die logische Objektposition konstant, d.h. im hegelsche Sinne "totes" Objekt bleibt, denn wir haben für das Objekt

$0 = f(1, 0)$

$0 = f(1, 0, 1)$

$0 = f(1, 0, 1, 0)$, usw.

und für das Subjekt

$1 = f(0, 1)$

$1 = f(0, 1, 0)$

$1 = f(0, 1, 0, 1)$, usw.,

d.h. vermöge Subjektanteile von Objekten und Objektanteile von Subjekten werden einander Objekt und Subjekt trotz zweiwertig bestehender Kontexturgrenze immer mehr in einem infiniten Regreß angenähert. Eine Logik, welche auf durch E vermittelten Kategorien basiert, vermittelt also nicht primär zwischen Kontexturen, die allein subjektfunktional sind, sondern zwischen Werten innerhalb jeder einzelnen von theoretisch ebenfalls unendlich vielen Kontexturen, ohne daß die zweiwertige aristotelische Basis aufgegeben werden muß.[11] Dagegen vermag die polykontexturale als Verbundsystem unendlich vieler zweiwertiger Logiken vermöge ihrer Transoperatoren zwar zwischen Kontexturen zu zählen, aber im Grunde ändert sich gegenüber der logischen Basis der quantitativen Mathematik überhaupt nichts, da die logische Zweiwertigkeit wegen Unvermitteltheit der Werte in jeder Kontextur bestehen bleibt. Das einzige, was sich ändert, ist die Verschiebung des Tertium non datur zu einem Quartum, Quintum, Sextum ... non datur. Ein Logik, die nur Subjekte, und zwar wohl verstanden absolute Subjekte, nicht aber Objekte, vermitteln kann, dürfte daher kaum als die revolutionäre Neuerung aufgefaßt werden, als die sie v.a. in den 1970er Jahren von einigen ihrer Exponenten gefeiert wurde.

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[11] Weshalb Günther auf die Idee gekommen war, ausgerechnet dem Subjekt Qualität zuzuschreiben, wo doch das Objekt per definitionem qualitativ ist, ist mir auf nach jahrzehntelanger Beschäftigung mit der polykontexturalen Logik völlig unklar.

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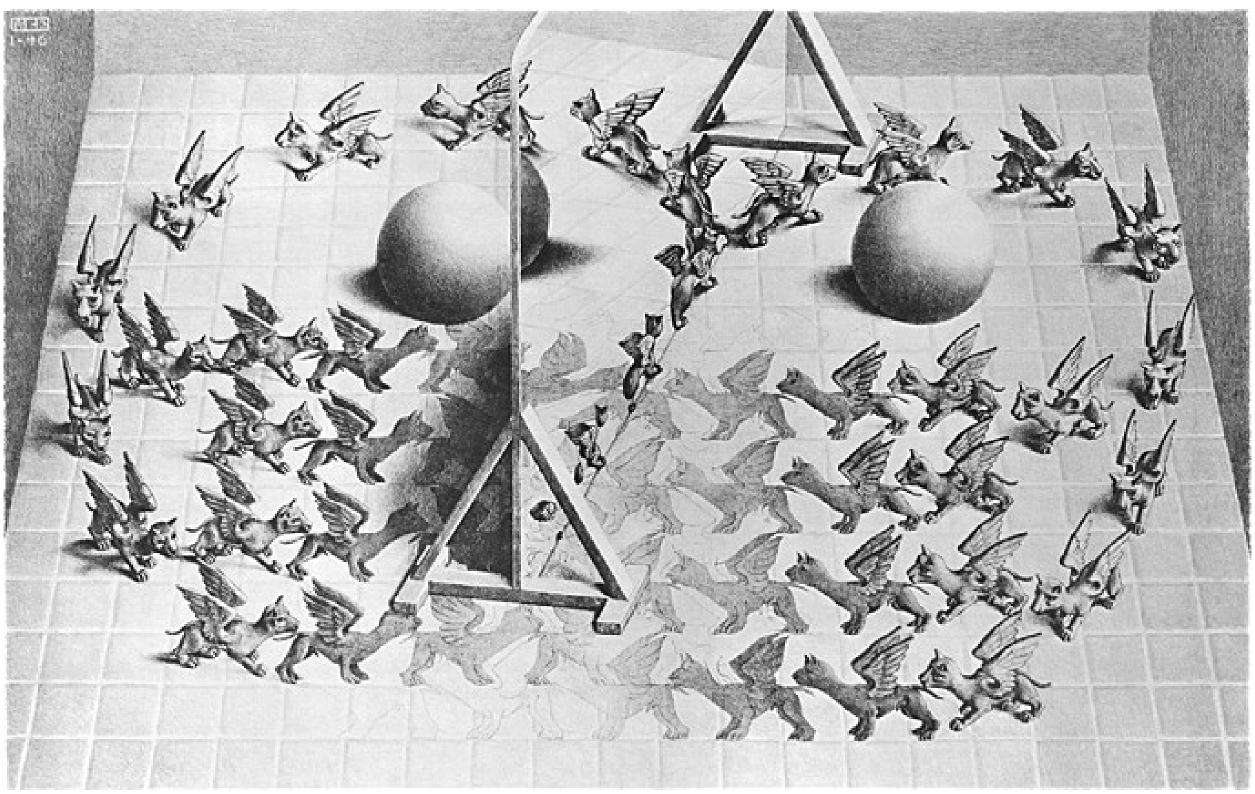
Die Aufhebung der coincidentia oppositorum

1. Die von Nikolaus von Kues postulierte coincidentia oppositorum (die sehr viel später in der polykontexturalen Logik Gotthard Günthers erneut eine wichtige Rolle spielen sollte) ist nichts anderes als der formale Ausdruck der Reflexionsidentität der beiden Werte in der aristotelischen logischen Basisdichotomie L = [0, 1]. Als ontische Modelle sollen im folgenden Eschers "Zauberspiegel" einerseits und Panizzas Erzählung "Die Kirche von Zinsblech" andererseits stehen, die den ersten und zweiten Teil dieser Abhandlung ausmachen. Als dritter Teil steht meine "Logik des Jägers Gracchus", worin gezeigt wird, daß logische Koinzidenz nur bei absoluten logischen Werten, d.h. bei objektiven Objekten und subjektiven Subjekten, möglich ist – und demzufolge aufgehoben wird, wenn man die ersteren durch subjektive Objekte und die letzteren durch objektive Subjekte, kurz gesagt also durch wahrgenommene Objekte und durch Zeichen, ersetzt.

2.1. M.C. Escher, Zauberspiegel (1946)



2.2. Oskar Panizza, Die Kirche von Zinsblech (1893)

Die Altäre waren geschmückt mit den in Landkirchen üblichen eingerahmten Tabletten, auf denen lateinische Sprüche waren, mit versilberten Leuchtern, Klingelspiel, alles in einfachster, wenig kostspieliger Form; auf Sockeln an der blanken, weißgetünchten Wand herum standen einige Apostel, Märtyrer und Ortsheilige mit ihren gewöhnlichen Werkzeugen und Symbolen.

(...)

Wie lange ich geschlafen, kann ich nicht sagen; ich erhielt plötzlich einen Stoß in die Seite, wie von einem harten Gegenstand. Erwachend bemerkte ich vor mir einen Mann in einem langen, roten Gewand. Unter dem Arm trug er ein großes, schiefes Holzkreuz; dieses Holzkreuz war an mich angestoßen. Der Mann kümmerte sich um mich gar nicht, sondern schritt ernst und gemessen dem Altare zu. Und nun erkannte ich, daß er nur einer unter vielen war, die in einer langen Reihe geordnet aus den Kirchenstühlen herauskamen in der Richtung zum Altar. Die ganze Kirche war taghell und prächtig erleuchtet. Auf allen Altären brannten Kerzen. Vom Chor herab tönte ein langsam-einschläferndes Gesumse der Orgel. Weihrauch und Kerzendampf lagerten sich in festen, bleigrauen Schwaden zwischen den weißgetünchten Pfeilern und der Wölbung. In dem Zug der geheimnisvoll dahinschleichenden Menschen bemerkte ich eine Menge seltsamer Gestalten. Da ging an der Spitze eine junge, prächtige Frau in einem blauen, sternbesäten Kleid, die Brüste offen, die linke halb entblößt. Durch Brust und Kleid hindurch ging ein Schwert, so zwar, daß das Kleid gerade noch getroffen war, als sollte es dadurch emporgehalten werden. Sie blickte fortwährend mit einem verzückten Lächeln an die weiße, kalkige Decke empor und hielt die Arme in brünstiger Gebärde über die Brust gekreuzt, so daß man den Eindruck gewann, als jubiliere sie innerlich über irgendeinen Gedanken. Wobei ich nochmals bemerke, daß das Schwert links, bei der linken Armbeuge, bis zum Heft fest in der Brust stak.

Dies war die vorderste Person. Aus der hinter ihr folgenden Reihe fielen manche durch ihre wunderliche Tracht auf. Die meisten hatten bestimmte Werkzeuge in der Hand. Der eine eine Säge, der andere ein Kreuz, der dritte einen Schlüssel, der vierte ein Buch, einer gar einen Adler, und ein anderer trug ein Lamm auf dem Arme mit herum. Niemand wunderte sich über den anderen, keiner sprach mit dem anderen. Aus dem Schiff der Kirche führten drei Stufen zu der erhöhten Estrade, wo der Altar stand. Jeder wartete mit seinem in bestimmter Haltung getragenen Werkzeug, bis der vordere die drei Stufen droben war, um nicht mit ihm zusammenzustoßen. Was mich am meisten wunderte: Niemand kümmerte sich um mich. Ich blieb völlig unbemerkt. Und selbst der Mann, der mit seinem schiefbalkigen Kreuz an mich angestoßen war, schien davon nichts bemerkt zu haben. Eine zweite weibliche Person fiel nur durch ihre pathetische Haltung im Zuge auf: eine blonde Frau, nicht mehr jung, mit hübschen aber abgewitterten, abgelebten Zügen. Sie trug ein ganz weißes Kleid, ohne Falbe oder Borde; in der Mitte mit einem Strick gebunden. Dieser Strick war aber vergoldet, die Brüste vollständig entblößt. Doch schaute niemand auf diese üppig quellenden Brüste hin. Reiche, blonde Flechten, vollständig aufgelöst, wallten den ganzen Rücken hinab. Sie trug den Kopf tief auf die Brust gesenkt und schaute verzweifelt auf ihre, nicht wie gewöhnlich gefalteten, sondern nach auswärts umgeknickten Hände – die Geste, die auf dem Theater Verzweiflung darstellt. Tränen perlten fortwährend von ihren Wimpern, fielen von da auf ihre Brüste, dann auf das Kleid und auch noch auf die manchmal unter dem Kleid hervorkommenden Füße. – Es wäre unmöglich, alle die aufzuzählen, die hier so still und selbstverständlich, wie zu einer regelmäßigen Übung, hinaufwanderten; aber der Mensch mit der verkniffenen Fratze, der anfangs seinen Schlüssel so energisch in das Mondlicht hielt und den ich vor dem Einschlafen unwillkürlich noch auf dem Postament betrachtet hatte, war auch dabei.

Trotz des eintönigen Orgelspiels war mir seit dem Erwachen ein zischelndes Geräusch hinter meinem Rücken am Altar nicht entgangen. Ich blickte mich jetzt um und bemerkte dort einen hochaufgeschossenen, ganz weiß gekleideten Menschen, der fortwährend in den an ihm vorbeiwandernden, teilweise vor ihm haltmachenden Zug hineinflüsterte: »Nehmet hin und esset! Nehmet hin und esset!« Es war eine unsäglich feine Figur: schlank, grazile Glieder, geistvolles Profil, griechische Nase. Dunkle, glattgescheitelte Lockenwellen fielen über Schläfe, Ohr und Nacken; ein durchsichtiger, jünglinghafter Flaum bedeckte Kinn und Lippen. Doch bemerkte ich an seinen Händen Blut. Er stand am äußersten linken Ende des Altars und schob den je zu zwei vor ihm stillstehenden und auf einem roten Schemel knienden Menschen des Zuges ein rundes, weiß angestrichenes Stück in den Mund, während diese unter brünstigem Augenaufschlag an die Decke blickten. Er flüsterte immerzu: »Nehmet hin und esset! Nehmet hin und esset!« Und »Nähmet hin und ässet!« prallte es von den halbkugelförmigen Hohlwänden hinter dem Altar zurück. Soweit war alles gut. Auffallend war mir zwar, woher dieser Mensch die weißen runden Stücke hernahm. Er langte wohl fortwährend in den Brustlatz seines Gewandes hinein, dort konnte aber ein Vorrat von den weißen Münzen unmöglich sein; einmal, weil dieses Austeilen ewig fortging und kein Ende nahm, ferner auch ein Unterkleid, wie man deutlich sehen konnte, nicht da war, und weil schließlich die Dünnbrüstigkeit dieses abgehärmten Menschen eine so exzessive war, daß, was sich im Profil darbot, notwendig dem Körper selbst angehören mußte. Auch bewegte er die feine, höchst schlankgebaute Hand so tief nach innen, daß für mich, soweit meine allerdings der Täuschung fähigen Sinne in Betracht kamen, kein Zweifel bestand, daß er die kreidigen Zwölfkreuzerstücke aus seinem Körper selbst nahm.

Ich sagte, soweit war alles gut: die Leute, die Frau mit dem Schwert in der Brust voraus, marschierten hinter dem Altar herum, um auf der rechten Seite wieder zu ihren Plätzen in den Kirchenbänken zurückzukehren. Aber was war denn auf dieser rechten Seite? – Dort stand ein ähnlicher Mensch – mehr ein mythologischer Zwitter als ein Mensch – in einem schwarzen, protestantischen Predigertalar, vorn am Hals die viereckigen, weißen Tabletten oder Bäffchen, hinter denen ein schwarz behaarter Hals zum Vorschein kam. Hinten am Gesäß teilte sich das Predigerkleid, und ein schwarzer, affenartiger Wickelschwanz rollte sich dort heraus, von so respektabler Länge, daß er, die Breite des Altars überspannend, mit dem Rücken des auf der linken Seite amtierenden weißen Menschen in stete Berührung kam. Unten guckten zwei hufartige Füße heraus, und oben auf dem Predigerhals saß ein Kopf, dessen wilder Haarwuchs, verbunden mit einem gelben Kolorit, eingefurchten, denkfaltigen Zügen und einer stumpfigen Nase einem deutschen Professorengesicht an Häßlichkeit wenig nachgab. Eine goldene Brille komplettierte diese aus Ärger, Bitterkeit und Ekel zusammengesetzte Physiognomie. – Eigentümlich war es, daß er fast pendelartig dieselben Bewegungen und Gesten machte, wie sein weißes Gegenüber auf der anderen Altarseite. – Er hielt einen schwarzen Becher in der Hand, aus dem er seiner ähnlich wie drüben vorbeiparadierenden Gesellschaft zu trinken gab. Dabei rief er in einem heiseren, grölenden Ton der jedesmal vor ihm knienden Person zu »Nehmet hin und trinket!« Und jedesmal führte er den Becher hinter sich herum, am Gesäß vorbei, um ihn dann der nächsten Person an die Lippen zu setzen. Was war nun aber das für eine Gesellschaft auf dieser rechten Seite! Eine merkwürdige und ganz anders geartete als drüben! Da war ganz vorne ein Mensch mit einer langen Nase und zurückweichendem Kinn, einen Dreimaster am Kopfe, den ausgemergelten Körper in eine französische Uniform à la Louis XV gesteckt, mit zurückgeschlagenen roten Rockflügeln, einen Degen zur Seite, in der rechten Hand einen Krückstock, und zu allem Überfluß noch unterm linken Arm eine Flöte. Er hielt den Kopf immer schief, sah sehr ausdrucksvoll drein, und schien genau zu wissen, was er tat. – Da war ferner ein feiner, eleganter Kerl in spanischem Kostüm, Trikots bis fast an die Lende, Pluderhosen, gestepptes, panzerartiges Wams, darüber einen goldbordierten kurzen Mantel à la Philipp II., Schnallenschuhe, Samthut mit Straußenfeder. Das Gesicht war gealtert, aber noch leichtfertig aufgelegt. Einen gezückten, blanken Degen in der Rechten tänzelte er, die Champagnerarie aus Mozart trällernd, die drei Stufen zum Altar hinauf, mit Wohlwollen auf die Zeremonien des schwarzgeschwänzten Predigers sich vorbereitend. Unter den Frauenzimmern bemerkte ich eine in einem weißen, griechischen Gewand mit goldener Falbel, die Arme nackt und nur goldenen Spangen, die Brüste verführerisch halb entblößt; auf dem blonden feingeschnittenen Haupt ein Königsdiadem, und unter dem Arm eine Lyra. Mit ihren fröhlichen, fast ausgelassenen Manieren bildete sie einen wirksamen Gegensatz zu der blonden, schluchzenden Frau auf der anderen Seite. – Es waren noch manche wunderbare, wie es schien, aus allen Gegenden und Zeiten zusammengewürfelte Gesellen da. Da war einer in einem langen, dunkeln, schleppenden Magistergewand, ein Barett über dem ernsten Gesicht, eine düstere, grübelnde Scholastenmiene, unter dem Arm ein geheimnisvolles Buch mit ägyptischen Lettern, der mit zu Boden gewandtem Blick schweigend in der Reihe einherging. Gleich hinter ihm ging ein junges Mädchen mit mildem, weichen Gesichtsausdruck, die einen abgehauenen, bärtigen Kopf auf einer Schüssel trug. Der Kopf schien der eines Denkers zu sein; das Mädchen lächelte und schien mit heiteren Gedanken beschäftigt zu sein. Aber weitaus die hervorragendste Figur in dem ganzen Zug war ein untersetzter, starkknochiger Mann mit rundem, glattrasierten Gesicht und Stiernacken im schwarzen Predigergewand, der mit emporgeworfenem Kopf und selbstbewußter Miene einherging, unter dem linken Arm eine Bibel, unter dem rechten eine Nonne; dies war überhaupt das einzige Paar im ganzen Zug.

Schon oben sagte ich: soweit war die Sache ganz gut. Und die Sache wäre auch weiterhin ganz gut gewesen: der linke Zug ging rechts um den Altar herum, der rechte links herum, um auf diese Weise in ihre Kirchenstühle zurückzukehren. Wie aber, wenn diese zwei Züge von so entgegengesetztem Charakter sich hinter dem Altar begegneten? Und das mußten sie! – Ich versäumte leider dieses Zusammentreffen. Fortwährend beschäftigt mit dem Durchmustern besonders des rechten Zuges, hörte ich plötzlich eine gelle heisere Lache aufschlagen. Ich wandte mich um und sah den schwarzgeschwänzten Menschen, der auf der rechten Seite den Kelch mit dem verdächtigen Inhalt kredenzte, sich mit einer höhnischen Fratze nach der anderen Seite umsehen, wo der weiße, sanfte Mann bleich und starr wie ein Toter stand. Hinter dem Altar sah ich die Spitzen beider Züge sich mit verdächtigen Mienen gegenseitig messen. In diesem Moment verlöschten sämtliche Kerzen. Ein dicker, schwefliger Dampf verbreitete sich im ganzen gewölbten Haus; das einschläfernde Summen der Orgel wurde von einem keifenden, gilfenden Aufschrei, wie von einem blechernen Akkord unterbrochen, als hätte man eine der Orgelpfeifen mit einem Beil verwundet. Es entstand ein fürchterlicher Tumult; ich hörte harte Körper stürzen, Werkzeuge aufschlagen, Leuchter und Schüsseln zu Boden fallen, vernahm weibliches Wehklagen, männliche Kernflüche, Lachen und Schreien. Dazwischen rief eine mokante, kropfige Stimme, die, glaube ich, dem Schwarzen angehörte, mit einem eigentümlichen, jodelnden Jargon: »Ja, ja! – Nähmet hin und ässet! – Ja, ja! – Nähmet hin und trinket!« – Halb aus Furcht erschlagen zu werden, halb aus Unmöglichkeit in der stickigen Luft weiter zu atmen, tappte ich mich im Finstern dem Ausgang zu, der, wie ich wußte, zur Rechten lag. Im Vorübergehen streifte ich am Weihkessel an, der mit einem »Spring Sau!« mir den Abschied gab, und gelangte glücklich ins Freie.

2.3. Die Logik des Jägers Gracchus (Toth 2015)

2.3.1. Für die 2-wertige aristotelische Logik gilt

$L = [0, 1] = L^{-1} = [1, 0]$,

denn das Gesetz vom Ausgeschlossenen Dritten verbietet die Annahme eines vermittelnden Wertes

$0 \vee \neg 0$

$1 \vee \neg 1$.

2.3.2. Allerdings gibt es neben der Möglichkeit substantieller dritter Werte die Erzeugung eines differentiellen Tertiums. Dafür benötigen wir einen Einbettungsoperator E (vgl. Toth 2014).

$E \rightarrow L = [0, 1] =$

$$\left(\begin{array}{ll} L_1 = [0, [1]] & L_1^{-1} = [[1], 0] \\ L_2 = [[0], 1] & L_2^{-1} = [1, [0]] \end{array} \right)$$

Anstelle von 0 und 1 bekommen wir somit in diesem minimalen Fall

0, [0]

1, [1],

d.h. für jedes L_i gilt

$0 = f(1)$

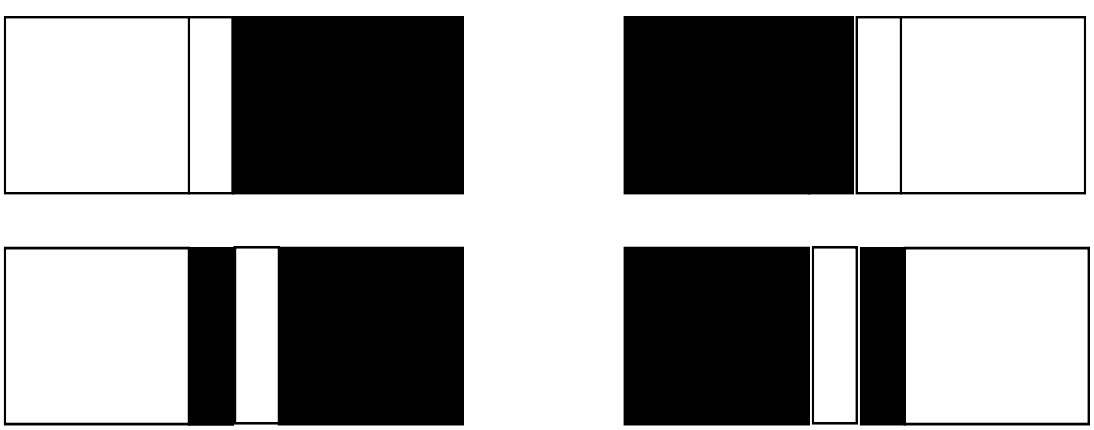
$1 = f(0)$,

und somit ist

$(x \in 0) \subset 1$

$(y \in 1) \subset 0$,

d.h. 0 hat 1-Anteile, und 1 hat 0-Anteile. Man kann dies schematisch wie folgt darstellen (vgl. Toth 2015a).



Die Werte in einer solchen Logik sind also vermöge eines differentiellen Tertiums vermittelt. In Sonderheit gilt also für den Rand R

$R[0, 1] \neq R[1, 0] \neq \emptyset$,

während für $L = [0, 1]$ natürlich gilt

$R[0, 1] = R[1, 0] = \emptyset$,

vgl. dazu die folgenden äußerst treffenden Feststellungen: "Beide Werte einer solchen Logik aber sind metaphysisch äquivalent. Das heißt, man kann sie beliebig miteinander vertauschen. Sie verhalten sich zueinander in einer totalen logischen Disjunktion, wie rechts und links. Es gibt keinen theoretischen Grund, welche Seite rechts und welche Seite links von der Zugspitze ist. Die Benennung beruht auf einer willkürlichen Entscheidung, und wenn man seinen Standpunkt wechselt, sind die rechte und die linke Seite miteinander vertauscht (Günther 2000, S. 230 f.).

2.3.3. Da entweder 0 oder 1 die logische Objekt- oder Subjektpositionen einnehmen, bedeutet die funktionelle Abhängigkeit beider Werte voneinander, daß das stillschweigend vorausgesetzte Axiom der 2-wertigen Logik, die, wie übrigens auch die polykonxexturale Logik Günthers, auf objektiven Objekten und subjektiven Subjekten basiert, suspendiert wird. Stattdessen sind subjektive Objekte und objekte Subjekte die neuen logischen Basiskategorien.

$\Omega = f(\Sigma)$	subjektives Objekt	Objekt
$\Sigma = f(\Omega)$	objektives Subjekt	Zeichen

Wie man erkennt, ist also das wahrgenommene subjektive Objekt gerade das Domänen- und das objektive Subjekt als dessen "Metaobjekt" (vgl. Bense 1967, S. 9) gerade das Codomänenelement der thetischen Einführung von Zeichen, d.h. die neue logische Basis ist gleichzeitig das vollständige Abbildungsschema der Zeichensetzung. Damit stehen Objekt und Zeichen in einer Dualrelation

$\Omega = f(\Sigma) \times \Sigma = f(\Omega)$,

und diese besagt, daß das Objekt – vermöge seiner Wahrnehmung, die selbstverständlich nur durch ein Subjekt erfolgen kann – Subjektanteile besitzt und daß das Subjekt – vermöge seiner Objektwahrnehmung – Objektanteile besitzt. Daraus folgt aber nicht mehr und nicht weniger, als daß es eine Brücke zwischen dem Diesseits des Subjektes bzw. Objektes und dem Jenseits des Objektes bzw. Subjektes gibt. Subjektanteile und Objektanteile werden also bei der Wahrnehmung vermöge einer Menge von Transformationen ausgetauscht

$[\Sigma = f(\Omega)] \rightleftarrows [\Omega = f(\Sigma)]$ subjektives Objekt ⇄ objektives Subjekt,

die als Partizipationsrelationen definierbar sind. Es nichtet nicht nur das Nichts im Sein des Seienden, sondern es west auch das Sein des Seienden im Nichts.

2.3.4. Da die Werte 0 und 1 auch als eingebettete in der Form [0] und [1] auftreten können, bedeutet dies, daß eine Linie zur Darstellung der Peanozahlen nicht mehr ausreicht. Die eingebetteten Zahlen können auch unter- oder oberhalb dieser Linie aufscheinen, d.h. sie bekommen erstens eine Menge von ontischen Orten und nicht nur einen "Stellenwert" (bzw. eine "Wertstelle") zugewiesen, und zweitens wird statt einer Zahlenlinie ein Zahlenfeld vorausgesetzt. In diesem gibt es somit nicht nur die horizontale, sondern auch eine vertikale und eine horizontale Zählweise, die wir in Toth (2015b-d) mit adjazenter, subjazenter und transjazenter Zählweise bezeichnet hatten. In den folgenden vollständigen Zahlenfeldern für die 2-elementige Menge P = (0, 1) sind nun alle partizipativen Austauschrelationen zwischen subjektiven Objekten und objektiven Subjekten qua 0 = f(1) und 1 = f(0) durch Doppelpfeile eingezeichnet.

2.3.4.1. Adjazente Zählweise

$$\begin{array}{ccccccccccc}
x_i & y_j & & y_i & x_j & & y_j & x_i & & x_j & y_i \\
\varnothing_i & \varnothing_j & \rightleftarrows & \varnothing_i & \varnothing_j & \rightleftarrows & \varnothing_j & \varnothing_i & \rightleftarrows & \varnothing_j & \varnothing_i \\
\uparrow\downarrow & & \times & \uparrow\downarrow & & \times & \uparrow\downarrow & & \times & \uparrow\downarrow & \\
\varnothing_i & \varnothing_j & & \varnothing_i & \varnothing_j & & \varnothing_j & \varnothing_i & & \varnothing_j & \varnothing_i \\
x_i & y_j & \rightleftarrows & y_i & x_j & \rightleftarrows & y_j & x_i & \rightleftarrows & x_j & y_i
\end{array}$$

2.3.4.2. Subjazente Zählweise

$$\begin{array}{ccccccccccc}
x_i & \emptyset_j & & \emptyset_i & x_j & & \emptyset_j & x_i & & x_j & \emptyset_i \\
y_i & \emptyset_j & \rightleftarrows & \emptyset_i & y_j & \rightleftarrows & \emptyset_j & y_i & \rightleftarrows & y_j & \emptyset_i \\
\updownarrows & & \times & \updownarrows & & \times & \updownarrows & & \times & \updownarrows & \\
y_i & \emptyset_j & & \emptyset_i & y_j & & \emptyset_j & y_i & & y_j & \emptyset_i \\
x_i & \emptyset_j & \rightleftarrows & \emptyset_i & x_j & \rightleftarrows & \emptyset_j & x_i & \rightleftarrows & x_j & \emptyset_i
\end{array}$$

2.3.4.3. Transjazente Zählweise

$$\begin{array}{ccccccccccc}
x_i & \emptyset_j & & \emptyset_i & x_j & & \emptyset_j & x_i & & x_j & \emptyset_i \\
\emptyset_i & y_j & \rightleftarrows & y_i & \emptyset_j & \rightleftarrows & y_j & \emptyset_i & \rightleftarrows & \emptyset_j & y_i \\
\updownarrows & & \times & \updownarrows & & \times & \updownarrows & & \times & \updownarrows & \\
\emptyset_i & y_j & & y_i & \emptyset_j & & y_j & \emptyset_i & & \emptyset_j & y_i \\
x_i & \emptyset_j & \rightleftarrows & \emptyset_i & x_j & \rightleftarrows & \emptyset_j & x_i & \rightleftarrows & x_j & \emptyset_i.
\end{array}$$

Da, wie bereits angedeutet, in der polykontexturalen Logik G. Günthers und der auf ihr beruhenden Mathematik der Qualitäten E. Kronthalers die 2-wertige aristotelische Logik L = (0, 1) für jede Einzelkontextur unangetastet bleibt und sich die Poly-Kontexturalität also lediglich der Iterierbarkeit des Subjektes verdankt, dieses aber weiterhin ein subjektives Subjekt ist, kann in dieser polykontexturalen Logik, Mathematik und Ontologie keine Rede davon sein, daß man Äpfel und Birnen addieren könne, wie dies ständig behauptet wird (vgl. z.B. Kronthaler 1990). 1 Apfel + 1 Birne ergeben bekanntlich 2 Früchte. Interessant an dieser qualitativen Gleichung ist aber nicht nur der angeblich Qualitätsverlust in der Summe, sondern die Tatsache, daß nur deswegen überhaupt eine Summe gebildet werden kann, weil Apfel und Birne ein vermittelndes Drittes gemeinsam haben, denn die weitere qualitative Gleichung 1 Apfel + 1 Stein hat beispielsweise keine angebbare Summe. Wenn es aber ein vermittelndes Drittes gibt, bedeutet dies natürlich wiederum, daß die Schnittmenge der Merkmalsmengen von Apfel und Birne nicht leer sein kann, und damit sind die Zahlen, welche Apfel und Birne vertreten, also 0 und 1 oder 1 und 0, natürlich vermittelt, d.h. folgend der oben skizzierten qualitativen ortsfunktionalen Arithmetik mit ihren drei 2-dimensionalen Zählweisen. Eine qualitative Mathematik, welche diesen Namen verdient, setzt also zwei fundamentale Änderungen der polykontexturallogischen Basis voraus:

1. die Ersetzung der logischen Basiskategorien des objektiven Objektes und des subjektiven Subjektes durch die vermittelten Kategorien des subjektiven Objektes und des objektiven Subjektes.

2. die daraus resultierende Möglichkeit, nicht nur das Subjekt, sondern auch das Objekt iterieren zu lassen. Damit ergeben sich ungeheuer komplexere "Permutogramme" (G.G. Thomas) bzw. Hamiltonkreise (G. Günther) als diejenigen, welche innerhalb der polykontexturalen Logik benutzt werden.

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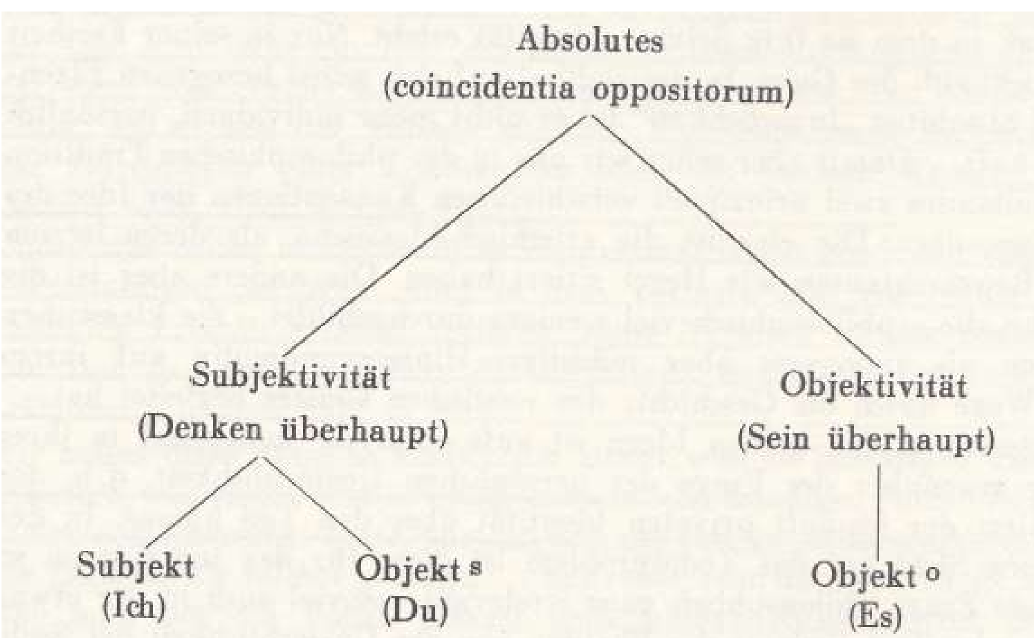
Transklassische Logik und ontische Einbettungsrelationen

1. Bekanntlich basiert die klassische 2-wertige aristotelische Logik auf der Dichotomie L = [P, N], darin Positivität (P) und Negativität (N) beliebig austauschbar sind. Gotthard Günther hatte dies sehr schön anhand eines Beispiels exemplifiziert: "Beide Werte einer solchen Logik aber sind metaphysisch äquivalent. Das heißt, man kann sie beliebig miteinander vertauschen. Sie verhalten sich zueinander in einer totalen logischen Disjunktion, wie rechts und links. Es gibt keinen theoretischen Grund, welche Seite rechts und welche Seite links von der Zugspitze ist. Die Benennung beruht auf einer willkürlichen Entscheidung, und wenn man seinen Standpunkt wechselt, sind die rechte und die linke Seite miteinander vertauscht (Günther 2000, S. 230 f.).

2. In Sonderheit ist eine Logik der Form L außerstande, die deiktische Differenz zwischen Ich- und Du-Subjektivität zu definieren. Beide müssen sich innerhalb von L wiederum wie Objekt (P) und Subjekt (N) zueinander verhalten, d.h. es kommt zu einem Subjekt-Objekt-Kollaps. Das folgende Schema, das Günther bereits in einem Rezensionsaufsatz vorgelegt hatte, der nichts weniger als die bisher einzige Grundlage zu einer aristotelisch natürlich ausgeschlossenen Metaphysik des Todes darstellt (Günther 1957 = 1980, S. 4), setzt dagegen eine 3-wertige Logik mit zwei statt nur einer Subjektposition voraus.



3. In einer solchen Logik gibt es nicht nun natürlich nicht nur die "klassische", d.h. 2-wertige logische Äquivalenz

$1 \equiv 2$,

sondern zusätzlich zwei "transklassische", in diesem Falle 3-wertige logische Äquivalenzen

$2 \equiv 3$

$1 \equiv 3$

(vgl. Günther 1980, S. 11). Dennoch bietet dieser bestechende Ansatz zu einer Metaphysik des Todes, der erstmals in der Geschichte der Philosophie in Frage stellt, "ob der Fortfall der ersten Identität im Tode wirklich die ichhafte Indeität des Individuums endgültig auflöst" (Günther 1980, S. 11 f.), ein formal schwerwiegendes Problem. Trotz der Möglichkeit, Identitätsrelationen zwischen dem Objekt (1) und den beiden Subjekten (2 und 3) herzustellen, ist nämlich in der güntherschen transklassischen Logik nur das Subjekt, nicht aber das Objekt iterierbar. Das Objekt bleibt, genauso wie bei Hegel, von dem Günther ausgeht, "totes" Objekt, d.h. es ist formal nicht iterierbar, und die von Günther zwar selbst eingeführten Vermittlungskategorien des subjektiven Objektes und des objektiven Subjektes sind wegen dieser Nicht-Iterierbarkeit in polykontexturalen Hamiltonkreisen (bzw. den ihnen korrespondieren thomasschen "Permutographen") ohne jede formale Relevanz. Die Basisrelation bleibt auch in der polykontexturalen Logik die 2-wertige aristotelische Logik der Form L, und die erstere ist lediglich ein theoretisch unendlich-wertiges Vermittlungssystem der zweiteren.

4. Man kann dieses Problem, wie bereits in Toth (2015a, b) gezeigt, jedoch auf formal ebenso elegante wie einfache Weise durch Einführung eines Einbettungsoperators E lösen, der durch

$E = (x \rightarrow [x])$

definiert ist. Dabei kann x sowohl Subjekt als auch Objekt sein, und damit wird nun auch das Objekt iterierbar. Ferner ermöglicht E, die drei obigen güntherschen Identitätsrelationen durch Einbettungsrelationen darzustellen. So wird eine elementare, d.h. nicht-eingebettete 3-wertige Logik der Form

R = [1, 2, 3]

auf die folgenden drei Einbettungsstrukturen abbildbar

[[1, 2], 3]

[1, [2, 3]]

[[1, 3], 2],

dazu kommen aber, da es sich hier um Ordnungsrelationen handelt, noch die drei konversen Relationen

[3, [1, 2]]

[[2, 3], 1]

[2, [1, 3]]

mit der weiteren Möglichkeit der Konversionen der eingebetteten logischen Werte innerhalb aller 6 Einbettungsrelationen, also z.B. [[2, 1], 3], ..., [2, [3, 1]]. Da es sich bei allen diesen Relationen um Einbettungsrelationen handelt, sind nun im Gegensatz zur polykontexturalen Logik nicht nur die Subjekte, sondern ist auch die Relation zwischen einem Objekt und einem Subjekt nicht mehr wie in L beliebig austauschbar.

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Jenseits von Wahr und Falsch

1. Die zweiwertige aristotelische Logik basiert, wie allgemein bekannt ist, auf den zwei Wahrheitswerten Wahr oder 0 und Falsch oder 1 und läßt sich durch die dichotomische Relation

$$L = [0, 1]$$

darstellen. Nun hatte bereits Gotthard Günther festgestellt: "Beide Werte einer solchen Logik aber sind metaphysisch äquivalent. Das heißt, man kann sie beliebig miteinander vertauschen. Sie verhalten sich zueinander in einer totalen logischen Disjunktion, wie rechts und links. Es gibt keinen theoretischen Grund, welche Seite rechts und welche Seite links von der Zugspitze ist. Die Benennung beruht auf einer willkürlichen Entscheidung, und wenn man seinen Standpunkt wechselt, sind die rechte und die linke Seite miteinander vertauscht (Günther 2000, S. 230 f.). Das bedeutet aber, daß gilt

$$[0, 1] \cong [1, 0]$$

und sogar, da L keine geordnete Menge ist,

$$[0, 1] = [1, 0]$$

gelten muß. Der Grund für die Austauschbarkeit der Werte beruht darin, daß 0 das objektive Objekt und 1 das subjektive Subjekt darstellt, da das Gesetz des Tertiums non datur die beiden möglichen "gemischten" erkenntnistheoretischen Kategorien des subjektiven Objekts und des objektiven Subjekts zum vornherein ausschließt.

2. Tatsächlich aber ist es so, daß uns weder objektive Objekte noch subjektive Subjekte zugänglich sind. Denn ein Objekt, das wahrgenommen wird, wird immer durch ein Subjekt wahrgenommen, und somit bekommt das Objekt Subjektanteile und das Subjekt Objektanteile. Dasselbe gilt für ein Subjekt, denn wir können nicht nur andere Subjekte, sondern auch uns selbst immer nur als Objekte wahrnehmen. Schreiben wir oO, sO, oS und sS für die vier erkenntnistheoretischen logischen Funktionen, bedeutet dies, daß

$$L = [oO, sS]$$

durch

L = [sO, oS]

ersetzt werden muß. Mathematisch kann man Subjektanteile von Objekten und Objektanteile von Subjekten durch den bereits in Toth (2014) eingeführten Einbettungsoperator E

E: $x \rightarrow [x]$

definieren. Wendet man E auf L an, dann bekommt man zunächst vier mögliche neue Wertkonstellationen

$E \rightarrow L =$

[0, [1]]	[[1], 0]
[[0], 1]	[1, [0]],

d.h. die beiden nun durch

0 := sO

1 := oS

interpretierten Werte können sowohl eingebettet als auch nicht-eingebettet an beiden logischen Positionen erscheinen. Die zweiwertige Basis der aristotelischen Logik wird somit durch f: (E → L) nicht angetastet.

3. Man kann nun diese vier Wertkonstellationen subjektiver Objekte und objektiver Subjekte wie folgt durch Graphen darstellen.

3.1. [0, [1]]

$$\begin{array}{ccc} 0 & \rightarrow & \emptyset \\ \downarrow & & \downarrow \\ \emptyset & \rightarrow & 1 \end{array}$$

3.2. [[1], 0]

$$\begin{array}{ccc} \emptyset & \rightarrow & 0 \\ \downarrow & & \downarrow \\ 1 & \rightarrow & \emptyset \end{array}$$

3.3. [[0], 1] 3.4. [1, [0]]

Ø → 1 1 → Ø

↓ ↓ ↓ ↓

0 → Ø Ø → 0.

Wie man leicht bemerkt, sind damit aber die Positionen von 0 und 1 innerhalb dieser 4 "Wertfelder" keineswegs ausgeschöpft, denn es gibt die folgenden 8 weiteren Wertfelder. Man beachte, daß sich diese im Gegensatz zu den vorstehenden 4 nicht durch lineare Einbettungsschemata notieren lassen.

3.5. 3.6.

0 → Ø 1 → Ø

↓ ↓ ↓ ↓

1 → Ø 0 → Ø

3.7. 3.8.

Ø → 0 Ø → 1

↓ ↓ ↓ ↓

Ø → 1 Ø → 0

Wie man bemerkt, sind damit die vertikalen Positionen von 0 und 1 in den Wertfeldern ausgeschöpft.

3.9. 3.10.

0 → 1 1 → 0

↓ ↓ ↓ ↓

Ø → Ø Ø → Ø

3.11. 3.12.

$$\begin{array}{ccccccc} \emptyset & \rightarrow & \emptyset & \quad & \emptyset & \rightarrow & \emptyset \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ 0 & \rightarrow & 1 & & 1 & \rightarrow & 0 \end{array}$$

Wie man bemerkt, sind damit die horizontalen Positionen von 0 und 1 in den Wertfeldern ausgeschöpft. Ferner erkennt man, daß mit den ersten 4 Wertfeldern, die den vier Einbettungsstrukturen korrespondieren, auch die diagonalen Positionen von 0 und 1 in den Wertfeldern ausgeschöpft sind.

4. Das bedeutet also, daß den 16 dyadischen Wahrheitswertfunktionen der auf L = [0, 1] aufgebauten Logik 12 dyadische Wahrheitswertfelder der auf der Abbildung f: E → L aufgebauten Logik korrespondieren. Da für die letztere oO und sS durch sO und oS ersetzt sind, sind allerdings die Werte 0 und 1 in den 12 Wahrheitswertfeldern nicht mehr relativ zu logischer Wahrheit und Falschheit unterscheidbar, denn es gilt ja z.B. [0, [1]] = (W = f(F)) und [[1], 0] = (F = f(W)). Wir befinden uns vermöge der Funktion f: E → L somit "Jenseits von Wahr und Falsch". Ferner sind die quadratischen Felder, die wir hier in Analogie zur klassischen Logik als Wahrheitswertfelder bezeichnet hatten, eher Zahlenfelder bzw. sie geben die Zählweisen der funktionalen, voneinander abhängigen Wahrheitswerte an, insofern die 4 Felder

$$\begin{array}{ccccccc} 0 & \rightarrow & 1 & \quad & 1 & \rightarrow & 0 \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ \emptyset & \rightarrow & \emptyset & & \emptyset & \rightarrow & \emptyset \end{array}$$

$$\begin{array}{ccccccc} \emptyset & \rightarrow & \emptyset & \quad & \emptyset & \rightarrow & \emptyset \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ 0 & \rightarrow & 1 & & 1 & \rightarrow & 0 \end{array}$$

genau den Zählschemata der adjazenten Zählweise, die 4 Felder

$$\begin{array}{ccccccc} 0 & \rightarrow & Ø & \quad & 1 & \rightarrow & Ø \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ 1 & \rightarrow & Ø & & 0 & \rightarrow & Ø \end{array}$$

$$\begin{array}{ccccccc} Ø & \rightarrow & 0 & \quad & Ø & \rightarrow & 1 \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ Ø & \rightarrow & 1 & & Ø & \rightarrow & 0 \end{array}$$

genau den Zählschemata der subjazenten Zählweise,

und die 4 Felder

$$\begin{array}{ccccccc} 0 & \rightarrow & Ø & \quad & Ø & \rightarrow & 0 \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ Ø & \rightarrow & 1 & & 1 & \rightarrow & Ø \end{array}$$

$$\begin{array}{ccccccc} Ø & \rightarrow & 1 & \quad & 1 & \rightarrow & Ø \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ 0 & \rightarrow & Ø & & Ø & \rightarrow & 0 \end{array}$$

genau den Zählschemata der tranjsazenten Zählweise, wie sie im Rahmen der qualitativen Arithmetik der ortsfunktionalen Peanozahlen eingeführt worden waren (vgl. Toth 2015a-c), entsprechen.

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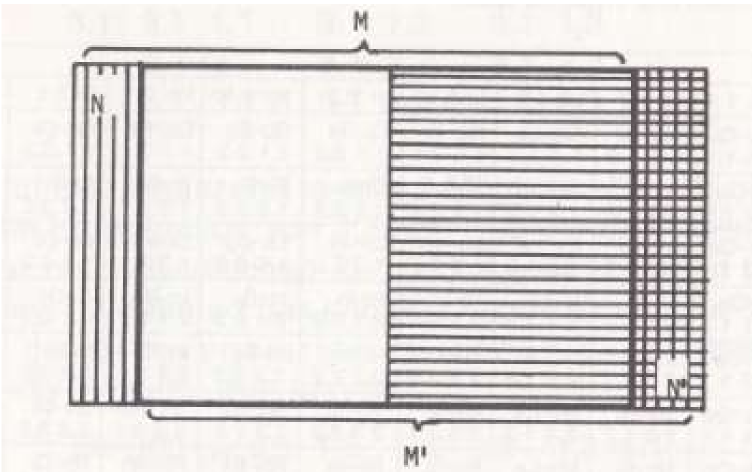
Benses "Grundfigur des ästhetischen Zustandes"

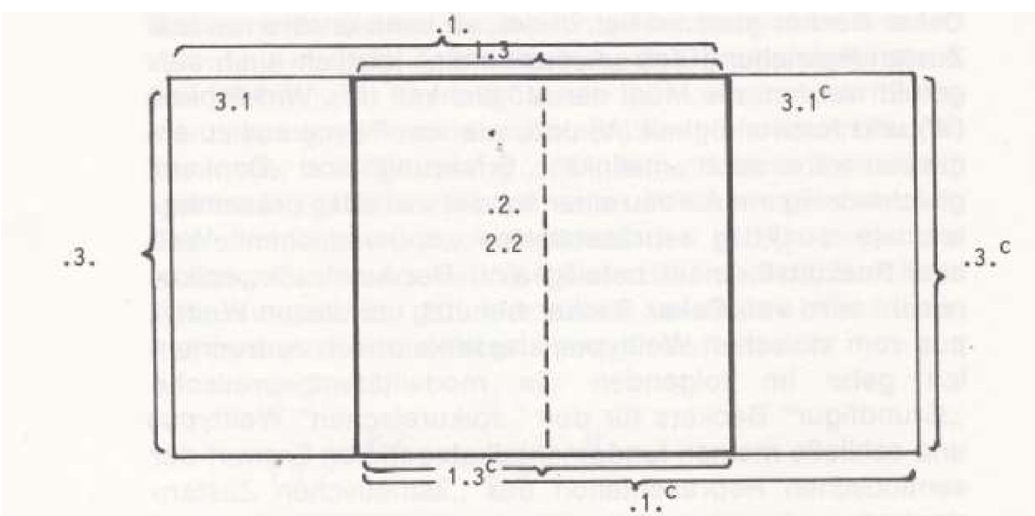
1. In seinem Buch "Die Unwahrscheinlichkeit des Ästhetischen" gab Bense (1979, S. 101) die von seinem mathematischen Lehrer Oskar stammende "modalitätentheoretische Grundfigur des epikuräischen Welttypus"



wieder und transformierte sie, wie im folgenden aus Bense (1979, S. 102) reproduziert, zur "Grundfigur des ästhetischen Zustandes", die durch das "eigenreale", dualinvariante semiotische Dualsystem

DS = [3.1, 2.2, 1.3] × [3.1, 2.2, 1.3]

repräsentiert wird (vgl. Bense 1992)



(„c“ als Index bedeutet das „Komplement“)

2. Wie zuletzt in Toth (2016) gezeigt, kann man durch Anwendung eines Einbettungsoperators

E: $x \rightarrow [x]$

die logische Basisdichotomie

L = [0, 1]

in ein Quadrupel von L-Relationen

E → L =

[0, [1]] [[1], 0]

[[0], 1] [1, [0]]

transformieren, in denen die beiden logischen Werte 0 und 1 erstens an beiden logischen Positionen und zweitens sowohl eingebettet als auch nicht-eingebettet aufscheinen. Setzt man, wie in der klassischen aristotelischen Logik üblich, als Wahrheitswerte

0 = W

1 = F

ein, so bekommt man also

[W, [F]] [[F], W]

[[W], F] [F, [W]],

d.h. es gibt zwar immer noch die funktional nicht-abhängigen Wahrheitswerte W und F an allen logischen Positionen, aber sie treten nun ebenfalls als funktional abhängige Wahrheitswerte der beiden Formen

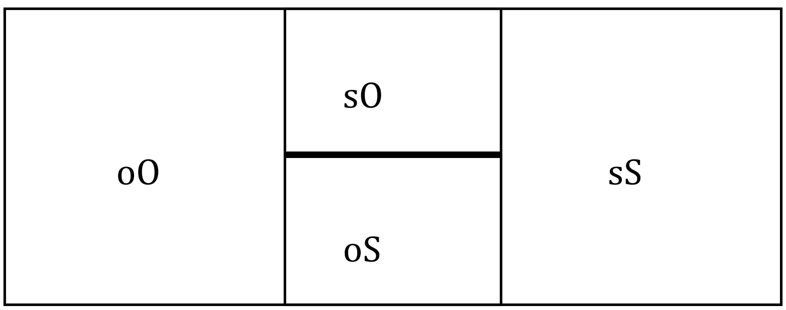
W = f(F)

F = f(W)

auf. Das bedeutet, daß wir es hier nicht nur, wie in der klassischen Logik, mit objektiven Objekten und subjektiven Subjekten zu tun haben, sondern daß wir vermöge des Einbettungsoperators E nun auch die beiden "gemischten" erkenntnistheoretischen Funktionen des subjektiven Objektes und des objektiven Subjektes bekommen, wie sie sich aus der folgenden erkenntnistheoretischen Matrix, darin O für Objekt und S für Subjekt stehen, ablesen lassen

	O	S
O	oO	oS
S	sO	sS.

Dadurch erhalten wir ein weiteres Beispiel für die Becker-Bense-Grundfiguren,



d.h. es gibt nun trotz Beibehaltung der logischen Zweiwertigkeit eine Vermittlung zwischen oO = O = 0 und sS = S = 1, nämlich O = f(S) und S = f(O).

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Einführung in die elementare qualitative Arithmetik

1. In der quantitativen Mathematik gibt es nur eine einzige Zählweise, die lineare, welche durch die Peano-Axiome festgelegt ist. Weshalb dies so ist, ist auch den meisten Mathematikern nicht bewußt. Der Grund liegt darin, daß die zweiwertige aristotelische Logik, welche die Grundlage für die quantitative Mathematik bildet, in ihrer Basisdichotomie

$L = [0, 1]$

keine Vermittlung zwischen den Werten 0 und 1 zuläßt, d.h. sie sind absolut. Nun hatte bereits Günther festgestellt: "Beide Werte einer solchen Logik aber sind metaphysisch äquivalent. Das heißt, man kann sie beliebig miteinander vertauschen. Sie verhalten sich zueinander in einer totalen logischen Disjunktion, wie rechts und links. Es gibt keinen theoretischen Grund, welche Seite rechts und welche Seite links von der Zugspitze ist. Die Benennung beruht auf einer willkürlichen Entscheidung, und wenn man seinen Standpunkt wechselt, sind die rechte und die linke Seite miteinander vertauscht (Günther 2000, S. 230 f.). Damit gilt natürlich

$L = [0, 1] = [1, 0] = L^{-1}$.

0 oder Position und 1 oder Negation (bzw. umgekehrt) stehen also erkenntnistheoretisch für das objektive Objekt einerseits und für das subjektive Subjekt andererseits.

2. Tatsächlich ist es aber so, daß die Begriffe Objekt und Subjekt ohne einander sinnlos sind. Ein Objekt kann nur dann ein solches sein, wenn es ein solches für ein Subjekt ist. Und ein Subjekt kann es nur dann geben, wenn es ein Objekt gibt, von dem es sich unterscheidet. Ontisch gesehen ist es daher nur dann sinnvoll, von einem Objekt zu sprechen, wenn es von einem Subjekt wahrgenommen wird. Durch die Wahrnehmung erhält aber das Objekt – als vom Subjekt Wahrgenommenes – Subjektanteile, und das Subjekt – als das Objekt Wahrnehmendes -. erhält Objektanteile. Es ist daher, wie bereits in Toth (2015a) ausgeführt, nötig, statt von L von einem Quadrupel von L-Funktionen der Form

$L_1 = [0, [1]]$ $\quad$ $L_1^{-1} = [[1], 0]$

$L_3 = [[0], 1]$ $\quad$ $L_2^{-1} = [1, [0]]$

auszugehen, die im Gegensatz zu $L = L^{-1}$ paarweise ungleich sind. Man kann diese vier L-Funktionen unter Vernachläßigung der Positionen der Werte in den vier Gleichungen durch

$0 = f(1)$

$1 = f(0)$

ausdrücken. Je nachdem, ob man 0 oder 1 die erkenntnistheoretische Funktion des Objektes und 1 oder 0 diejenige des Subjektes zuweist, formalisieren daher die beiden letzten Gleichungen das subjektive Objekt und das objektive Subjekt. Rein formal wird für die Transformation

τ: $\quad L \to (L_1, L_1^{-1}, L_2, L_2^{-1})$

lediglich ein Einbettungsoperator der Form

E: $\quad x \to [x]$ (mit $x \in (0, 1)$)

benötigt, d.h. kein dritter Wert, welcher als (substantielles) Tertium die aristotelische Basis der Logik zerstört. Wenn man E als Tertium bezeichnen will, dann handelt es sich um ein differentielles Tertium, das tatsächlich innerhalb der aristotelischen Logik nicht vorgesehen ist.

3. Wie man sieht, spielt innerhalb des Quadrupels von L-Funktionen allerdings nicht nur die Tatsache, ob ein Wert eingebettet oder nicht-eingebettet ist, eine Rolle, sondern auch, wo der Wert, d.h. die Zahl, steht, denn wie bereits gesagt, gilt

$[0, [1]] \neq [[1], 0]$

$[[0], 1] \neq [1, [0]]$

$[1, [0]] \neq [[0], 1]$

$[[1], 0] \neq [0, [1]]$.

Jede Zahl ist somit nicht nur von E, sondern auch von einem Ort ω abhängig, d.h. für jede Peanozahl x gilt

$$x = f(E, \omega),$$

und diese Abhängigkeit ist es, was sie zur qualitativen Zahl macht (vgl. Toth 2015b-d) und nicht etwa die Orthogonalität von Paaren von Peanozahlen (vgl. Günther 1991, S. 419 ff.). In der in diesem Aufsatz zu skizzierenden qualitativen Arithmetik erhält man für Paare von Peanozahlen P = (x, y) unter Anwendung von x = f(E, ω) und y = f(E, ω) statt der einen, linearen Zählweise der quantitativen Arithmetik drei Zählweisen mit je acht verschiedenen qualitativen Zahlen.

3.1. Sind x und y linear, so liegt in meiner Terminologie die adjazente Zählweise vor

$$\begin{array}{ccccccccccc}
x_i & y_j & & y_i & x_j & & y_j & x_i & & x_j & y_i \\
\emptyset_i & \emptyset_j & & \emptyset_i & \emptyset_j & & \emptyset_j & \emptyset_i & & \emptyset_j & \emptyset_i \\
& & \times & & & \times & & & \times & & \\
\emptyset_i & \emptyset_j & & \emptyset_i & \emptyset_j & & \emptyset_j & \emptyset_i & & \emptyset_j & \emptyset_i \\
x_i & y_j & & y_i & x_j & & y_j & x_i & & x_j & y_i.
\end{array}$$

3.2. Sind x und y orthogonal, so liegt in meiner Terminologie die subjazente Zählweise vor

$$\begin{array}{ccccccccccc}
x_i & \emptyset_j & & \emptyset_i & x_j & & \emptyset_j & x_i & & x_j & \emptyset_i \\
y_i & \emptyset_j & & \emptyset_i & y_j & & \emptyset_j & y_i & & y_j & \emptyset_i \\
& & \times & & & \times & & & \times & & \\
y_i & \emptyset_j & & \emptyset_i & y_j & & \emptyset_j & y_i & & y_j & \emptyset_i \\
x_i & \emptyset_j & & \emptyset_i & x_j & & \emptyset_j & x_i & & x_j & \emptyset_i.
\end{array}$$

3.3. Sind x und y diagonal, so liegt in meiner Terminologie die transjazente Zählweise vor

$$\begin{array}{ccccccccccc}
x_i & \emptyset_j & & \emptyset_i & x_j & & \emptyset_j & x_i & & x_j & \emptyset_i \\
\emptyset_i & y_j & & y_i & \emptyset_j & & y_j & \emptyset_i & & \emptyset_j & y_i \\
& & \times & & & \times & & & \times & & \\
\emptyset_i & y_j & & y_i & \emptyset_j & & y_j & \emptyset_i & & \emptyset_j & y_i \\
x_i & \emptyset_j & & \emptyset_i & x_j & & \emptyset_j & x_i & & x_j & \emptyset_i.
\end{array}$$

(In diesen Schemata referieren die Indizes auf die SUBJEKTSTANDPUNKTE. Diese ermöglichen die Kompatibilisierung unserer qualitativen Arithmetik mit der von Kronthaler geschaffenen Mathematik der Qualitäten (vgl. Kronthaler 1986), die auf der polykontexturalen Logik beruht, welche ein Verbundsystem von zweiwertigen aristotelischen Logiken ist und jedem Subjekt seine eigene zweiwertige Logik zugesteht. Da hier aber nur das Subjekt iterierbar ist, während das Objekt, wie Hegel sagte, totes Objekt bleibt, gibt es in der Mathematik der Qualitäten im Gegensatz zu unserer qualitativen Arithmetik keine Vermittlung der Werte innerhalb der auch in der Mathematik der Qualitäten unangetasteten Dichotomie $L = [0, 1]$.)

4. In der qualitativen Arithmetik kann also jede Peanozahl x vermöge $x = f(E, \omega)$ entweder adjazent, subjazent oder transjazent gezählt werden, wobei es acht Möglichkeiten in jeder der drei Zählweisen gibt. Das bedeutet, daß bereits die elementare und nur quantitativ eindeutige Peano-Addition $(0 + 1)$ qualitativ in 24 Möglichkeiten mehrdeutig ist. Jede qualitative Zahl ist somit in der allgemeinen Form

$$x_{n,\, m}$$

notierbar, darin n den Wert von E und m den Wert von ω angibt. Die quantitative Addition $(0 + 1)$ ist somit ein Spezialfall für $n = m$. Beschränken wir uns zur Illustration auf das L-Quadrupel, so erhält man zunächst die folgenden Ungleichungen

$$[0, [1]] \neq [[1], 0] \quad \rightarrow \quad (0, 1_{-1}) \neq (1_{-1}, 0)$$

$$[[0], 1] \neq [1, [0]] \quad \rightarrow \quad (0_{-1}, 1) \neq (1, 0_{-1})$$

$[1, [0]] \neq [[0], 1] \quad \rightarrow \quad (1, 0_{-1}) \neq (0_{-1}, 1)$

$[[1], 0] \neq [0, [1]] \quad \rightarrow \quad (1_{-1}, 0) \neq (0, 1_{-1})$.

Damit entsteht allerdings eine Ambiguität zwischen Subjazenz und Transjazenz, denn z.B. kann $(0, 1_{-1})$

0

1

oder

0

  1

bedeuten. Daher muß auch bei Zahlenpaaren mit festgelegter Ordnung nicht nur die E-, sondern auch die ω-Position indiziert werden, d.h.

0

1 = $(0_{n, m}, 1_{n-1, m})$,

aber

0

  1 = $(0_{n, m}, 1_{n-1, m+1})$,

wogegen

  0

1 = $(0_{n, m}, 1_{n-1, m-1})$, usw.

Es ist somit möglich, alle 3 mal 8 Zählweisen der qualitativen Arithmetik für jede quantitative Peanozahl x durch x = f(E, ω) qualitativ darzustellen. Da ferner alle 24 Zählweisen paarweise voneinander verschieden sind, ist die Abbildung von quantitativen auf qualitative Zahlen bijektiv. Man braucht also nicht wie in der Mathematik der Qualitäten auf die Korzybski-Mehrdeutigkeit auszuweichen, welche dazu führt, daß kein einziger Satz der Mathematik der

Qualitäten beweisbar ist. Dagegen werden alle Sätze einer qualitativen Mathematik, welche auf der Basis der qualitativen Arithmetik errichtet werden wird, auch beweisbar sein.

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Kontexturgrenzen zwischen der quantitativen und der qualitativen zweiwertigen Logik

1. In Toth (2016a) hatten wir die Relationen der quantitativen und der qualitativen zweiwertigen Logik in 4! = 24 relationalen Quadraten angegeben, welche also sowohl die quantitative aristotelische Logik der Form

$L = [0, 1]$

als auch die qualitative Logik (vgl. Toth 2015) der Form

$L_1 = [0, [1]]$ $\quad$ $L_3 = [[1], 0]$

$L_2 = [[0], 1]$ $\quad$ $L_4 = [1, [0]]$

enthalten. Zur Vereinfachung sehen wir im folgenden von den positionsbedigten Varianten ab, d.h. wir unterscheiden nicht zwischen den je zwei Möglichkeiten für das subjektive Objekt und für das objektive Subjekt

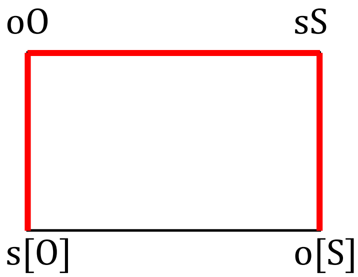
s[O] und [O]s

o[S] und [S]o,

da diese durch die Definition der qualitativen Zahl $P = f(\omega, E)$ induzierte Differenzierung für unsere folgenden Überlegungen keine Rolle spielt.

2. Was wir anhand der 24 relationalen logischen Quadrate bestimmen wollen, sind die Kontexturgrenzen, denn solche bestehen lediglich innerhalb von $L = [0, 1]$, da die aristotelische Logik keine Vermittlung zuläßt, und dies ist nur deshalb der Fall, weil das Gesetz des Tertium non datur stets substantiell gemeint ist, d.h. einen dritten Wert verbietet. In der qualitativen und immer noch zweiwertigen Logik findet sich nun zwar ein Tertium, aber dieses ist differentiell, nämlich induziert durch den Einbettungsoperator E von $P = f(\omega, E)$, und dieser erst ermöglicht ja die Ersetzung der Dichotomie von absolutem Objekt und Subjekt durch relatives, d.h. subjektives, Objekt und relatives, d.h. objektives, Objekt. Im folgenden zeigen wir kontexturelle Grenzen an, indem wir die die entsprechenden Relationen anzeigenden Linien zwischen den erkenntnistheoretischen Kategorien rot einfärben.

2.1.



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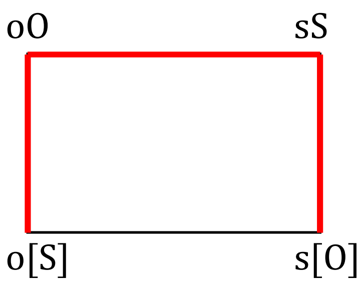

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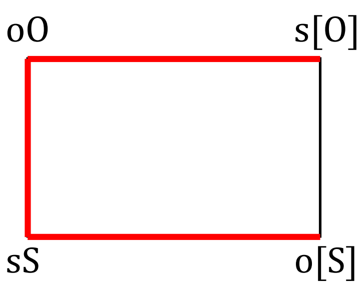
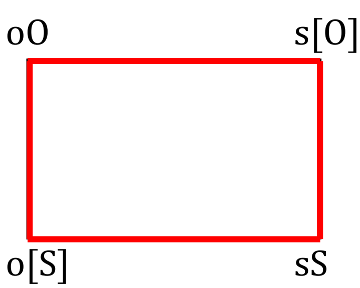


2.4.



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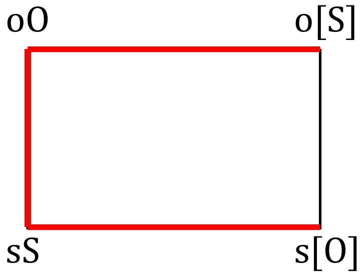

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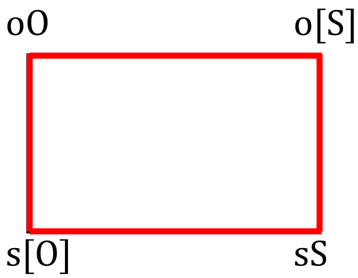

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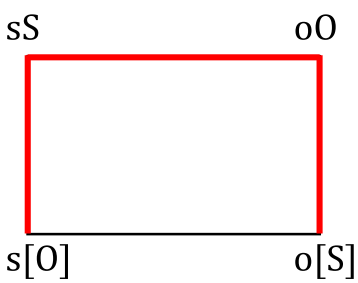

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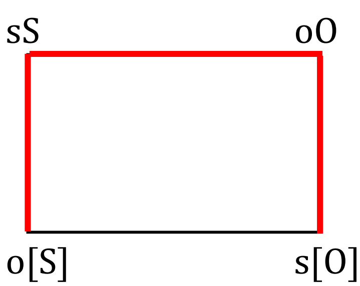

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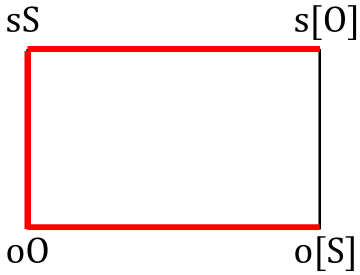

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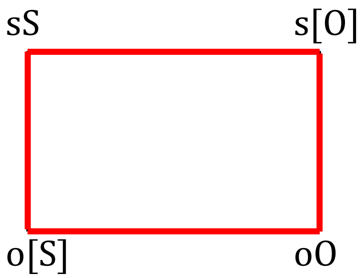

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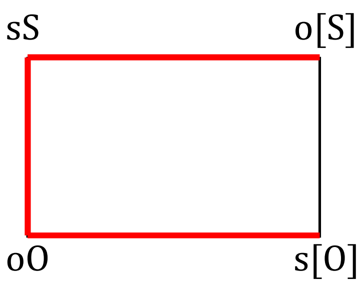

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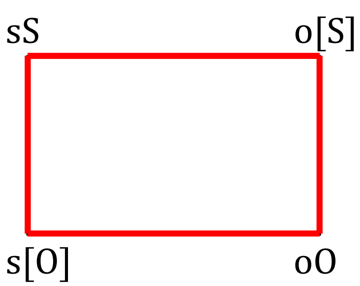

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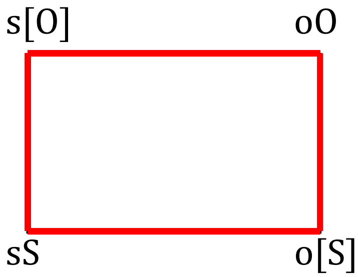

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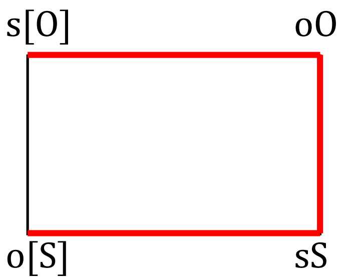

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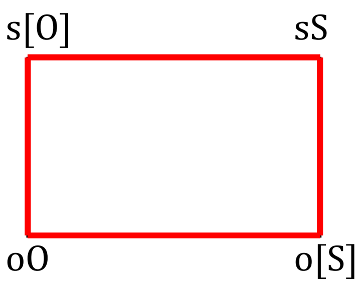

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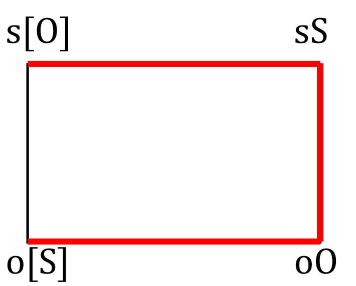

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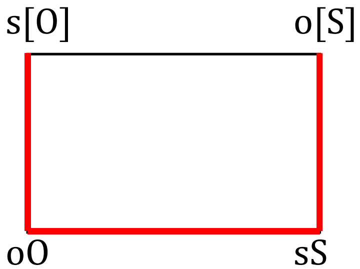

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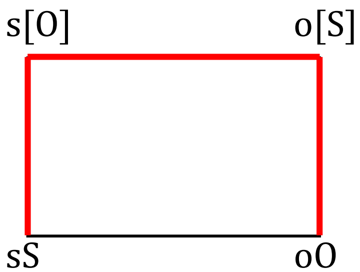

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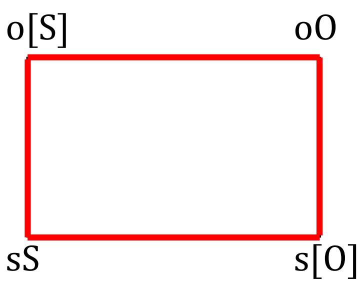

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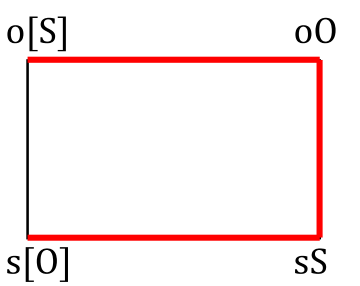

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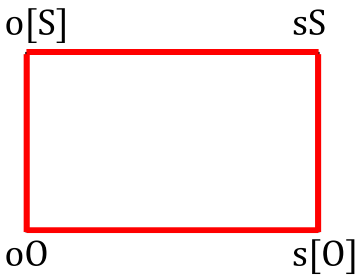

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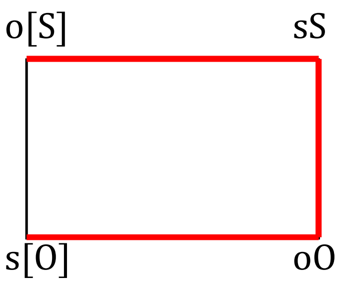

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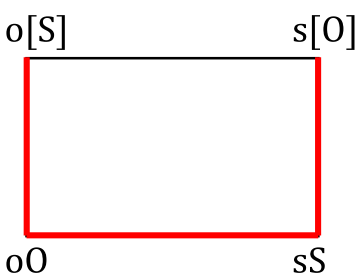

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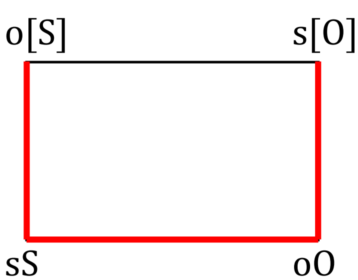


Keine Kontexturengrenzen finden sich also genau in denjenigen relationalen Quadraten, in denen s[O] und o[S] in einer linearen, d.h. horizontalen oder vertikalen, Relation zueinander stehen, oder, um es mit den Begriffen der qualitativen Arithmetik auszudrücken, wenn s[O] und o[S] adjazent oder subjazent sind. Die Ergebnisse unserer Untersuchung lassen sich somit als Satz der qualitativen Arithmetik formulieren

SATZ. In logischen quadratischen Graphen, deren Ecken durch die quantitativen Kategorien oO und sS sowie durch die qualitativen Kategorien s[O] und o[S] besetzt sind, sind alle Kanten Kontexturgrenzen gdw. s[O] und o[S] transjazent sind.

Literatur

Toth, Alfred, Jenseits von wahr und falsch. In: Electronic Journal for Mathematical Semiotics, 2015

Toth, Alfred, Einführung in die elementare qualitative Arithmetik. In: Electronic Journal for Mathematical Semiotics, 2016a

Toth, Alfred, Die Relationen der quantitativen und der qualitativen zweiwertigen Logik. In: Electronic Journal for Mathematical Semiotics, 2016b

Ein Gesetz für semiotische Designation

1. In Toth (2016a) hatten wir semiotische Palindrome für die in Toth (2016b) eingeführten semiotischen Zahlen eingeführt. Die Idee dazu stammt von Kaehr (2012a, b), statt Morphogramme "Morphe", d.h. durch Reidemeister-Bewegungen formal faßbare dynamische Bewegungen in Zöpfen (breads) als tiefste Ebene der Polykontexturalitätstheorie zu setzen. Kaehrs Kritik beruht auf der korrekten Einsicht, Günther widerspreche sich selbst, wenn er erst Folgen von Leerstellen als Abstraktionen von logischen Wertfolgen einführe, später aber in seinen Negationszyklen wieder mit Werten operiere. Auf der anderen Seite unterscheiden sich die von uns entdeckten semiotischen Zahlen in zweifacher Hinsicht von den polykontexturalen Gestaltzahlen, insofern in ihrer logischen Basis nicht nur die Subjekt-, sondern auch die Objektposition iterierbar ist und insofern die Werte der 2-wertigen Basis durch einen Einbettungsoperator, d.h. funktional und nicht substantiell (durch einen dritten Wert) vermittelbar sind.

2. Für jede n-stelle Zeichenrelation Z^n hängt die Anzahl der Palindrome natürlich von der Kardinalität $K[Z^n]$, d.h. von ihrer "Kontexturen"-Länge, ab, d.h. es besteht die Beziehung

$$pal[Z^n] = f(K[Z^n]).$$

Wegen der Möglichkeit, die Abwesenheit von Zeichen als 1-stellige Zeichen zu definieren, verschiebt sich damit das Verhältnis von Z^n zu $K[Z^n]$ um 1 Stufe. $K[Z^n]$ selbst ist die OEIS-Folge A095121 (Expansion von (1-x+2x^2)/((1-x)(1-2x))). Wie bereits in Toth (2016a) festgestellt wurde, kommt es jedoch bemerkenswerterweise mit steigendem n in Z^n nicht zu einem Wechsel von Unter- und Überbalanciertheit zwischen designierten und nicht-designierten semiotischen Palindromen. Im folgenden wird gezeigt, daß die Differenz zwischen den beiden sogar konstant ist.

Man vgl. die folgende Tabelle für n = 1 bis n = 10.

Z^n	$K[Z^n]$	$pal[Z^n]$	
		designiert	nicht-designiert
1	Ø	—	—
2	1	2	0
3	6	4	2
4	14	8	6
5	30	16	14
6	62	32	30
7	126	64	62
8	254	128	126
9	510	256	254
10	1022	512	510,

d.h., wie man durch einfaches Ablesen "beweisen" kann, ist

$pal[Z^n] = (K[Z^n] - 2)$.

Literatur

Kaehr, Rudolf, Gunther's Negation Cycles and Morphic Palindromes. In: ThinkArtLab, 2012 (2012a)

Kaehr, Rudolf, Morphosphere(s): Asymmetric Palindromes as Keys. In: ThinkArtLab, 2012 (2012b)

Toth, Alfred, Semiotische Palindrome. In: Electronic Journal for Mathematical Semiotics 2016a

Toth, Alfred, Die qualitativ-mathematische Unvollständigkeit der triadischen Zeichenrelation. In: Electronic Journal for Mathematical Semiotics 2016b

Zu einer kategorial-einbettungstheoretischen Semiotik

1. Die für die Ontik zuständige qualitative Arithmetik (vgl. Toth 2015) kennt bekanntlich nicht nur die quantitative Zählweise der Peanozahlen, sondern drei 2-dimensionale Zählweisen, die als adjazente, subjazente und transjazente bezeichnet worden waren, deren allgemeine Zählschemata wie folgt sind.

1.1. Adjazente Zählweise

$$\begin{array}{ccccccccccc}
x_i & y_j & & y_i & x_j & & y_j & x_i & & x_j & y_i \\
\emptyset_i & \emptyset_j & & \emptyset_i & \emptyset_j & & \emptyset_j & \emptyset_i & & \emptyset_j & \emptyset_i \\
 & & \times & & & \times & & & \times & & \\
\emptyset_i & \emptyset_j & & \emptyset_i & \emptyset_j & & \emptyset_j & \emptyset_i & & \emptyset_j & \emptyset_i \\
x_i & y_j & & y_i & x_j & & y_j & x_i & & x_j & y_i
\end{array}$$

1.2. Subjazente Zählweise

$$\begin{array}{ccccccccccc}
x_i & \emptyset_j & & \emptyset_i & x_j & & \emptyset_j & x_i & & x_j & \emptyset_i \\
y_i & \emptyset_j & & \emptyset_i & y_j & & \emptyset_j & y_i & & y_j & \emptyset_i \\
 & & \times & & & \times & & & \times & & \\
y_i & \emptyset_j & & \emptyset_i & y_j & & \emptyset_j & y_i & & y_j & \emptyset_i \\
x_i & \emptyset_j & & \emptyset_i & x_j & & \emptyset_j & x_i & & x_j & \emptyset_i
\end{array}$$

1.3. Transjazente Zählweise

$$\begin{array}{ccccccccccc}
x_i & \emptyset_j & & \emptyset_i & x_j & & \emptyset_j & x_i & & x_j & \emptyset_i \\
\emptyset_i & y_j & & y_i & \emptyset_j & & y_j & \emptyset_i & & \emptyset_j & y_i \\
 & & \times & & & \times & & & \times & & \\
\emptyset_i & y_j & & y_i & \emptyset_j & & y_j & \emptyset_i & & \emptyset_j & y_i \\
x_i & \emptyset_j & & \emptyset_i & x_j & & \emptyset_j & x_i & & x_j & \emptyset_i
\end{array}$$

2. Man kann also, sehr vereinfacht ausgedrückt, jedes horizontale, vertikale oder diagonale Paar von qualitativen Zahlen der Form Z = (x, y) durch Anwendung eines Einbettungsoperators E vermöge einer Funktion

e: E → Z

definieren. Im folgenden sollen einige allgemeine Grundlagen einer (im übrigen äußerst komplexen) qualitativ-arithmetischen Einbettungstheorie dargestellt werden.

2.1. Keine Einbettung

Z = R(1, 2, 3)

2.2. Einbettungen

2.2.1. Gleichstufige Einbettungen

2.2.1.1. Nur 1 Relatum eingebettet

Z = R((1), 2, 3)

Z = R(1, (2), 3)

Z = R(1, 2, (3))

2.2.1.2. 2 Relata eingebettet

Z = R((1), (2,) 3)

Z = R(1, (2), (3))

Z = R((1), 2, (3))

2.2.1.3. Alle 3 Relata eingebettet

Z = R((1), (2), (3))

2.2.2. Verschiedenstufige Einbettungen

2.2.2.1. Nur 1 Relatum eingebettet

Z = R((1), 2, 3) ≠ R(((1)), 2, 3) ≠ R((((1))), 2, 3), usw.

$Z = R(1, (2), 3) \neq R(1, ((2)), 3) \neq R(1, (((2))), 3)$, usw.

$Z = R(1, 2, (3)) \neq R(1, 2, ((3))) \neq R(1, 2, (((3))))$, usw.

2.2.2.2. 2 Relata eingebettet

$Z = R((1, (2)), 3)$

$Z = R(1, (2, (3)))$

$Z = R((1), 2, ((3)))$

$Z = R((1, ((2))), 3)$

$Z = R(1, (2, ((3))))$

$Z = R((1), 2, ()(3))))$

$Z = R(((1), ((2))), 3)$

$Z = R(1, ((2), ((3))))$

$Z = R(((1)), 2, (((3))))$

2.2.2.3. Alle 3 Relata eingebettet

$Z = ((1), ((2)), (((3))))$

und jeweils alle Permutationen, d.h. also nicht nur die zu den peirce-bense-schen Zeichenklassen (Z = ZKl) der Form

ZKl = (3.x, 2.y, 1.z)

dualen Realitätsthematiken (Z = RTh) der Form

×ZKl = RTh = (z.1, y.2, x.3),

sondern auch weiteren Permutationen der Formen

Z = (3.x, 1.y, 2.z) mit Z^{-1} = (z.2, y.1, x.3),

Z = (2.x, 3.y, 1.z) mit Z^{-1} = (z.1, y.3, x.2),

Z = (2.x, 1.y, 3.z) mit Z^{-1} = (z.3, y.1, x.2).

Die von Bense (1979, S. 53 u. 67) definierte Zeichenrelation hat demnach die Form

$Z = (1, (2, (3)))$

und besteht aus einer Folge von Einbettungen der Form

$E = (E^1, E^2, E^3)$.

Literatur

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1979

Toth, Alfred, Qualitative Arithmetik des Zählens auf 3 In: Electronic Journal for Mathematical Semiotics, 2015

Ränder und Grenzen in semiotischen Dualsystemen

1. In Toth (2015) waren ontische Grenzen als Teilmengen von ontischen Rändern definiert worden

$G \subset R.$

Für Ränder gelten ferner folgende zwei Möglichkeiten

$R[A, B] = R[A, B] = \emptyset$

$R[A, B] \neq R[B, A] \neq \emptyset.$

Anders als in der Ontik sind jedoch in der Semiotik Grenzen innerhalb von Rändern in eindeutiger Weise bestimmbar, und zwar vermöge der semiosischen Inklusion (vgl. Bense/Walther 1973, S. 42).

2. Damit können wir semiotische Grenzen und Ränder für alle 10 peirce-benseschen Dualsysteme bestimmen. Man beachte, daß es die beiden Haupttypen $G = R$ und $G \neq R$ und beim ersteren Typ nur ein Dualsystem gibt, das die Kardinalität 3 besitzt, nämlich die bekannte eigenreale, d.h. dualidentische Zeichen-Realitäts-Thematik (vgl. Bense 1992). Es gibt hingegen für $G = R$ kein Dualsystem mit Kardinalität 2, nur mit Kardinalität 1, und im Falle des zweiten Typus nur Dualsysteme mit Kardinalität 2.

$DS\ 1 = \quad (3.1 \quad 2.1 \quad \underline{1.1}) \quad \times \quad (\underline{1.1} \quad 1.2 \quad 1.3)$

$G = R = (1.1)$

$DS\ 2 = \quad (3.1 \quad \underline{2.1 \quad 1.2}) \quad \times \quad (\underline{2.1 \quad 1.2} \quad 1.3)$

$G \subset R = (1.2 \subset 2.1)$

$DS\ 3 = \quad (\underline{3.1} \quad 2.1 \quad \underline{1.3}) \quad \times \quad (\underline{3.1} \quad 1.2 \quad \underline{1.3})$

$G \subset R = (1.3 \subset 3.1)$

$DS\ 4 = \quad (3.1 \quad \underline{2.2} \quad 1.2) \quad \times \quad (2.1 \quad \underline{2.2} \quad 1.3)$

$G = R = (2.2)$

DS 5 = $(\underline{3.1 \quad 2.2 \quad 1.3}) \times (\underline{3.1 \quad 2.2 \quad 1.3})$

G = R = (3.1, 2.2, 1.3)

DS 6 = $(\underline{3.1} \quad 2.3 \quad \underline{1.3}) \times (\underline{3.1} \quad 3.2 \quad \underline{1.3})$

G ⊂ R = (1.3 ⊂ 3.1)

DS 7 = $(3.2 \quad \underline{2.2} \quad 1.2) \times (2.1 \quad \underline{2.2} \quad 2.3)$

G = R = (2.2)

DS 8 = $(3.2 \quad \underline{2.2} \quad 1.3) \times (3.1 \quad \underline{2.2} \quad 2.3)$

G = R = (2.2)

DS 9 = $(\underline{3.2 \quad 2.3} \quad 1.3) \times (3.1 \quad \underline{3.2 \quad 2.3})$

G ⊂ R = (2.3 ⊂ 3.2)

DS 10 = $(\underline{3.3} \quad 2.3 \quad 1.3) \times (3.1 \quad 3.2 \quad \underline{3.3})$

G = R = (3.3)

Literatur

Bense, Max, Die Eigenrealität der Zeichen. Baden-Baden 1992

Bense, Max/Walther, Elisabeth, Wörterbuch der Semiotik. Köln 1973

Toth, Alfred, Rand einer Grenze und Grenze eines Randes. In: Electronic Journal for Mathematical Semiotics, 2015

Semiotische Abhängigkeit von Dualsystemen

1. Bekanntlich stellt innerhalb der Ontik die Objektabhängigkeit eine Invariante dar und kann in dreifacher Gradation, d.h. 0-seitig, 1-seitig oder 2-seitig für jedes Paar von Objekten innerhalb eines n-tupels auftreten (vgl. Toth 2012). Innerhalb der Semiotik hingegen gehört Abhängigkeit nicht zum Katalog der von Bense bestimmten semiotischen Invarianten (vgl. Bense 1975, S. 39 ff.). In Toth (2015) hatten wir bereits semiotische Abhängigkeit von Subzeichen untersucht und waren zum Schluß gekommen, daß es nur 2-, 3- und 4-seitige Abhängigkeit gibt, sofern man diagonale Semiosen ausschließt.

2. Für das sog. peircesche Zehnersystem, das besser als bensesches Zehnersystem bezeichnet werden sollte, da die numerische Einführung der Primzeichenrelation auf Bense zurückgeht, ist es bekanntlich so, daß innerhalb des mathematischen Verbandes alle 10 semiotischen Dualsysteme, d.h. also sowohl die Zeichenklassen als auch ihre dualen Realitätsthematiken, in mindestens einem und maximal zwei Subzeichen paarweise miteinander zusammenhängen. Da diese Subzeichen Teilrelationen der eigenrealen, d.h. dualinvarianten Zeichenklasse/Realitätsthematik sind, spricht Bense im Anschluß an Walther vom Zehnersystem als einem determinantensymmetrischen Dualitätssystem, vgl. die folgende Darstellung aus Bense (1992, S. 76).

Zkl			Rth			Rpw	
3.1	2.1	1.1	1.1	1.2	1.3	9	
3.1	2.1	1.2	2.1	1.2	1.3	10	Mittel
3.1	2.1	1.3	3.1	1.2	1.3	11	
3.1	2.2	1.2	2.1	2.2	1.3	11	
3.2	2.2	1.2	2.1	2.2	2.3	12	Objekt
3.2	2.2	1.3	3.1	2.2	2.3	13	
3.1	2.3	1.3	3.1	3.2	1.3	13	
3.2	2.3	1.3	3.1	3.2	2.3	14	Interpretant
3.3	2.3	1.3	3.1	3.2	3.3	15	
3.1	2.2	1.3	3.1	2.2	1.3	12	Eigenrealität

3. Sobald jedoch Paare von Zeichenklassen aus diesem Verband herausgelöst werden, kann man zwischen 0-seitiger, 1-seitiger, 2-seitiger und 3-seitiger semiotischer Abhängigkeit unterscheiden. Es gibt somit im Gegensatz zur Ontik ein 4-stufiges und kein 3-stufiges Gradationssystem von semiotischer Abhängigkeit.

3.1. Beispiele für 0-seitige semiotische Abhängigkeit

(3.1, 2.1, 1.1)	(3.2, 2.2, 1.2)	(3.1, 2.1, 1.1)
(3.2, 2.2, 1.2)	(3.3, 2.3, 1.3)	(3.3, 2.3, 1.3)

3.2. Beispiele für 1-seitige semiotische Abhängigkeit

(3.1, 2.1, 1.1)	(3.1, 2.1, 1.2)	(3.1, 2.1, 1.3)
(3.1, 2.2, 1.2)	(3.1, 2.2, 1.3)	(3.1, 2.2, 1.2)

3.3. Beispiele für 2-seitige semiotische Abhängigkeit

(3.1, 2.1, 1.1)	(3.1, 2.1, 1.2)	(3.1, 2.1, 1.1)
(3.1, 2.1, 1.2)	(3.1, 2.1, 1.3)	(3.1, 2.1, 1.3)

3.4. Beispiele für 3-seitige semiotische Abhängigkeit

Hier gibt es nur den Fall der semiotischen Selbstabhängigkeit, der formal direkt aus der Eigenschaft der Dualinvarianz der Eigenrealität folgt

(3.1, 2.2, 1.3)

(3.1, 2.2, 1.3).

Nimmt man neben dieser die Nebendiagonale der Kleinen Matrix bildenden Zeichenklasse auch die Zeichenrelation, welche deren Hauptdiagonale bildet, hinzu, ergibt sich ferner

(3.3, 2.2, 1.1)

(3.3, 2.2, 1.1),

d.h. die von Bense (1992) so genannte Kategorienklasse.

Literatur

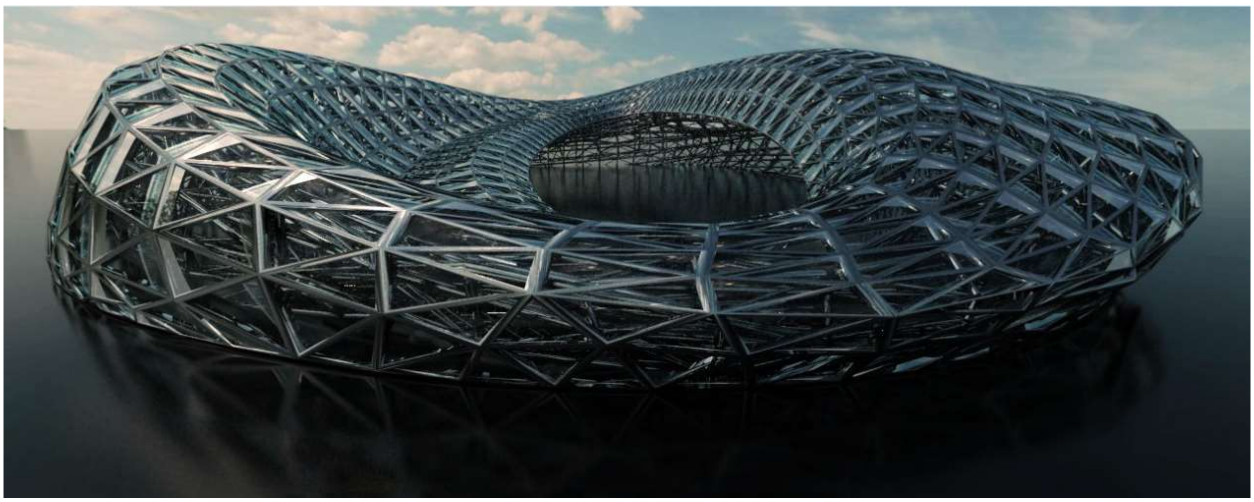
Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Bense, Max, Die Eigenrealität der Zeichen. Baden-Baden 1992

Toth, Alfred, Systeme, Teilsysteme und Objekte I-V. In: Electronic Journal for Mathematical Semiotics 2012

Die semiotische Struktur des Transit-Korridors

1. In Toth (2006) hatte ich die qualitativ-mathematischen Grundlagen des Transit-Korridors darzustellen versucht. Zahlreiche Einzelaufsätze folgten später, doch mir war seinerzeit kein Torus-Modell bekannt, welches der semiotischen Struktur des Transit-Korridors auch nur nahe gekommen wäre. Obwohl mir leider die Quelle dieses mir zugesandten und nachstehend reproduzierten Bildes unbekannt ist, liegt nun ein perfektes ontisches Modell vor, welches die mathematischen Eigenschaften des semiotischen Transit-Korridor aufweist.



2. Um die Struktur dieses ontischen Modelles anhand der Semiotik aufzuzeigen, ist es natürlich nötig, auf Benses letztes Buch (vgl. Bense 1992) zurückzukommen, worin die Eigenrealität durch das dualinvariante Dualsystem

DS(ER) = [3.1, 2.2, 1.3] × [3.1, 2.2, 1.3]

und die Kategorienrealität durch das nicht-dualinvariante, jedoch spiegelsymmetrische Dualsystem

DS(KR) = [3.3, 2.2, 1.1] × [1.1, 2.2, 3.3],

definiert und einschließlich des transformatorischen Zusammenhangs von DS(ER) und DS(KR) ausführlich dargelegt worden waren.

2. Es genügt allerdings nicht, von der in Bense (1975, S. 37) eingeführten kleinen semiotischen Matrix auszugehen, denn diese enthält zwar selbstver-

ständlich die beiden semiotischen Diagonalen, aber nicht den Zusammenhang zwischen ihnen auf der Ebene der Subzeichen bzw. Subrealitäten. Hingegen bietet sich die von Bense (1975, S. 105) eingeführte große semiotische Matrix an. Im folgenden (schiefen und in die Anfänge der Scanner der 1980er Jahre zurückgehenden) Modell sind alle Paare von Subzeichen gelb markiert, welche Subzeichen der Form SZ ⊂ (DS(ER) ∪ DS(KR)) enthalten.

		M			O			I		
		Qu 1.1	Si 1.2	Le 1.3	Ic 2.1	In 2.2	Sy 2.3	Rh 3.1	Di 3.2	Ar 3.3
M	Qu 1.1	Qu-Qu 1.1 1.1	Qu-Si 1.1 1.2	Qu-Le 1.1 1.3	Qu-Ic 1.1 2.1	Qu-In 1.1 2.2	Qu-Sy 1.1 2.3	Qu-Rh 1.1 3.1	Qu-Di 1.1 3.2	Qu-Ar 1.1 3.3
	Si 1.2	Si-Qu 1.2 1.1	Si-Si 1.2 1.2	Si-Le 1.2 1.3	Si-Ic 1.2 2.1	Si-In 1.2 2.2	Si-Sy 1.2 2.3	Si-Rh 1.2 3.1	Si-Di 1.2 3.2	Si-Ar 1.2 3.3
	Le 1.3	Le-Qu 1.3 1.1	Le-Si 1.3 1.2	Le-Le 1.3 1.3	Le-Ic 1.3 2.1	Le-In 1.3 2.2	Le-Sy 1.3 2.3	Le-Rh 1.3 3.1	Le-Di 1.3 3.2	Le-Ar 1.3 3.3
O	Ic 2.1	Ic-Qu 2.1 1.1	Ic-Si 2.1 1.2	Ic-Le 2.1 1.3	Ic-Ic 2.1 2.1	Ic-In 2.1 2.2	Ic-Sy 2.1 2.3	Ic-Rh 2.1 3.1	Ic-Di 2.1 3.2	Ic-Ar 2.1 3.3
	In 2.2	In-Qu 2.2 1.1	In-Si 2.2 1.2	In-Le 2.2 1.3	In-Ic 2.2 2.1	In-In 2.2 2.2	In-Sy 2.2 2.3	In-Rh 2.2 3.1	In-Di 2.2 3.2	In-Ar 2.2 3.3
	Sy 2.3	Sy-Qu 2.3 1.1	Sy-Si 2.3 1.2	Sy-Le 2.3 1.3	Sy-Ic 2.3 2.1	Sy-In 2.3 2.2	Sy-Sy 2.3 2.3	Sy-Rh 2.3 3.1	Sy-Di 2.3 3.2	Sy-Ar 2.3 3,3
I	Rh 3.1	Rh-Qu 3.1 1.1	Rh-Si 3.1 1.2	Rh-Le 3.1 1.3	Rh-Ic 3.1 2.1	Rh-In 3.1 2.2	Rh-Sy 3.1 2.3	Rh-Rh 3.1 3.1	Rh-Di 3.1 3.2	Rh-Ar 3.1 3.3
	Di 3.2	Di-Qu 3.2 1.1	Di-Si 3.2 1.2	Di-Le 3.2 1.3	Di-Ic 3.2 2.1	Di-In 3.2 2.2	Di-Sy 3,2 2.3	Di-Rh 3.2 3.1	Di-Di 3.2 3.2	Di-Ar 3.2 3.3
	Ar 3.3	Ar-Qu 3.3 1.1	Ar-Si 3.3 1.2	Ar-Le 3.3 1.3	Ar-Ic 3.3 2.1	Ar-In 3.3 2.2	Ar-Sy 3,3 2.3	Ar-Rh 3.3 1.3	Ar-Di 3.3 3.2	Ar-Ar 3.3 3.3

Wie man erkennt, sind der obere und der untere Teil der Matrix asymmetrisch; die entsprechenden zwei Positionen von Paaren von Subzeichen wurden von mir seinerzeit rot umrandet. Diese beiden Positionen sind also Subzeichen, für die gilt SZ ⊄ (DS(ER) ∪ DS(KR)), und ihnen entsprechen im ontischen Torus-Modell die beiden Verschlingungsebenen. Man beachte dabei den mathematischen Zusammenhang zwischen Torus und Möbiusband. Das letztere hatte Bense (1992) ja im Zusammenhang mit der kleinen semiotischen Matrix bereits selbst als topologisches Modell benutzt. Der Transit-Korridor ist somit nichts anderes als der dem 2-dimensionalen Möbiusband semiotisch korrespondierende 3-dimensionale Torus-Raum.

Literatur

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Bense, Max, Eigenrealität der Zeichen. Baden-Baden 1992

Toth, Alfred, In Transit. Klagenfurt 2006

Partitionen der semiotischen Matrix durch die qualitative Arithmetik

1. Wie in Toth (2015a-c) und zahlreichen weiteren Arbeiten zur qualitativen Arithmetik gezeigt wurde, kennen ortsfunktionale Zahlen im Gegensatz zu Peanozahlen die drei zweidimensionalen Zählweisen der Adjazenz, der Subjazenz und der Transjazenz. Im folgenden wird gezeigt, daß man durch diese drei Zählweisen die von Bense (1975, S. 37) eingeführte kleine semiotische Matrix partitionieren kann.

2. Adjazente Matrix-Partition

Adjazent sind in der semiotischen Matrix genau die Trichotomien, d.h. wir bekommen

1.1 1.2 1.3,

2.1 2.2 2.3,

3.1 3.2 3.3.

3. Subjazente Matrix-Partition

Subjazent sind in der semiotischen Matrix genau die Triaden, d.h. wir bekommen

1.1 2.1 3.1

1.2 2.2 2.3

1.3 2.3 3.3.

4. Transjazente Matrix-Partition

Transjazent sind in der semiotischen Matrix die beiden Diagonalen, d.h. die hauptdiagonale Kategorienklasse und die nebendiagonale Eigenrealitätsklasse.

3.3 2.2 1.1

3.1 2.2 1.3.

5. Kombinierte Matrix-Partitionen

5.1. Adjazent-subjazente Partitionen

1.1	1.2	1.1	1.2	1.2	1.3	1.2	1.3
2.1			2.2	2.2			2.3

2.1	2.2	2.1	2.2	2.2	2.3	2.2	2.3
3.1			3.2	3.2			3.3

5.2. Subjazent-transjazente Partitionen

1.1			1.3
2.1	2.2	2.2	2.3
3.1			3.3

5.3. Adjazent-transjazente Partitionen

1.1	1.2	1.3	2.1	2.2	2.3
	2.2			3.2	

Literatur

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Toth, Alfred, Zur Arithmetik der Relationalzahlen I-II. In: Electronic Journal for Mathematical Semiotics, 2015a

Toth, Alfred, Qualitative Arithmetik des Zählens auf drei. In: Electronic Journal for Mathematical Semiotics, 2015b

Toth, Alfred, Qualitative Zahlenfelder, Zahlenschemata und ontische Modelle. In: Electronic Journal for Mathematical Semiotics, 2015c

Notation semiotischer Dualsysteme mit qualitativen Morphismen

1. Wir gehen aus von der in Toth (2016) für die Raumsemiotik eingeführten qualitativen Arithmetik sowie dem folgenden vollständigen System aller 3^3 = 27 über der allgemeinen Form von semiotischen Dualsystemen

DS = [3.x, 2.y, 1.z] × [z.1, y.2, x.3]

mit x, y, z ∈ {1, 2, 3}

konstruierbaren triadischen Relationen. Man beachte, daß hier die Ordnung

x ≦ y ≦ z,

durch welche die "regulären" zehn peirce-benseschen Zeichenklassen aus der Gesamtmenge der semiotischen Relationen herausgefiltert werden, nicht verlangt wird.

DS 1 =	[3.1, 2.1, 1.1]	×	[1.1, 1.2, 1.3]
DS 2 =	[3.1, 2.1, 1.2]	×	[2.1, 1.2, 1.3]
DS 3 =	[3.1, 2.1, 1.3]	×	[3.1, 1.2, 1.3]
DS 4 =	[3.1, 2.2, 1.1]	×	[1.1, 2.2, 1.3]
DS 5 =	[3.1, 2.2, 1.2]	×	[2.1, 2.2, 1.3]
DS 6 =	[3.1, 2.2, 1.3]	×	[3.1, 2.2, 1.3]
DS 7 =	[3.1, 2.3, 1.1]	×	[1.1, 3.2, 1.3]
DS 8 =	[3.1, 2.3, 1.2]	×	[2.1, 3.2, 1.3]
DS 9 =	[3.1, 2.3, 1.3]	×	[3.1, 3.2, 1.3]

DS 10 =	[3.2, 2.1, 1.1]	×	[1.1, 1.2, 2.3]
DS 11 =	[3.2, 2.1, 1.2]	×	[2.1, 1.2, 2.3]

DS 12 =	[3.2, 2.1, 1.3]	×	[3.1, 1.2, 2.3]
DS 13 =	[3.2, 2.2, 1.1]	×	[1.1, 2.2, 2.3]
DS 14 =	[3.2, 2.2, 1.2]	×	[2.1, 2.2, 2.3]
DS 15 =	[3.2, 2.2, 1.3]	×	[3.1, 2.2, 2.3]
DS 16 =	[3.2, 2.3, 1.1]	×	[1.1, 3.2, 2.3]
DS 17 =	[3.2, 2.3, 1.2]	×	[2.1, 3.2, 2.3]
DS 18 =	[3.2, 2.3, 1.3]	×	[3.1, 3.2, 2.3]

DS 19 =	[3.3, 2.1, 1.1]	×	[1.1, 1.2, 3.3]
DS 20 =	[3.3, 2.1, 1.2]	×	[2.1, 1.2, 3.3]
DS 212 =	[3.3, 2.1, 1.3]	×	[3.1, 1.2, 3.3]
DS 223 =	[3.3, 2.2, 1.1]	×	[1.1, 2.2, 3.3]
DS 23 =	[3.3, 2.2, 1.2]	×	[2.1, 2.2, 3.3]
DS 24 =	[3.3, 2.2, 1.3]	×	[3.1, 2.2, 3.3]
DS 25 =	[3.3, 2.3, 1.1]	×	[1.1, 3.2, 3.3]
DS 26 =	[3.3, 2.3, 1.2]	×	[2.1, 3.2, 3.3]
DS 27 =	[3.3, 2.3, 1.3]	×	[3.1, 3.2, 3.3]

2. Im folgenden benutzen wir die drei qualitativen Zählweisen, d.h. die adjazente, subjazente und transjazente, um die "regulären" Dualsysteme innerhalb der von Bense (1975, S. 37) eingeführten semiotischen Matrix darzustellen. Da in semiotischen Dualsystemen nur die x, y und z variabel sind, ergibt sich eine maximal redundanzfreie Notation jedes Dualsystems mit Hilfe von qualitativen Morphismen. Als Zeichen für adjazente Abbildungen wird "→", als Zeichen für

subjazente Abbildungen wird "↑", und als Zeichen für transjazente Abbildungen wird "↗" verwendet.

2.1. DS 1 = [3.1, 2.1, 1.1] × [1.1, 1.2, 1.3]

1.1	Ø	Ø		1.1	1.2	1.3
2.1	Ø	Ø	×	Ø	Ø	Ø
3.1	Ø	Ø		Ø	Ø	Ø

DS 1 = [.1↑] × [1.→]

2.2. DS 2 = [3.1, 2.1, 1.2] × [2.1, 1.2, 1.3]

Ø	1.2	Ø		Ø	1.2	1.3
2.1	Ø	Ø	×	2.1	Ø	Ø
3.1	Ø	Ø		Ø	Ø	Ø

DS 2 = [.1↑, .2↗] × [2.↗, 1.→]

2.3. DS 3 = [3.1, 2.1, 1.3] × [3.1, 1.2, 1.3]

Ø	Ø	1.3		Ø	1.2	1.3
2.1	Ø	Ø	×	Ø	Ø	Ø
3.1	Ø	Ø		3.1	Ø	Ø

DS 3 = [.1↑,.3↗] × [3.↗, 1.→]

2.4. DS 5 = [3.1, 2.2, 1.2] × [2.1, 2.2, 1.3]

Ø	1.2	Ø		Ø	Ø	1.3
Ø	2.2	Ø	×	2.1	2.2	Ø
3.1	Ø	Ø		Ø	Ø	Ø

DS 5 = [.1↗, .2↑] × [2.→, 1.↗]

2.5. DS 6 = [3.1, 2.2, 1.3] × [3.1, 2.2, 1.3]

Ø	Ø	1.3		Ø	Ø	1.3
Ø	2.2	Ø	×	Ø	2.2	Ø
3.1	Ø	Ø		3.1	Ø	Ø

DS 6 = [.1↗, .2↗] × [3.↗, 2.↗]

2.6. DS 9 = [3.1, 2.3, 1.3] × [3.1, 3.2, 1.3]

Ø	Ø	1.3		Ø	Ø	1.3
Ø	Ø	2.3	×	Ø	Ø	Ø
3.1	Ø	Ø		3.1	3.2	Ø

DS 9 = [.1↗, .3↑] × [3.→, 1.↗]

2.7. DS 14 = [3.2, 2.2, 1.2] × [2.1, 2.2, 2.3]

Ø	1.2	Ø		Ø	Ø	Ø
Ø	2.2	Ø	×	2.1	2.2	2.3
Ø	3.2	Ø		Ø	Ø	Ø

DS 14 = [.2↑] × [2.→]

2.8. DS 15 = [3.2, 2.2, 1.3] × [3.1, 2.2, 2.3]

Ø	Ø	1.3		Ø	Ø	Ø
Ø	2.2	Ø	×	Ø	2.2	2.3
Ø	3.2	Ø		3.1	Ø	Ø

DS 15 = [.2↑, .3↗] × [3.↗, 2.→]

2.9. DS 18 = [3.2, 2.3, 1.3] × [3.1, 3.2, 2.3]

Ø	Ø	1.3		Ø	Ø	Ø
Ø	Ø	2.3	×	Ø	Ø	2.3
Ø	3.2	Ø		3.1	3.2	Ø

DS 18 = [.2↗, .3↑] × [3.→, 2.↗]

2.10. DS 27 = [3.3, 2.3, 1.3] × [3.1, 3.2, 3.3]

Ø	Ø	1.3		Ø	Ø	Ø
Ø	Ø	2.3	×	Ø	Ø	Ø
Ø	Ø	3.3		3.1	3.2	3.3

DS 27 = [.3↑] × [3.→]

Man beachte also, daß einzig die Notation in qualitativen Morphismen des selbstdualen semiotischen Systems (vgl. dazu Bense 1992) nicht-symmetrisch ist, während sie, in quantitativen Morphismen dargestellt, natürlich symmetrisch ist, denn es ist ja

×[3.1, 2.2, 1.3] = [3.1, 2.3, 1.3] =

×[α°β°, id2, βα] = [α°β°, id2, βα].

Der Grund dafür liegt natürlich darin, daß Identität eine rein quantitative Hypostase ist, d.h. qualitativ nicht vorkommt, es sei denn als Selbstidentität. Dies ist aber bei vorausgesetzter Transzendenz zwischen Zeichen und Objekt unmöglich, es sei denn, mit der Differenz beider falle die Semiotik in sich zusammen.

Literatur

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Nullstellen bei Paaren dualer semiotischer Relationen

1. Bekanntlich ist das dualsymmetrische, durch die Eigenrealitätsklasse determinierte sog. peirce-bensesche Zehnersystem

Zkl			Rth			Rpw	
3.1	2.1	1.1	1.1	1.2	1.3	9	
3.1	2.1	1.2	2.1	1.2	1.3	10	Mittel
3.1	2.1	1.3	3.1	1.2	1.3	11	
3.1	2.2	1.2	2.1	2.2	1.3	11	
3.2	2.2	1.2	2.1	2.2	2.3	12	Objekt
3.2	2.2	1.3	3.1	2.2	2.3	13	
3.1	2.3	1.3	3.1	3.2	1.3	13	
3.2	2.3	1.3	3.1	3.2	2.3	14	Interpretant
3.3	2.3	1.3	3.1	3.2	3.3	15	
3.1	2.2	1.3	3.1	2.2	1.3	12	Eigenrealität

(vgl. Bense 1992, S. 76) nur ein Ausschnitt aus der Gesamtmenge der über S = (3.x, 2.y, 1.z) mit x, y, z ∈ {1, 2, 3) erzeugbaren $3^3 = 27$ semiotischen Relationen, die, vermöge der bereits durch Bense (1975) eingeführten Dualitätsoperation, in zweifacher Form, nämlich als eine die Subjektposition kodierende Zeichenklasse und eine die Objektposition kodierende Realitätsklasse, aufscheint.

2. Während im semiotischen 10er-System nur die drei Zeichenklassen, deren Realitätsklassen homogene entitätische Realitäten thematisieren, paarweise vollständig Nullstellen aufweisen, d.h.

(3.1, 2.1, 1.1) ∩ (3.2, 2.2, 1.2) = Ø

(3.2, 2.2, 1.2) ∩ (3.3, 2.3, 1.3) = Ø

(3.1, 2.1, 1.1) ∩ (3.3, 2.3, 1.3) = Ø,

weisen die (27 mal 26 / 2) = 351 möglichen Paarrelationen des vollständigen semiotischen 27er-Systems zahlreiche Nullstellen, d.h. semiotische Diskonnexitäten, auf, deren Struktur, Verteilung und semiotische Relevanz bislang überhaupt nicht entdeckt, geschweige denn untersucht worden ist. Im vorliegenden ersten Teil unserer Untersuchungen zu Nullstellen bei Paaren dualer semiotischer Relationen geben wir das System in generativ-semiosischer Ordnung aller 351 Paare wieder.

2.1. Semiotische Konnexionen zwischen DS(1) und DS(n) mit n > 1

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
				Ø		Ø		
DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
				Ø		Ø		
DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
			Ø				Ø	
DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
			Ø	Ø		Ø	Ø	
DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
			Ø	Ø		Ø	Ø	
DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
			Ø				Ø	
DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
			Ø	Ø		Ø	Ø	
DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
			Ø	Ø		Ø	Ø	
DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø						Ø
DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø		Ø		Ø		Ø
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø		Ø		Ø		Ø
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø	Ø				Ø	Ø
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø	Ø				Ø	Ø
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø						Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3
DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø		Ø		Ø		Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø		Ø		Ø		Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø						Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø	Ø				Ø	Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

2.2. Semiotische Konnexionen zwischen DS(2) und DS(n) mit $n > 2$

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
				Ø		Ø		
DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
			Ø	Ø		Ø	Ø	
DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
			Ø				Ø	
DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
			Ø	Ø		Ø	Ø	
DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
			Ø	Ø		Ø	Ø	
DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
			Ø				Ø	
DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
			Ø	Ø		Ø	Ø	
DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø		Ø		Ø		Ø
DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø						Ø
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø		Ø		Ø		Ø
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø	Ø				Ø	Ø
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø	Ø				Ø	Ø
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø		Ø		Ø		Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø						Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø		Ø		Ø		Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø	Ø				Ø	Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø	Ø				Ø	Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

2.3. Semiotische Konnexionen zwischen DS(3) und DS(n) mit n > 3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
			Ø	Ø		Ø	Ø	
DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
			Ø	Ø		Ø	Ø	
DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
			Ø				Ø	
DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
			Ø	Ø		Ø	Ø	
DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
			Ø	Ø		Ø	Ø	
DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
			Ø				Ø	
DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø		Ø		Ø		Ø
DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø		Ø		Ø		Ø
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø						Ø
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø	Ø				Ø	Ø
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø	Ø				Ø	Ø
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø		Ø		Ø		Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø		Ø		Ø		Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø						Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø	Ø				Ø	Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø	Ø				Ø	Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

2.4. Semiotische Konnexionen zwischen DS(4) und DS(n) mit $n > 4$

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
				Ø		Ø		
DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
				Ø		Ø		
DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
			Ø				Ø	
DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
			Ø	Ø		Ø	Ø	
DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
			Ø	Ø		Ø	Ø	
DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø	Ø				Ø	Ø
DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø						Ø
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø		Ø		Ø		Ø
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø		Ø		Ø		Ø
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø	Ø				Ø	Ø
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø	Ø				Ø	Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø						Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø		Ø		Ø		Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø		Ø		Ø		Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø	Ø				Ø	Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

2.5. Semiotische Konnexionen zwischen DS(5) und DS(n) mit $n > 5$

DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
				Ø		Ø		
DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3

DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
			Ø	Ø		Ø	Ø	
DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
			Ø				Ø	
DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
			Ø	Ø		Ø	Ø	
DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø	Ø				Ø	Ø
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3

DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3

DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø		Ø		Ø		Ø
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3

DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø						Ø
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3

DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø		Ø		Ø		Ø
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3

DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3

DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø	Ø				Ø	Ø
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3

DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3

DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3

DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø	Ø				Ø	Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3

DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø		Ø		Ø		Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3
DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø						Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3
DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø		Ø		Ø		Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3
DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3
DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø	Ø				Ø	Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

2.6. Semiotische Konnexionen zwischen DS(6) und DS(n) mit $n > 6$

DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
			Ø	Ø		Ø	Ø	
DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3

DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
			Ø	Ø		Ø	Ø	
DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3

DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
			Ø				Ø	
DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3

DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3

DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø	Ø				Ø	Ø
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø		Ø		Ø		Ø
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø		Ø		Ø		Ø
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3
DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø						Ø
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3

DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø	Ø				Ø	Ø
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3
DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3

DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø	Ø				Ø	Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø		Ø		Ø		Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø		Ø		Ø		Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø						Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø	Ø				Ø	Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

2.7. Semiotische Konnexionen zwischen DS(7) und DS(n) mit n > 7

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
				Ø		Ø		
DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
				Ø		Ø		
DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø	Ø				Ø	Ø
DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø	Ø				Ø	Ø
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø						Ø
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø		Ø		Ø		Ø
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø		Ø		Ø		Ø
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø	Ø				Ø	Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø	Ø				Ø	Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø						Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø		Ø		Ø		Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø		Ø		Ø		Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

2.8. Semiotische Konnexionen zwischen DS(8) und DS(n) mit n > 8

DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
				Ø		Ø		
DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3

DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3

DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø	Ø				Ø	Ø
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3

DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3
DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø		Ø		Ø		Ø
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3

DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø						Ø
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø		Ø		Ø		Ø
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3
DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø	Ø				Ø	Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3
DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø	Ø				Ø	Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø		Ø		Ø		Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø						Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø		Ø		Ø		Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

2.9. Semiotische Konnexionen zwischen DS(9) und DS(n) mit $n > 9$

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø	Ø				Ø	Ø
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3
DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø	Ø				Ø	Ø
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø		Ø		Ø		Ø
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø		Ø		Ø		Ø
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø						Ø
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø	Ø				Ø	Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø	Ø				Ø	Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø		Ø		Ø		Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø		Ø		Ø		Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø						Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

2.10. Semiotische Konnexionen zwischen DS(10) und DS(n) mit $n > 10$

DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
				Ø		Ø		
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3

DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
				Ø		Ø		
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3

DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
			Ø				Ø	
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3

DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
			Ø	Ø		Ø	Ø	
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3

DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
			Ø	Ø		Ø	Ø	
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3

DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
			Ø				Ø	
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3

DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
			Ø	Ø		Ø	Ø	
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3

DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
			Ø	Ø		Ø	Ø	
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3

DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
		Ø						Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3

DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
				Ø		Ø		
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3

DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
				Ø		Ø		
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
		Ø	Ø				Ø	Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
		Ø	Ø				Ø	Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

2.11. Semiotische Konnexionen zwischen DS(11) und DS(n) mit $n > 11$

DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
				Ø		Ø		
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3

DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
			Ø	Ø		Ø	Ø	
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3

DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
			Ø				Ø	
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3

DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
			Ø	Ø		Ø	Ø	
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3

DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
			Ø	Ø		Ø	Ø	
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
			Ø				Ø	
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
			Ø	Ø		Ø	Ø	
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
		Ø		Ø		Ø		Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
		Ø						Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3

DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
		Ø		Ø		Ø		Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
		Ø	Ø				Ø	Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
		Ø	Ø				Ø	Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

2.12. Semiotische Konnexionen zwischen DS(12) und DS(n) mit $n > 12$

DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
			Ø	Ø		Ø	Ø	
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3

DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
			Ø	Ø		Ø	Ø	
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3

DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
			Ø				Ø	
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3

DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
			Ø	Ø		Ø	Ø	
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
			Ø	Ø		Ø	Ø	
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
			Ø				Ø	
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
		Ø		Ø		Ø		Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
		Ø		Ø		Ø		Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3

DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
		Ø						Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
		Ø	Ø				Ø	Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
		Ø	Ø				Ø	Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

2.13. Semiotische Konnexionen zwischen DS(13) und DS(n) mit n > 13

DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
				Ø		Ø		
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3

DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
				Ø		Ø		
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3

DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
			Ø				Ø	
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3

DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
			Ø	Ø		Ø	Ø	
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3

DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
			Ø	Ø		Ø	Ø	
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3

DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
		Ø	Ø				Ø	Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3

DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3

DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
		Ø						Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
		Ø		Ø		Ø		Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
		Ø		Ø		Ø		Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
		Ø	Ø				Ø	Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

2.14. Semiotische Konnexionen zwischen DS(14) und DS(n) mit n > 14

DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3
				Ø		Ø		
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3

DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3
			Ø	Ø		Ø	Ø	
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3

DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3
			Ø				Ø	
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3

DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3
			Ø	Ø		Ø	Ø	
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3

DS(14)	=	3.2	2.2	1.2	$\times$	2.1	2.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(19)	=	3.3	2.1	1.1	$\times$	1.1	1.2	3.3

DS(14)	=	3.2	2.2	1.2	$\times$	2.1	2.2	2.3
		Ø	Ø				Ø	Ø
DS(20)	=	3.3	2.1	1.2	$\times$	2.1	1.2	3.3

DS(14)	=	3.2	2.2	1.2	$\times$	2.1	2.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(21)	=	3.3	2.1	1.3	$\times$	3.1	1.2	3.3

DS(14)	=	3.2	2.2	1.2	$\times$	2.1	2.2	2.3
		Ø		Ø		Ø		Ø
DS(22)	=	3.3	2.2	1.1	$\times$	1.1	2.2	3.3

DS(14)	=	3.2	2.2	1.2	$\times$	2.1	2.2	2.3
		Ø						Ø
DS(23)	=	3.3	2.2	1.2	$\times$	2.1	2.2	3.3

DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3
		Ø		Ø		Ø		Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3
		Ø	Ø				Ø	Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

2.15. Semiotische Konnexionen zwischen DS(15) und DS(n) mit n > 15

DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
			Ø	Ø		Ø	Ø	
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3

DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
			Ø	Ø		Ø	Ø	
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
			Ø				Ø	
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
		Ø	Ø				Ø	Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
		Ø		Ø		Ø		Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
		Ø		Ø		Ø		Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
		Ø						Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
		Ø	Ø				Ø	Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

2.16. Semiotische Konnexionen zwischen DS(16) und DS(n) mit n > 16

DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
				Ø		Ø		
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3

DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
				Ø		Ø		
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3

DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
		Ø	Ø				Ø	Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3

DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3

DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
		Ø	Ø				Ø	Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
		Ø						Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
		Ø		Ø		Ø		Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
		Ø		Ø		Ø		Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

2.17. Semiotische Konnexionen zwischen DS(17) und DS(n) mit $n > 17$

DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
				Ø		Ø		
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3

DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3

DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
		Ø	Ø				Ø	Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3

DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
		Ø	Ø				Ø	Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
		Ø		Ø		Ø		Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
		Ø						Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
		Ø		Ø		Ø		Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

2.18. Semiotische Konnexionen zwischen DS(18) und DS(n) mit $n > 18$

DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3

DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3

DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
		Ø	Ø				Ø	Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
		Ø	Ø				Ø	Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
		Ø		Ø		Ø		Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
		Ø		Ø		Ø		Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
		Ø						Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

2.19. Semiotische Konnexionen zwischen DS(19) und DS(n) mit $n > 19$

DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3
				Ø		Ø		
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3

DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3
				Ø		Ø		
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3
			Ø				Ø	
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3
			Ø	Ø		Ø	Ø	
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3
			Ø	Ø		Ø	Ø	
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3
			Ø				Ø	
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3
			Ø	Ø		Ø	Ø	
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3
			Ø	Ø		Ø	Ø	
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

2.20. Semiotische Konnexionen zwischen DS(20) und DS(n) mit $n > 20$

DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3
				Ø		Ø		
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3
			Ø	Ø		Ø	Ø	
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3
			Ø				Ø	
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3
			Ø	Ø		Ø	Ø	
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3
			Ø	Ø		Ø	Ø	
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3
			Ø				Ø	
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3
			Ø	Ø		Ø	Ø	
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

2.21. Semiotische Konnexionen zwischen DS(21) und DS(n) mit n > 21

DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3
			Ø	Ø		Ø	Ø	
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3
			Ø	Ø		Ø	Ø	
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3
			Ø				Ø	
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3
			Ø	Ø		Ø	Ø	
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3
			Ø	Ø		Ø	Ø	
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3
			Ø				Ø	
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

2.22. Semiotische Konnexionen zwischen DS(22) und DS(n) mit n > 22

DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3
				Ø		Ø		
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3
				Ø		Ø		
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3
			Ø				Ø	
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3
			Ø	Ø		Ø	Ø	
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3
			Ø	Ø		Ø	Ø	
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

2.23. Semiotische Konnexionen zwischen DS(23) und DS(n) mit $n > 23$

DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3
				Ø		Ø		
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3
			Ø	Ø		Ø	Ø	
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3
			Ø				Ø	
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3
			Ø	Ø		Ø	Ø	
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

2.24. Semiotische Konnexionen zwischen DS(24) und DS(n) mit n > 24

DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3
			Ø	Ø		Ø	Ø	
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3
			Ø	Ø		Ø	Ø	
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3
			Ø				Ø	
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

2.25. Semiotische Konnexionen zwischen DS(25) und DS(n) mit n > 25

DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3
				Ø		Ø		
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3
				Ø		Ø		
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

2.26. Semiotische Konnexionen zwischen DS(26) und DS(n) mit $n > 26$

DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3
				Ø		Ø		
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

Literatur

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Bense, Max, Die Eigenrealität der Zeichen. Baden-Baden 1992

Distribution der Strukturen von Nullstellen in semiotischen Dualsystemen

1. Bekanntlich ist das dualsymmetrische, durch die Eigenrealitätsklasse determinierte sog. peirce-bensesche Zehnersystem (vgl. Bense 1992, S. 76) nur ein Ausschnitt aus der Gesamtmenge der über S = (3.x, 2.y, 1.z) mit x, y, z ∈ {1, 2, 3) erzeugbaren 3^3 = 27 semiotischen Relationen, die, vermöge der bereits durch Bense (1975) eingeführten Dualitätsoperation, in zweifacher Form, nämlich als eine die Subjektposition kodierende Zeichenklasse und eine die Objektposition kodierende Realitätsklasse, aufscheint. Während im semiotischen 10er-System nur die drei Zeichenklassen, deren Realitätsklassen homogene entitätische Realitäten thematisieren, paarweise vollständig Nullstellen aufweisen, d.h.

(3.1, 2.1, 1.1) ∩ (3.2, 2.2, 1.2) = Ø

(3.2, 2.2, 1.2) ∩ (3.3, 2.3, 1.3) = Ø

(3.1, 2.1, 1.1) ∩ (3.3, 2.3, 1.3) = Ø,

weisen die (27 mal 26 / 2) = 351 möglichen Paarrelationen des vollständigen semiotischen 27er-Systems zahlreiche Nullstellen, d.h. semiotische Diskonnexitäten, auf, deren Struktur, Verteilung und semiotische Relevanz bislang überhaupt nicht entdeckt, geschweige denn untersucht worden ist. Im Anschluß an unsere Untersuchungen zu Nullstellen bei Paaren dualer semiotischer Relationen (vgl. Toth 2016) geben im folgenden die Distribution der Strukturen von Nullstellen in semiotischen Dualsystemen für alle 351 Paare wieder.

2.1. Einfache semiotische Nullstellen

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
				Ø		Ø		
DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
				Ø		Ø		
DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
			Ø				Ø	
DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
			Ø				Ø	
DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø						Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3
DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø						Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø						Ø
DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
				Ø		Ø		
DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
			Ø				Ø	
DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
			Ø				Ø	
DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø						Ø
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø						Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3
DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
			Ø				Ø	
DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
			Ø				Ø	
DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø						Ø
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø						Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
				Ø		Ø		
DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
				Ø		Ø		
DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
			Ø				Ø	
DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø						Ø
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø						Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
				Ø		Ø		
DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3

DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
			Ø				Ø	
DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3

DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø						Ø
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3

DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø						Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
			Ø				Ø	
DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3

DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø						Ø
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø						Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3
DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
				Ø		Ø		
DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
				Ø		Ø		
DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø						Ø
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø						Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3
DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
				Ø		Ø		
DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø						Ø
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø						Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3
DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø						Ø
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø						Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3
DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
				Ø		Ø		
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
				Ø		Ø		
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
			Ø				Ø	
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
			Ø				Ø	
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3

DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
		Ø						Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3

DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
				Ø		Ø		
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3

DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
				Ø		Ø		
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
				Ø		Ø		
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3

DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
			Ø				Ø	
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3

DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
			Ø				Ø	
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
		Ø						Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
			Ø				Ø	
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
			Ø				Ø	
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
		Ø						Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
				Ø		Ø		
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3

DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
				Ø		Ø		
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3

DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
			Ø				Ø	
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3

DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
		Ø						Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3
				Ø		Ø		
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3

DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3
			Ø				Ø	
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3

DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3
		Ø						Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
			Ø				Ø	
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3

DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
		Ø						Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
				Ø		Ø		
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3

DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
				Ø		Ø		
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
		Ø						Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
				Ø		Ø		
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
		Ø						Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
		Ø						Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3
				Ø		Ø		
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3

DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3
				Ø		Ø		
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3
			Ø				Ø	
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3
			Ø				Ø	
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3
				Ø		Ø		
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3
			Ø				Ø	
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3
			Ø				Ø	
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3
			Ø				Ø	
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3
			Ø				Ø	
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3
				Ø		Ø		
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3
				Ø		Ø		
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3
			Ø				Ø	
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3
				Ø		Ø		
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3
			Ø				Ø	
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3
			Ø				Ø	
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3
				Ø		Ø		
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3
				Ø		Ø		
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3
				Ø		Ø		
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

2.2. Doppelte semiotische Nullstellen

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
			Ø	Ø		Ø	Ø	
DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
			Ø	Ø		Ø	Ø	
DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
			Ø	Ø		Ø	Ø	
DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
			Ø	Ø		Ø	Ø	
DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø		Ø		Ø		Ø
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø		Ø		Ø		Ø
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø	Ø				Ø	Ø
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø	Ø				Ø	Ø
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø		Ø		Ø		Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3
DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø		Ø		Ø		Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3
DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø	Ø				Ø	Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3
DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
			Ø	Ø		Ø	Ø	
DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
			Ø	Ø		Ø	Ø	
DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
			Ø	Ø		Ø	Ø	
DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
			Ø	Ø		Ø	Ø	
DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø		Ø		Ø		Ø
DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø		Ø		Ø		Ø
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø	Ø				Ø	Ø
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3
DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø	Ø				Ø	Ø
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø		Ø		Ø		Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3
DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø		Ø		Ø		Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3
DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø		Ø		Ø		Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø	Ø				Ø	Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø	Ø				Ø	Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
			Ø	Ø		Ø	Ø	
DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
			Ø	Ø		Ø	Ø	
DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
			Ø	Ø		Ø	Ø	
DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
			Ø	Ø		Ø	Ø	
DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø		Ø		Ø		Ø
DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø		Ø		Ø		Ø
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø	Ø				Ø	Ø
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø	Ø				Ø	Ø
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø		Ø		Ø		Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø		Ø		Ø		Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø	Ø				Ø	Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø	Ø				Ø	Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
			Ø	Ø		Ø	Ø	
DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
			Ø	Ø		Ø	Ø	
DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø	Ø				Ø	Ø
DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø		Ø		Ø		Ø
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3
DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø		Ø		Ø		Ø
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø	Ø				Ø	Ø
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø	Ø				Ø	Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø		Ø		Ø		Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø		Ø		Ø		Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø	Ø				Ø	Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
			Ø	Ø		Ø	Ø	
DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3

DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
			Ø	Ø		Ø	Ø	
DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3

DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø	Ø				Ø	Ø
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3

DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø		Ø		Ø		Ø
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3

DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø		Ø		Ø		Ø
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3

DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø	Ø				Ø	Ø
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3

DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø	Ø				Ø	Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3
DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø		Ø		Ø		Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3
DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø		Ø		Ø		Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3
DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø	Ø				Ø	Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3
DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
			Ø	Ø		Ø	Ø	
DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3

DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
			Ø	Ø		Ø	Ø	
DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3

DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø	Ø				Ø	Ø
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3

DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø		Ø		Ø		Ø
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3

DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø		Ø		Ø		Ø
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3

DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø	Ø				Ø	Ø
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3

DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø	Ø				Ø	Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3
DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø		Ø		Ø		Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3
DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø		Ø		Ø		Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3
DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø	Ø				Ø	Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3
DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø	Ø				Ø	Ø
DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø	Ø				Ø	Ø
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø		Ø		Ø		Ø
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø		Ø		Ø		Ø
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø	Ø				Ø	Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø	Ø				Ø	Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø		Ø		Ø		Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3
DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø		Ø		Ø		Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3
DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø	Ø				Ø	Ø
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø		Ø		Ø		Ø
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø		Ø		Ø		Ø
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3

DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø	Ø				Ø	Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3

DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø	Ø				Ø	Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø		Ø		Ø		Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø		Ø		Ø		Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø	Ø				Ø	Ø
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø	Ø				Ø	Ø
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø		Ø		Ø		Ø
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø		Ø		Ø		Ø
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø	Ø				Ø	Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø	Ø				Ø	Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø		Ø		Ø		Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø		Ø		Ø		Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
			Ø	Ø		Ø	Ø	
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3

DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
			Ø	Ø		Ø	Ø	
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3

DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
			Ø	Ø		Ø	Ø	
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3

DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
			Ø	Ø		Ø	Ø	
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
		Ø	Ø				Ø	Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3
DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
		Ø	Ø				Ø	Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
			Ø	Ø		Ø	Ø	
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
			Ø	Ø		Ø	Ø	
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3

DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
			Ø	Ø		Ø	Ø	
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3

DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
			Ø	Ø		Ø	Ø	
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3

DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
		Ø		Ø		Ø		Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3

DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
		Ø		Ø		Ø		Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
		Ø	Ø				Ø	Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
		Ø	Ø				Ø	Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
			Ø	Ø		Ø	Ø	
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
			Ø	Ø		Ø	Ø	
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
			Ø	Ø		Ø	Ø	
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
			Ø	Ø		Ø	Ø	
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3

DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
		Ø		Ø		Ø		Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3

DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
		Ø		Ø		Ø		Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3

DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
		Ø	Ø				Ø	Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
		Ø	Ø				Ø	Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
			Ø	Ø		Ø	Ø	
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3

DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
			Ø	Ø		Ø	Ø	
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
		Ø	Ø				Ø	Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
		Ø		Ø		Ø		Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
		Ø		Ø		Ø		Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
		Ø	Ø				Ø	Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3
			Ø	Ø		Ø	Ø	
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3
			Ø	Ø		Ø	Ø	
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3
		Ø	Ø				Ø	Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3
		Ø		Ø		Ø		Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3
		Ø		Ø		Ø		Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3
		Ø	Ø				Ø	Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
			Ø	Ø		Ø	Ø	
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3

DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
			Ø	Ø		Ø	Ø	
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3

DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
		Ø	Ø				Ø	Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
		Ø		Ø		Ø		Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
		Ø		Ø		Ø		Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
		Ø	Ø				Ø	Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
		Ø	Ø				Ø	Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3

DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
		Ø	Ø				Ø	Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
		Ø		Ø		Ø		Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
		Ø		Ø		Ø		Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
		Ø	Ø				Ø	Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3

DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
		Ø	Ø				Ø	Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
		Ø		Ø		Ø		Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
		Ø		Ø		Ø		Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
		Ø	Ø				Ø	Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
		Ø	Ø				Ø	Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
		Ø		Ø		Ø		Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
		Ø		Ø		Ø		Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3
			Ø	Ø		Ø	Ø	
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3
			Ø	Ø		Ø	Ø	
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3
			Ø	Ø		Ø	Ø	
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3
			Ø	Ø		Ø	Ø	
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3
			Ø	Ø		Ø	Ø	
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3
			Ø	Ø		Ø	Ø	
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3
			Ø	Ø		Ø	Ø	
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3
			Ø	Ø		Ø	Ø	
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3
			Ø	Ø		Ø	Ø	
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3
			Ø	Ø		Ø	Ø	
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3
			Ø	Ø		Ø	Ø	
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3
			Ø	Ø		Ø	Ø	
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3
			Ø	Ø		Ø	Ø	
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3
			Ø	Ø		Ø	Ø	
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3
			Ø	Ø		Ø	Ø	
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3
			Ø	Ø		Ø	Ø	
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3
			Ø	Ø		Ø	Ø	
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3
			Ø	Ø		Ø	Ø	
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

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DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(1)	=	3.1	2.1	1.1	×	1.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
DS(2)	=	3.1	2.1	1.2	×	2.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(2)	=	3.1	2.1	1.2	$\times$	2.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(24)	=	3.3	2.2	1.3	$\times$	3.1	2.2	3.3
DS(2)	=	3.1	2.1	1.2	$\times$	2.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(25)	=	3.3	2.3	1.1	$\times$	1.1	3.2	3.3
DS(2)	=	3.1	2.1	1.2	$\times$	2.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(27)	=	3.3	2.3	1.3	$\times$	3.1	3.2	3.3
DS(3)	=	3.1	2.1	1.3	$\times$	3.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(13)	=	3.2	2.2	1.1	$\times$	1.1	2.2	2.3
DS(3)	=	3.1	2.1	1.3	$\times$	3.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(14)	=	3.2	2.2	1.2	$\times$	2.1	2.2	2.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(3)	=	3.1	2.1	1.3	×	3.1	1.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3
DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(4)	=	3.1	2.2	1.1	×	1.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3

DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3
DS(5)	=	3.1	2.2	1.2	×	2.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(5)	=	3.1	2.2	1.2	$\times$	2.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(25)	=	3.3	2.3	1.1	$\times$	1.1	3.2	3.3

DS(5)	=	3.1	2.2	1.2	$\times$	2.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(27)	=	3.3	2.3	1.3	$\times$	3.1	3.2	3.3

DS(6)	=	3.1	2.2	1.3	$\times$	3.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(10)	=	3.2	2.1	1.1	$\times$	1.1	1.2	2.3

DS(6)	=	3.1	2.2	1.3	$\times$	3.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(11)	=	3.2	2.1	1.2	$\times$	2.1	1.2	2.3

DS(6)	=	3.1	2.2	1.3	$\times$	3.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(16)	=	3.2	2.3	1.1	$\times$	1.1	3.2	2.3

DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3

DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3

DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3

DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(6)	=	3.1	2.2	1.3	×	3.1	2.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(7)	=	3.1	2.3	1.1	×	1.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3

DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3

DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3

DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3

DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3

DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3

DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(8)	=	3.1	2.3	1.2	×	2.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(9)	=	3.1	2.3	1.3	×	3.1	3.2	1.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3
DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3
DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3
DS(10)	=	3.2	2.1	1.1	×	1.1	1.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3
DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(11)	=	3.2	2.1	1.2	×	2.1	1.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(22)	=	3.3	2.2	1.1	×	1.1	2.2	3.3

DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3
DS(12)	=	3.2	2.1	1.3	×	3.1	1.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3
DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(13)	=	3.2	2.2	1.1	×	1.1	2.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3
DS(14)	=	3.2	2.2	1.2	×	2.1	2.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(27)	=	3.3	2.3	1.3	×	3.1	3.2	3.3

DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3

DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3

DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(25)	=	3.3	2.3	1.1	×	1.1	3.2	3.3

DS(15)	=	3.2	2.2	1.3	×	3.1	2.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(26)	=	3.3	2.3	1.2	×	2.1	3.2	3.3

DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(20)	=	3.3	2.1	1.2	×	2.1	1.2	3.3

DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

DS(16)	=	3.2	2.3	1.1	×	1.1	3.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(24)	=	3.3	2.2	1.3	×	3.1	2.2	3.3

DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(19)	=	3.3	2.1	1.1	×	1.1	1.2	3.3

DS(17)	=	3.2	2.3	1.2	×	2.1	3.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(21)	=	3.3	2.1	1.3	×	3.1	1.2	3.3

$$
\begin{array}{llllllllll}
DS(17) & = & 3.2 & 2.3 & 1.2 & \times & 2.1 & 3.2 & 2.3 \\
 & & \emptyset & \emptyset & \emptyset & & \emptyset & \emptyset & \emptyset \\
DS(22) & = & 3.3 & 2.2 & 1.1 & \times & 1.1 & 2.2 & 3.3
\end{array}
$$

$$
\begin{array}{llllllllll}
DS(17) & = & 3.2 & 2.3 & 1.2 & \times & 2.1 & 3.2 & 2.3 \\
 & & \emptyset & \emptyset & \emptyset & & \emptyset & \emptyset & \emptyset \\
DS(24) & = & 3.3 & 2.2 & 1.3 & \times & 3.1 & 2.2 & 3.3
\end{array}
$$

$$
\begin{array}{llllllllll}
DS(18) & = & 3.2 & 2.3 & 1.3 & \times & 3.1 & 3.2 & 2.3 \\
 & & \emptyset & \emptyset & \emptyset & & \emptyset & \emptyset & \emptyset \\
DS(19) & = & 3.3 & 2.1 & 1.1 & \times & 1.1 & 1.2 & 3.3
\end{array}
$$

$$
\begin{array}{llllllllll}
DS(18) & = & 3.2 & 2.3 & 1.3 & \times & 3.1 & 3.2 & 2.3 \\
 & & \emptyset & \emptyset & \emptyset & & \emptyset & \emptyset & \emptyset \\
DS(20) & = & 3.3 & 2.1 & 1.2 & \times & 2.1 & 1.2 & 3.3
\end{array}
$$

$$
\begin{array}{llllllllll}
DS(18) & = & 3.2 & 2.3 & 1.3 & \times & 3.1 & 3.2 & 2.3 \\
 & & \emptyset & \emptyset & \emptyset & & \emptyset & \emptyset & \emptyset \\
DS(22) & = & 3.3 & 2.2 & 1.1 & \times & 1.1 & 2.2 & 3.3
\end{array}
$$

DS(18)	=	3.2	2.3	1.3	×	3.1	3.2	2.3
		Ø	Ø	Ø		Ø	Ø	Ø
DS(23)	=	3.3	2.2	1.2	×	2.1	2.2	3.3

Literatur

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Bense, Max, Die Eigenrealität der Zeichen. Baden-Baden 1992

Toth, Alfred, Nullstellen bei Paaren dualer semiotischer Relationen. In: Electronic Journal for Mathematical Semiotics, 2016

Neben- und hauptdiagonale semiotische Transjazenz

1. Die folgende Tabelle aus Bense (1992, S. 76) zeigt das von Walther (1982) entdeckte determinantentensymmetrische Dualitätssystem.

Zkl	Rth	Rpw	
3.1 2.1 1.1	1.1 1.2 1.3	9	
3.1 2.1 1.2	2.1 1.2 1.3	10	Mittel
3.1 2.1 1.3	3.1 1.2 1.3	11	
3.1 2.2 1.2	2.1 2.2 1.3	11	
3.2 2.2 1.2	2.1 2.2 2.3	12	Objekt
3.2 2.2 1.3	3.1 2.2 2.3	13	
3.1 2.3 1.3	3.1 3.2 1.3	13	
3.2 2.3 1.3	3.1 3.2 2.3	14	Interpretant
3.3 2.3 1.3	3.1 3.2 3.3	15	
3.1 2.2 1.3	3.1 2.2 1.3	12	Eigenrealität

Es besagt, daß die dualinvariante Zeichenklasse, die somit mit ihrer Realitätsthematik identisch ist, in mindestens einem und höchstens zwei Subzeichen mit jeder Zeichenklasse und jeder Realitätsthematik des Systems der 10 peirce-benseschen Zeichenklassen und Realitätsthematiken zusammenhängt. Diese Eigenschaft bewog Bense bekanntlich, in der nebendiagonalen Determinanten der kleinen semiotischen Matrix die Zeichenklasse bzw. Realitätsthematik des Zeichens selbst zu sehen und deren Eigenschaft als semiotische Eigenrealtität zu bestimmen.

2. Daß die hauptdiagonale Diskriminante nicht die gleichen Eigenschaften aufweist, wurde in Toth (2008) bewiesen. Das bedeutet somit, daß es kein diskriminantensymmetrisches Dualsystem gibt, insofern die Hauptdiagonale der kleinen semiotischen Matrix nicht mit jeder Zeichenklasse auch nur in einem Subzeichen zusammenhängt. Diese Asymmetrie zwischen determinanter Nebendiagonale und diskriminanter Hauptdiagonale zeigt sich auch in der ortsfunktionalen, d.h. qualitativ-arithmetischen Eigenschaft der Transjazenz,

denn beide semiotischen Diagonalen stellen transjazente Zählstrukturen dar (vgl. Toth 2015).

2.1. Nebendiagonale Transjazenz

3.1	2.1, 2.2, 2.3			1.1, 1.2, 1.3	
	2.2	3.1, 3.2		1.2, 1.3	
		1.3	3.1, 3.2, 3.3		2.1, 2.2, 2.3

2.2. Hauptdiagonale Transjazenz

		1.1	3.1	2.1
	2.2	3.1, 3.2		1.2, 1.3
3.3	2.3	1.3		

Wie man leicht erkennt, sind die beiden transjazenten Darstellungen der Eigen- und der Kategorienrealität relativ zu den zu Zeichenklassen kombinierbaren Subzeichen weder symmetrisch noch kemplementär.

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Objekte, Zeichen und Zahlen

1. Wir benutzen Objekte wie z.B. Kleider, Besen und Autos, wir kommunizieren in Zeichen, weil man nur mit Zeichen kommunizieren kann, und wir rechnen, indem wir etwa unser Geld zählen oder ausrechnen, wieviel Rente wir nach der Pensionierung bekommen. Die drei Entitäten Objekte, Zeichen und Zahlen dürften damit die drei grundlegenden Entitäten überhaupt sein, und alle weiteren Kategorien sind von ihnen abgeleitet. So ist etwa die Substanz ein Teil von Objekten, Relationen sind Beziehungen zwischen Objekten (zu denen auch die Subjekte gehören), Zahlen oder Zeichen. Ich plädiere hier also für drei neue "Fundamentalkategorien" als Substitute der von Peirce eingeführten, die klassischen Kategorientafeln reduzierenden Kategorien der Mittelrealität, der Objektrealität und der Interpretantenrealität. Daß es sich hier nicht um "fundamentale" Kategorien handeln kann, müßte eigentlich bereits Peirce bemerkt haben, denn die Mittelrelaität ist sowohl in der Objekt- als auch in der Interpretantenrealität eingeschlossen, und die Objektrealität ist in der Interpretantenrealität eingeschloßen. Wie ferner Bense (1979, S. 53 u. 67) mit Hilfe der Kategorietheorie bewiesen hatte, enthält sich die triadische Zeichenrelation im ebenfalls triadischen Interpretantenbezug selbst (sonst wäre das Zeichen nicht autoreproduktiv und das peircesche Dualsystem wäre nicht vermöge Eigenrealität determinantensymmetrisch). Bereits bei Peirce gibt es also streng genommen nur eine einzige unserer drei vorgeschlagenen Fundamentalkategorien, nämlich diejenige des Zeichens selbst.

2. Die Vorstellung, daß die Semiotik ein im modelltheoretischen Sinne abgeschlossenes Universum bildet (vgl. Bense 1983), wurde bereits von Bense selbst an zahlreichen Stellen seines Frühwerkes widerlegt. So liest man bereits 1971: "Schließlich stellen Grenzpfähle und Grenzwege, Schlagbäume bzw. Niemandslandstreifen iconische Differenziations- und Vermittlungszeichen zwischen zwei (staatlichen) Situationssystemen dar, denn als Berührungszonen gehören Grenzphänomene zu beiden Situationssystemen, d.h. jeder Grenzpunkt gehört zugleich auch jedem begrenzten Gebiet an und hat als bezeichnendes Zeichen mit seinem Objekt übereinstimmende Merkmale" (Bense 1971, S. 87). Das bedeutet also, daß es zwischen Objekt und Zeichen ein Tertium datur gibt, d.h. einen Rand, der zwar tatsächlich leer sein kann – im

Falle von Arbitrarität -, der aber auch nicht-leer sein kann, wie in Benses Beispiel der Niemandslandstreifen, der sich somit als Menge von Partizipationsrelationen zwischen den Paaren der ihm angrenzenden Gebiete entpuppt. Dasselbe gilt für die Ontik selbst: Ein Zaun, der eine Wiese in zwei separate Teilwiesen trennt, liegt nicht im Nirgendwo und gehört auch nicht einer der beiden Wiesen an, sondern beiden gleichzeitig, d.h. er partizipiert an beiden von ihm getrennten Wiesen – damit aber trennt er diese nicht nur, sondern verbindet sie gleichzeitig.

3. Die den drei fundamentalen Entitäten Objekt, Zeichen und Zahl zugehörigen Wissenschaften sind die Ontik (Objekttheorie), die Semiotik (Zeichentheorie) und die Mathematik (die man entweder auf der Zahl, der Menge oder der Kategorie als Basisbegriff fundieren kann). Während jedoch die Mathematik seit Jahrhunderten an unseren Universitäten durch Lehrstühle und weitere Stellen fest institutionalisiert ist, fristet die Semiotik heute allenfalls noch als eine besondere Form der Hermeneutik ein Schattendasein innerhalb der Literaturwissenschaft einerseits und innerhalb der Architektur andererseits. Die Ontik ist überhaupt nicht vertreten, und in diesem Falle liegt die Schuld bei der Semiotik peircescher Provenienz, die behauptet, wir könnten "Realität" nicht anders als durch Zeichen wahrnehmen. In Wahrheit ist aber ein wahrgenommenes Objekt noch kein Zeichen, denn die thetische Setzung von Zeichen bedingt einen willentlichen Akt, die Wahrnehmung von Objekten geschieht aber unwillentlich. Wahrgenommene Objekte sind subjektive Objekte (da sie ja nur von Subjekten wahrgenommen werden können), Zeichen aber sind objektive Subjekte (da sie ja nur von Subjekten auf Objekte abgebildet werden können). Statt der primitiven aristotelischen logischen Dichotomie von Objekten und Zeichen bzw. Objekten und Subjekten sollte man also eine Logik konstruieren, die von subjektiven Objekten und objektiven Subjekten ausgeht. Erst eine solche Logik wäre mit der Trias von Objekten, Zeichen und Zahlen und dadurch mit Ontik, Semiotik und Mathematik kompatibel.

4. Die übrigen Wissenschaften sind folglich von Ontik, Semiotik und Mathematik abgeleitet. Es besteht somit eine ähnliche Situation wie sie die Bourbakis in der Mathematik geschaffen haben, indem sie alle mathematischen Einzeldisziplinen aus Kombinationen von Algebra, Ordnungstheorie und Topologie

abgeleitet eingeführt hatten. Daß wir heute soweit sind, daß wir Fächer wie "Umweltnaturwissenschaften" haben, die alleine auf praktische Zwecke hin zusammengeschustert sind aus Fragmenten von zahlreichen und selbst wiederum abgeleiteten Wissenschaften, deren Grundlagen die Studierenden gar nicht nachvollziehen können, läßt sich mit einem Koch vergleichen, der keine Ahnung von der Herstellung der von ihm verwendeten Grundprodukte hat. Jemand, der nur weiß, wie man Fertigprodukte hantiert, versteht damit auch nicht, was er überhaupt tut. Erkenntnis und nicht Kenntnis ist aber der Zweck aller Wissenschaft – und selbst der Gebiete, die zu den Nicht-Wissenschaften gehören. An der Misere nicht nur der deutschen, sondern der internationalen Wissenschaft wird sich also nichts ändern, bevor nicht nur die Mathematik, sondern auch die Semiotik und die Ontik durch Lehrstühle institutionalisiert werden und Studenten aller abgeleiteten Fächer wenigstens deren Grundlagen vermittelt werden. Man kann in der Fächertrias von Ontik, Semiotik und Mathematik somit eine neue Kybernetik sehen, welche, jenseits des Geschwätzes der sog. inter- und transdisziplinären Wissenschaft angesiedelt, eine gemeinsame formal-abstrakte Basis nicht nur aller abgeleiteten Einzelwissenschaften, sondern durch Systeme von Isomorphien die zwischen Ontik, Semiotik und Mathematik bestehenden Zusammenhänge liefert.

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Eine imaginäre Zeichenrelation

1. Bekanntlich steht bei de Saussure: "Die Sprache ist sozusagen eine Algebra, die nur komplexe Termini enthält" (1967, S. 146). Helmar Frank hatte sogar die These vertreten, das Zeichen sei eine komplexe Funktion, die zu einem imaginären und einem reellen Grenzwert konvergiere (vgl. dazu Toth 2013). Die Idee einer imaginären Zeichenrelation scheint mir besser zu passen, denn das Zeichen ist seiner Natur nach eine Kopie des realen und damit auch reellen Objektes, mit dem es durch Referenz verbunden ist. Man könnte somit die Semiotik als imaginäre Gegenwelt der reellen Ontik bstimmen.

2. Wir gehen aus von den durch Bense (1981, S. 17 ff.) definierten Zeichenzahlen, von Bense etwas unglücklich als Primzeichen bezeichnet

$Z_{re} = (1, 2, 3)$

und ersetzen diese reelle durch die folgende imaginäre Zeichenzahlenrelation

$Z_{im} = (1, i, -1)$.

Damit können wir folgende neue semiotische Matrix konstruieren

	1	i	-1
1	1	i	-1
i	i	-1	-i
-1	-1	-i	1.

Wie man erkennt, bestehen sowohl die Determinante als auch die Diskriminante der Matrix ausschließlich aus reellen Zeichenzahlen. Es gibt also offenbar weder eine Eigenrealität noch eine Kategorienrealität der Imaginarität.

3. Weil die über Z_{im} im Gegensatz zu der über Z_{re} konstruierten Matrix in Bezug auf die Matrixeinträge redundant ist, ist es möglich, über Z_{im} folgende drei kategorialen Identifikationen zu konstruieren.

3.1. $(1 \equiv i) = (M \equiv O)$

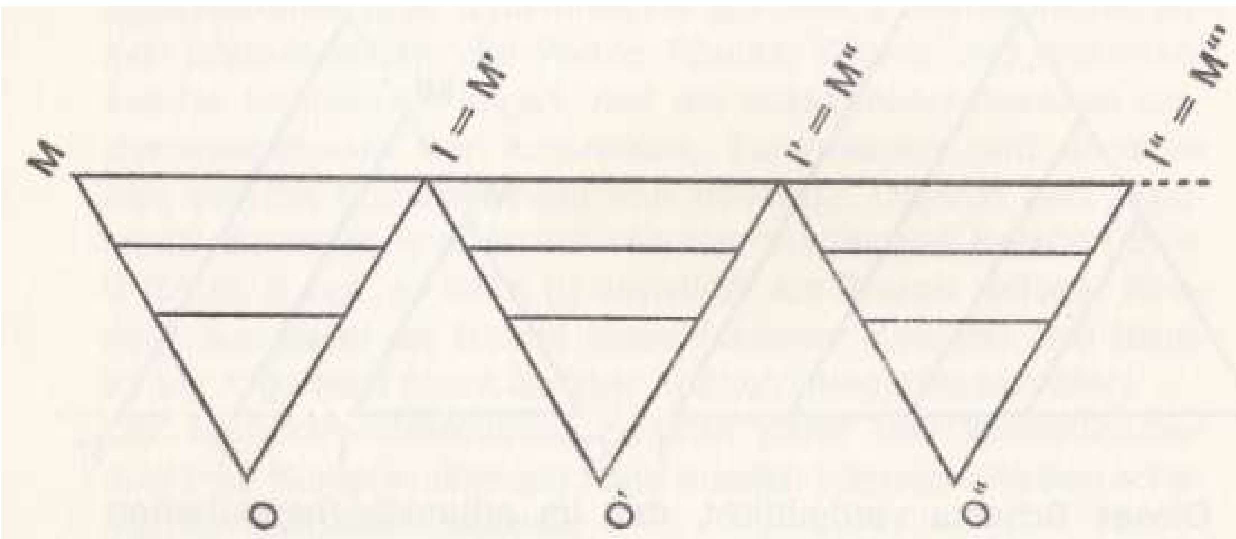
Natürliche Zeichen, Anzeichen, Symptome, Signale, Spuren, Reste.

3.2. (i ≡ -1) = (O ≡ I)

Sprecher-Hörer-Union, Kommunikationstheorie (Informationstheorie).

3.3. (1 ≡ -1) = (M ≡ I)

Allein diese kategoriale Identifikation taucht in Benses Werk auf, und zwar seit Bense (1971, S. 54) in Form des Superisationsschemas



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Bijektion der imaginären und der rellen semiotischen Dualysteme

1. In Toth (2016) hatten wir eine imaginäre Zeichenrelation der Form

Z = (-1, i, 1)

definiert, welche die reelle Zeichenrelation, die Bense (1981, S. 17 ff.) eingeführt hatte, ersetzen könnte, zumal sich de Saussure (1967, S. 146), Frank (2001) und Toth (2013) für die Imaginarität der Zeichenrelation ausgesprochen hatten.

Die zu Z gehörige Matrix ist

	-1	i	1
-1	1	-i	-1
i	-i	-1	i
1	-1	i	1,

2. Im folgenden gehen wir von den dualen Realitätsthematiken der zehn peirce-benseschen Zeichenklassen aus und transformieren sie durch die ebenfalls primen Zeichenzahlen von Z. Diese werden so geordnet, daß die ersten Gruppe alle Tripel umfaßt, deren erstes Glied -1 ist, deren zweite Gruppe alle Tripel umfaßt, deren erstes Glied –i ist, und deren dritte Gruppe alle Tripel umfaßt, deren erstes Glied 1 ist.

3.1 1.2 1.3 → -1, -i, -1

3.1 2.2 1.3 → -1, -1, -1

3.1 2.2 2.3 → -1, -1, i

3.1 3.2 2.3 → -1, i, i

3.1 3.2 1.3 → -1, i, -1

2.1 2.2 1.3 → -i, -1, -1

2.1 1.2 1.3 → -i, -i, -1

2.1 2.2 2.3 → -i, -1, i

1.1 1.2 1.3 → 1, -i, -1

Obwohl zur ersten Gruppe von Tripel gehörig, nimmt

3.1 3.2 3.3 → -1, i, 1 = Z

eine Sonderstellung ein, denn der vollständige Interpretantenbezug fällt in der Notation mit imaginären Primzeichenzahlen mit der Definition des Zeichens zusammen. Man beachte, daß dies bei der imaginären Korrespondenz der Eigenrealitätsklasse 3.1 2.2 1.3 → -1, -1, -1 nicht der Fall ist.

Wie man sogleich erkennt, folgt trotz der "Redundanz" der imaginären Zahlenwerte in der Matrix die Bijektion der imaginären und der reellen Realitätsthematiken, dadurch vermöge Dualität auch die Bijektion imaginärer und reeller Zeichensystemen und somit der imaginären und der reellen semiotischen Dualsysteme.

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Die qualitative Zahl des Zeichens

1. Es gibt zwei völlig verschiedene Ansätze einer semiotischen Mathematik von Max Bense, die, wenn ich recht sehe, sogar dem Großteil seiner Studenten entgangen ist.

1.1. Die Konzeption einer quantitativen semiotischen Mathematik, die etwas bekannter ist, weil sie Bense nicht nur in (1975, S. 168 ff.), sondern auch in seinem Nachweis, daß Peirce die Peano-Axiome vorweggenommen hatte (vgl. Bense 1983, S. 192 ff.), vorgebracht hatte. Darin wird gezeigt, daß man das Zeichen als triadische Relationen mit Hilfe der Peano-Axiome einführen kann. Die Kulmination dieser Vorstellung des Zeichens als quantitativer Zahl stellt dann Benses letztes Werk dar, der Nachweis, daß das semiotische eigenreale Dualsystem auch die "Zahl" selbst repräsentiert (vgl. Bense 1992).

1.2. Die Konzeption einer qualitativen semiotischen Mathematik, die sich hinter der erst 1980 eingeführten Relation der Primzeichen verbirgt (vgl. Bense 1981, S. 17 ff.). Darin nimmt Bense folgende Abbildungen zwischen den Zeichenzahlen und ihren Qualitäten vor, die wir hier in der Form von Sätzen formulieren.

1.2.1. Qualitäten (1) werden kardinal gezählt.

1.2.2. Objekte (2) werden ordinal gezählt.

1.2.3. Konnexe (3) werden relational gezählt.

Die Qualitäten machen in Peirces Einführung des Zeichens den Mittelbezug des Zeichens aus, es wird zwischen reinen, singulären und gesetzmäßig verwendeten Qualitäten unterschieden.

Die Objekte machen in Peirces Einführung des Zeichens den Objektbezug des Zeichens aus, es wird zwischen abbildenden, hinweisenden und arbiträren Objekten unterschieden.

Die Konnexe machen in Peirces Einführung des Zeichens den Interpretantenbezug des Zeichens aus, es wird zwischen offenen, abgeschlossenen und vollständigen Konnexen unterschieden.

2. Tatsächlich ist es aber so, daß alle drei Entitäten, d.h. Qualitäten, Objekte und Konnexe, selbst Qualitäten sind, denn Objekte sind Qualitäten per se, und wenn in der Semiotik von Konnexen die Rede ist, dann von interpretierenden und nicht von rein topologischen, daher auch der Name des Interpretantenbezuges, der die Subjektbeteiligung voraussetzt. Fragen wir uns deshalb, was für Zahlen es sind, welche durch Benses Primzeichen oder besser: Zeichenzahlen gezählt werden.

2.1. Qualitäten, die kardinal gezählt werden

Wir wollen die Entitäten, welche Qualitäten kardinal zählen, einfach als Zahlen bezeichen. Es kann sich um einen Apfel, eine Birne oder eine Pflaume, d.h. um qualitativ differente Objekte, oder um die Zahlen 1, 2 oder 3, d.h. um qualitativ gleiche Objekte, handeln.

2.2. Qualitäten, die ordinal gezählt werden

Wir wollen die Entitäten, welche Qualitäten ordinal zählen, als Anzahlen bezeichnen. Damit kann zum Beispiel eine Menge von Äpfeln, eine Menge von Birnen oder eine Menge von Pflaumen bei qualitativen Objekten oder die Mengen der natürlichen, rationalen oder reellen Zahlen bei quantitativen Objekten abgezählt werden. Die Differenz wischen kardinaler und ordinaler Zählung ist daher diejenige zwischen zählen und abzählen.

2.3. Qualitäten, die relational gezählt werden

Wir wollen die Entitäten, welche Qualitäten relational zählen, als Nummern bezeichnen. Damit kann man zum Beispiel eine Menge von Häusern entlang einer Straße, d.h. Objekte, deren kardinale und ordinale Stellung bereits vorgegeben ist, durch die Abbildung von Nummern identifizieren. Man beachte, daß die Numerierung nicht mit der Abzählung übereinstimmen muß. Daß also zum Beispiel die letzte Haus-Nummer einer Straße 96 ist, bedeutet nicht, dass diese Straße 96 Häuser hat.

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Polyrepräsentativität, Polykontexturalität, Polyvariabilität

1. Monokontexturalität

1.1. Definition

"Beide Werte einer solchen Logik aber sind metaphysisch äquivalent. Das heißt, man kann sie beliebig miteinander vertauschen. Sie verhalten sich zueinander in einer totalen logischen Disjunktion, wie rechts und links. Es gibt keinen theoretischen Grund, welche Seite rechts und welche Seite links von der Zugspitze ist. Die Benennung beruht auf einer willkürlichen Entscheidung, und wenn man seinen Standpunkt wechselt, sind die rechte und die linke Seite miteinander vertauscht (Günther 2000, S. 230 f.).

1.2. Hamiltonkreis

S O

O S.

2. Polyrepräsentativität

"Die für die Ontologie entscheidende Weiterentwicklung, die in diesem Dualisierungsschema liegt, ist, im Vergleich zu allen früheren Ansätzen und auch zu Peirce selbst darin zu sehen, daß der mehr oder minder einheitliche Begriff der Realität ersetzt wird durch eine genaue, formalisierte Ausdifferenzierung in die abzählbar-endliche Anzahl von zehn möglichen Realitätsthematiken" (Bayer 1994, S. 16).

Nach einer Entdeckung von Walther (1982) determiniert dabei eine dieser Realitäten, die mit ihrer Realitätsthematik dualinvariante und damit – monokontextural gesehen – dualidentische Eigen-Realität des Zeichens die übrigen neun Realitätsstufen des zehnstufigen Realitätssystems. Die jüngste originale Darstellung stammt von Bense (1992, S. 76).

Zkl			Rth			Rpw	
3.1	2.1	1.1	1.1	1.2	1.3	9	
3.1	2.1	1.2	2.1	1.2	1.3	10	Mittel
3.1	2.1	1.3	3.1	1.2	1.3	11	
3.1	2.2	1.2	2.1	2.2	1.3	11	
3.2	2.2	1.2	2.1	2.2	2.3	12	Objekt
3.2	2.2	1.3	3.1	2.2	2.3	13	
3.1	2.3	1.3	3.1	3.2	1.3	13	
3.2	2.3	1.3	3.1	3.2	2.3	14	Interpretant
3.3	2.3	1.3	3.1	3.2	3.3	15	
3.1	2.2	1.3	3.1	2.2	1.3	12	Eigenrealität

Die Tatsache, daß es gerade 10 und nicht, wie zu erwarten wäre, $3^3 = 27$ stufige Realitäten gibt, liegt an dem von Peirce eingeführten und völlig willkürlichen Axiom, daß innerhalb einer triadischen Zeichenrelation der Ordnung Z = (3.x, 2.y, 1.z) $x \leqq y \leqq z$ sein muß. Übersehen hat Peirce allerdings, daß die (z.B. von Bense 1992 extensiv behandelte) "Kategorienklasse", die Hauptdiagonale der semiotischen Matrix (3.3, 2.2, 1.1), dieser Inklusionsordnung bereits widerspricht. Da die Semiotik nur über einen Subjektbegriff verfügt, also denjenigen des Ich-Subjektes der aristotelischen Logik, auf der sie beruht, ist sie hinsichtlich Du- und Er-Deixis defektiv, d.h. auch das weitere Peircesche "Axiom", alle n-adischen Relationen mit $n > 3$ ließen sich auf 3-adische reduzieren, ist angesichts der logischen Subjektdeixis falsch. Pikanterweise ist das "Axiom" auch ohne Berücksichtigung von Subjektdeixis falsch, denn Ernst Schröder hatte in einem bekannten Theorem schon zu Peirces Lebenszeit nachgewiesen, daß n-adische Relationen auf dyadische reduzierbar sind.

3. Polykontexturalität

3.1. Definitionen

Die Arithmetik mußte ganz anderes und Wunderbares leisten können, weshalb er an seinen Lehrer die Frage stellte: Wenn das Zusammensein von vielen Bergen ein Gebirge ergab, was ergäbe dann zahlenmäßig das Zusammensein, wenn man eine Kirche zu einem Krokodil addierte und dazu noch seine Mutter und obendrein ein Zahnweh.

Daraus läßt sich nun folgender Schluß ziehen: die primordialen Qualitäten sind ontologische Schnittpunkte ebenso vieler zweiwertiger Universalkontexturen wie wir Qualitätsdifferenzen zählen können. Jede ist von der gleichen Allgemeinheit und Durchgängigkeit wie die monokontexturale Welt des klassischen Universums. jede hat ihre eigene Objektivität; und zwischen je zweien klafft immer wieder der gleiche ontologische Abgrund wie zwischen dem einmaligen Diesseits und dem supranaturalen Jenseits der älteren Philosophie. Der Anspruch der klassischen Logik, die Objektivität der Welt als eine einzige bruchlose Universalkontextur, jenseits der nur das Absolute west, zu verstehen, wird damit ein für allemal bestritten. Die Wirklichkeit, in der wir leben, besitzt keine solche ungebrochene Kontinuität. An jeder Kontexturschranke erlischt ein Gültigkeitsbereich der klassischen Logik, aber in jeder neuen Universalkontextur tritt er mit *verändertem Positionswert* wieder auf. Eine transklassische Logik hat es im wesentlichen mit der Veränderung dieser Positionswerte zu tun.

(aus: Günther 1975)

3.2. Hamiltonkreis

O	O	S^1	S^1	S^2	S^2
S^1	S^2	O	S^2	O	S^1
S^2	S^1	S^2	O	S^1	O

Die polykontexturale Logik hat, wie in Toth (2016) gezeigt wurde, zwei Haupt-Defizite: 1. In ihr läßt sich nur das Subjekt, nicht aber das Objekt iterieren. 2. Somit wird jedem Subjekt eine eigene Logik zuweisbar, aber diese Logiken sind allesamt die 2-wertigen aristotelischen Logiken, es gibt also i.S. keine Vermittlung innerhalb, sondern nur zwischen Kontexturen, da das Tertium non datur die Annahme subjektiver Objekte und objektiver Subjekte anstelle der längst überkommenen objektiven Objekte und subjektiven Subjekte der Monokontexturalität explizit ausschließt.

4. Polyvariabilität

4.1. Definitionen

$L^2 = (O, S) \rightarrow$

$L^2 = [O, S] \neq L^2 = [S, O]$

$L^2 = [[O], S] \neq L\ L^2 =^2 = [S, [O]]$

$L^2 = [O, [S]] \neq L^2 = [[S], O]$.

$E \rightarrow L^n$ entsprechend für $n > 2$.

$L = (O^1, S^1, S^2) \neq (S^1, O^1, O^2)$

$L = (O^1, O^2, S^1, S^2) \neq (O^1, S^1, S^2, S^3) \neq (S^1, O^1, O^2, O^3)$

4.2. Hamiltonkreis

O^1	O^1	O^2	O^2	S^1	S^1
O^2	S^1	O^1	S^1	O^1	O^2
S^1	O^2	S^1	O^1	O^2	O^1

Erst in einer Logik, die in Toth (2016) als "polyvariabel" bezeichnet wurde, lassen sich sich nicht nur das Subjekt, sondern auch das Objekt iterieren, d.h. es gibt nicht nur Negationszyklen der Subjekte, sondern auch Positionszyklen der Objekte. Die polyvariable Logik wird von immenser Bedeutung als Grundlage der längst begonnenen Ontik sein, d.h. der der Semiotik als Zeichentheorie an die Seite gestellten Objekttheorie, denn nach peirce-bensescher Auffassung ist die Semiotik als "Universum der Zeichen" (Bense 1983) ja ein im modelltheoretischen Sinne abgeschlossenes Universum, in dem es keine Objekte und keine Subjekte, sondern nur ihre Relationen gibt. Daß eine solche Pansemiotik bereits Axiom 1 aus Bense (1967), dem ersten wissenschaftlichen Buch der Semiotik, widerspricht, wonach jedes Objekt zum Zeichen erklärt werden kann, wurde sehr überraschenderweise gar nicht bemerkt. Woher soll in einem abgeschlossenen Universum von Objekrelationen ein Objekt kommen? Und genau genommen ist das Axiom ja ohnedies überflüssig, denn eine thetische Setzung eines Objektes als Metaobjekt ist allein deswegen überflüssig, weil wir ja angeblich alles, was wir wahrnehmen, als Zeichen wahrnehmen, eine Auffassung, deren Falschheit wir in zahlreichen Arbeiten nachgewiesen hatten.

Wir können die Ergebnisse dieser Studie also wie folgt kurz zusammenfassen

Logik	Iterierbarkeit von O	Iterierbarkeit von S
aristotelisch	nein	nein
semiotisch	ja	nein
polykontextural	nein	ja
polyvariabel	ja	ja.

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Transrelationale Systeme als Basen für semiotische Matrizen

1. Unter transrelationalen Systemen wollen wir solche verstehen, welche die quantitative Dreiteilung symmetrischer Relationen

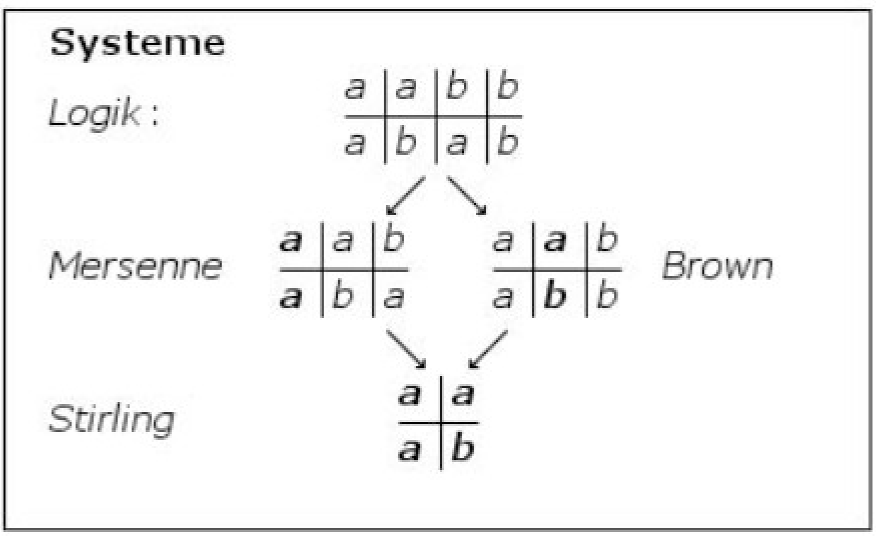
symmetrisch: $\forall x, y \in M : (xRy \Rightarrow yRx)$

asymmetrisch: $\forall x, y \in M : xRy \Rightarrow \neg(yRx)$

antisymmetrisch: $\forall x, y \in M : xRy \wedge yRx \Rightarrow x = y$

verwerfen, indem sie mindestens eine der drei Relationstypen NICHT aufweisen. Dieses Problem gesehen und behandelt zu haben, ist das Verdienst Rudolf Kaehrs (vgl. Kaehr 2012).

2. Von den folgenden vier, von Kaehr (2012, S. 4) dargestellten Systemen erfüllt nur dasjenige von Leibniz bzw. Boole, das der aristotelischen Logik zugrunde liegt, die Dreiteilung symmetrischer Relationen



Dagegen fehlt im System von Mersenne die Differenzierung zwischen [a, a] und [b, b], d.h. es findet ein Identitätenkollaps statt, und im System von Brown gibt es wegen des Fehlens der Differenz von [a, b] und [b, a] keine Antisymmetrie. Das Trito-System der Stirlingzahlen schließlich vereinigt Identitätskollaps und Elimination von Antisymmetrie.

3.1. Es dürfte unmittelbar einleuchten, daß die von Bense (1975, S. 37) eingeführte semiotische Matrix

1.1	1.2	1.3
2.1	2.2	2.3
3.1	3.2	3.3

sowohl Identitätsdifferenz, da

$(1.1) \neq (2.2) \neq (3.3)$

gilt, als auch Antisymmetrie kennt, da

$(1.2) \neq (2.1), (1.3) \neq (3.1), (2.3) \neq (3.2)$

gilt. Dank Kaehrs Entdeckung der vier Systemtypen können wir ferner sagen, daß die von Bense (1992) eingehend behandelte Eigenrealitätsklasse

ZKl = (3.1, 2.1, 1.3)

ihre monokontexturale Dualinvarianz exakt den beiden, Leibniz-boolesche Systemen charakterisierenden, Eigenschaften der Identitätsdifferenzierung und der Antisymmetrie verdankt.

3.2. Dagegen würde eine mersennesche semiotische Matrix wie folgt aussehen

X	1.2	1.3
2.1	X	2.3
3.1	3.2	X

mit X = (1.1, 2.2, 3.3).

3.3. Eine brownsche semiotische Matrix dagegen sähe wie folgt aus

1.1	X	Y
X	2.2	Z
Y	Z	3.3

mit X ∈ (1.2, 2.1), Y ∈ (1.3, 3.1) und Z ∈ (2.3, 3.2).

3.4. Daß von einer stirlingschen semiotischen Matrix keine Rede sein kann, versteht sich von selbst. Eine solche Semiotik könnte man durch

$S = (<x.x>, <x.y>)$

mit x, y ∈ (1, 2, 3) definieren. Dabei würden allerdings vermöge Trito-Äquivalenz folgende Kollapse eintreten

$(1.1) \equiv_t (2.2) \equiv_t (3.3)$

$(1.2) \equiv_t (2.1)$

$(1.3) \equiv_t (3.1)$

$(2.3) \equiv_t (3.2)$.

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Kategorifizierung in der Semiotik

1. Wir gehen aus von der von Kaehr publizierten Tafel kategorietheoretischer Hierarchien (vgl. Kaehr 2007, S. 11)

Viewpoints to Categorification

$$\begin{tikzcd}
\mathit{Set} \arrow[r] \arrow[d, dash] & \mathit{Function} \arrow[d, dash] & : \mathit{Set\ Theory} \\
\mathit{Object} \arrow[r] \arrow[d, dash] & \mathit{Morphism} \arrow[d, dash] & : \mathit{Basic\ Category\ Theory} \\
\mathit{Morphism} \arrow[r] \arrow[d, dash] & \mathit{Functor} \arrow[d, dash] & : \mathit{Category\ Theory} \\
\mathit{Functor} \arrow[r] \arrow[d, dash] & \mathit{Nat\ Transf} \arrow[d, dash] & : \mathit{Functor\ Theory} \\
\mathit{Nat\ Transf} \arrow[r] \arrow[d, dash] & n-\mathit{Category} \arrow[d, dash] & : \mathit{Categorification} \\
n-\mathit{category} \arrow[r, shift left] & (n+1)-\mathit{categorie} \arrow[l, shift left] & : (\mathit{De}-)\mathit{Categorification}
\end{tikzcd}$$

und fragen uns, ob dieses Schema innerhalb der kaehrschen "Graphematik", zu der ja auch die Semiotik gehört, wirklich universell ist.

2.1. Menge → Funktion

Da dieses Thema bereits extensiv von Bense (vgl. Bense 1981, S. 76 ff.) behandelt wurde, können wir es mit diesem Verweis darauf belassen.

2.2. Objekt → Morphismus

2.3. Morphism → Funktor

Morphismen wurden ebenfalls von Bense (1981, S. 124 ff.) in die Semiotik eingeführt, vgl. die zusammenfassende Darstellung in Toth (1997, S. 21 ff.). Grundsätzlich ist zu sagen, daß der mathematische Unterschied zwischen Objekt und Abbildung bereits im Subzeichen angelegt ist, auf dessen Doppelnatur Bense wiederholt hingewiesen hatte, einerseits statisch-entitätisch, andererseits dynamisch-semiosisch zu sein. Z.B. bezeichnet das Icon (2.1) eine Abbildung, aber auch die dyadische Retrosemiose $\alpha° = (2 \rightarrow 1)$. Die kleine semiotische Matrix läßt sich mit Hilfe semiotischer Morphismen wie folgt darstellen

$$\begin{pmatrix} id1 & \alpha & \beta\alpha \\ \alpha° & id2 & \beta \\ \alpha°\beta° & \beta° & id3 \end{pmatrix},$$

und demnach stellt jede Abbildung der 9 Morphismen auf sich selbst oder einen anderen Morphismus in der Semiotik einen Funktor dar.

2.4. Funktor → natürliche Transformation

Wie in Toth (1997, S. 21 ff.) gezeigt, können die Zeichenklassen und Realitätsthematiken als natürliche Transformationen definiert werden. Diese haben also die allgemeine Form

$$\text{ZKl} = \quad ([3. \rightarrow .x] \rightarrow [2. \rightarrow .y]) \rightarrow ([2. \rightarrow .y] \rightarrow [1. \rightarrow .z])$$

$$\text{RTh} = \quad ([z. \rightarrow .1] \rightarrow [y. \rightarrow .2]) \rightarrow ([y. \rightarrow .2] \rightarrow [x. \rightarrow .3]).$$

2.5. Natürliche Transformation → n-Kategorie

Die jüngste Entwicklung innerhalb der Kategorietheorie (vgl. Leinster 2004) findet ihre Entsprechung in der Determination des peirce-benseschen Zehnersystems der Semiotik durch die eigenreale (dual-invariante) Zeichenklasse/ Realitätsthematik, wodurch sich das System als "deerminantensymmetrisches Dualitätssystem" (E. Walther) wie folgt in der Notation von Bense (1992, S. 76) darstellen läßt

Zkl			Rth			Rpw	
3.1	2.1	1.1	1.1	1.2	1.3	9	
3.1	2.1	1.2	2.1	1.2	1.3	10	Mittel
3.1	2.1	1.3	3.1	1.2	1.3	11	
3.1	2.2	1.2	2.1	2.2	1.3	11	
3.2	2.2	1.2	2.1	2.2	2.3	12	Objekt
3.2	2.2	1.3	3.1	2.2	2.3	13	
3.1	2.3	1.3	3.1	3.2	1.3	13	
3.2	2.3	1.3	3.1	3.2	2.3	14	Interpretant
3.3	2.3	1.3	3.1	3.2	3.3	15	
3.1	2.2	1.3	3.1	2.2	1.3	12	Eigenrealität

Der Frage, wie viele triadische Trichotomien bzw. trichotomische Triaden es innerhalb der Semiotik gibt, wird ausführlich in Toth (2008) nachgegangen.

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Zeichen als Systeme und ihre internen Umgebungen

1. Was ist die Umgebung eines Zeichens? Das Objekt, das es bezeichnet, denn nicht umsonst wurde das Zeichen durch Bense (1967, S. 9) als "Meta-Objekt" eingeführt. Allerdings stehen Objekt und Zeichen lediglich in einer äußeren Austauschrelation, insofern Zeichen und Objekt beide als System und Umgebung fungieren können. Denn Zeichen werden ja seit Bense (1975, S. 37) durch semiotische 3×3 Matrizen definiert, und da Zeichen triadisch-trichotomische Relationen sind, sind pro Zeichenklasse bzw. Realitätsthematik von einer Matrix mit 9 Plätzen genau 3 Plätze belegt. Die Frage, die wir uns jetzt stellen, kann man also wie folgt formulieren: Welche und wie viele Zeichenklassen (oder im dualen Falle, Realitätsthematiken) enthält die Komplementärmenge der belegten Plätze (Einträge) einer semiotischen Matrix? Als Symbole für unbelegte und belegte Plätze verwenden wir □ und ■.

2. Ferner müssen wir uns im Sinne des peirce-benseschen Zehnersystems auf sog. reguläre Zeichenklassen beschränken, d.h. auf solche, für die

ZKl = (3.x, 2.y, 1.z)

$x \leqq y \leqq z$

gilt, wodurch aus den theoretisch möglichen $3^3 = 27$ semiotischen Relationen genau die 10 Zeichenklassen herausgefiltert werden. Semiotische Relationen wie z.B.

(3.2 2.1 1.3), (3.2, 2.2, 1.1), (3.1, 2.2, 1.1) usw.

sind also irregulär (obwohl die semiotische Matrix mit ihrer Hauptdiagonale eine dieser irregulären Relationen besitzt). Für unser Vorgehen bedeutet dies, daß U(ZKl) wie folgt definiert werden muß

U(ZKl) = U(3.x, 2.y, 1.z) = {(3.(x+1), 2.(y+1), 1.(z+1))}.

Leider werden durch diese Definition aber nicht nur die irregulären Relationen wie z.B.

S^4 $\quad\quad$ $U(S^4)$

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□■□ $\quad\quad$ □■□

■□□ $\quad\quad$ ■□□ ,

sondern auch reguläre wie z.B.

S^7 $\quad\quad$ $U(S^7) = S^1$

□■□ $\quad\quad$ ■□□

□■□ $\quad\quad$ ■□□

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ausgeschieden. Schließlich folgt aus der Definition der Umgebung, daß keine semiotische Relation ihre eigene Umgebung sein kann. Man beachte, daß dies in Sonderheit auch für die eigenreale, dualinvariante Zeichenklasse gilt. Sowohl gegen die Definition von ZKln als auch gegen das Verbot der Selbstumgebung verstößt z.B.

S^7 $\quad\quad$ $U(S^7)$

□■□ $\quad\quad$ ■□□

□■□ $\quad\quad$ □■□

□■□ $\quad\quad$ □■□ .

3. Die folgende Liste enthält somit für alle 10 Zeichenklassen genau jene Umgebungen, die wiederum Zeichenklassen sind, d.h. der Umgebungsoperator ist extensiv, monoton und abgeschlossen und entspricht damit der von Bense (1983) definierten Modelltheorie eines "Universums der Zeichen".

S^1 $U^1(S^1)$ $U^2(S^1)$ $U^3(S^1)$ $U^4(S^1)$

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S^2 $U^1(S^2)$ $U^2(S^2)$ $U^3(S^2)$ $U^4(S^2)$

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S^3 $U^1(S^3)$ $U^2(S^3)$ $U^3(S^3)$ $U^4(S^3)$

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S^4 $U^1(S^4)$ $U^2(S^4)$ $U^3(S^4)$ $U^4(S^4)$

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S^5 $U^1(S^5)$ $U^2(S^5)$ $U^3(S^5)$ $U^4(S^5)$

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S^6 $U^1(S^6)$ $U^2(S^6)$ $U^3(S^6)$ $U^4(S^6)$

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S^7 $U^1(S^7)$ $U^2(S^7)$ $U^3(S^7)$ $U^4(S^7)$

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S^8 $U^1(S^8)$ $U^2(S^8)$ $U^3(S^8)$ $U^4(S^8)$

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S^9 $U^1(S^9)$ $U^2(S^9)$ $U^3(S^9)$ $U^4(S^9)$

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S^{10} $U^1(S^{10})$ $U^2(S^{10})$ $U^3(S^{10})$ $U^4(S^{10})$

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Zeichenklassen können somit maximal 4 Umgebungen haben. Besonders auffällig sind jene Zeichenklassen, die 0 Umgebungen haben; es sind per definitionem genau diejenigen, die mindestens eine drittheitliche Subrelation aufweisen.

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ε/ν-Analyse von Paaren peircescher Zeichenrelationen

1. Im folgenden wird die ε/ν-Analyse auf das System der 10 peirce-benseschen Zeichenklassen (vgl. Bense 1975, S. 36 ff.) angewandt, so zwar, daß die 1. Zeichenklasse mit sich selbst und jeder anderen Relation in Paarrelation gesetzt wird.

2. Wie die ε/ν-Analyse deutlich macht, tritt die Folge $F = <\nu, \nu, \nu>$ erstmals beim Übergang vom Rhema zum Dicent auf. Damit wird der bereits in Toth (2007, S. 107) festgestellte Trichotomienwechsel als innere semiotische Grenze des sog. semiotischen Zehnersystems bestätigt.

3.1	2.1	1.1	3.1	2.1	1.1	3.1	2.1	1.1
ε	ε	ε	ε	ε	ν	ε	ε	ν
3.1	2.1	1.1	3.1	2.1	1.2	3.1	2.1	1.3

3.1	2.1	1.1	3.1	2.1	1.1	3.1	2.1	1.1
ε	ν	ν	ε	ν	ν	ε	ν	ν
3.1	2.2	1.2	3.1	2.2	1.3	3.1	2.3	1.3

3.1	2.1	1.1	3.1	2.1	1.1	3.1	2.1	1.1
ε	ν	ν	ν	ν	ν	ν	ν	ν
3.2	2.2	1.2	3.2	2.2	1.3	3.2	2.3	1.3

3.1 2.1 1.1

ν ν ν

3.3 2.3 1.3.

3. Nimmt man nun die ε/ν-Folgen und bildet wiederum Folgen aus Identitäten und Nichtidentitäten aus ihnen

ε	ε	ε						
			ε	ε	ν			
ε	ε	ν						
						ε	ν	ε
ε	ε	ν						
			ε	ν	ν			
ε	ν	ν						
						ε	ν	ν
ε	ν	ν						
			ε	ε	ε			
ε	ν	ν						
						ν	ε	ε
ε	ν	ν						
			ν	ε	ε			
ν	ν	ν						
						ν	ε	ε
ν	ν	ν						

			ε	ε	ε			
ν	ν	ν						
ε	ν	ε						
			ε	ε	ν			
ε	ν	ν				ν	ν	ε
			ν	ν	ν			
ν	ε	ε				ν	ν	ν
			ε	ε	ε			
ν	ε	ε,						

so erhält man bei algorithmischer Durchführung dieses Verfahrem am Ende die Folge <ε, ε, ν>, die nicht-identisch ist.

ν	ν	ε			
			ε	ε	ν.
ν	ν	ν			

Der Grund dürfte darin liegen, daß sich zwischen der 9. und der 10. Zeichenklasse ein weiterer Trichotomienwechsel befindet, welcher das semiotische Zehnersystem mit dem bereits genannten Trichotomienwechsel zwischen der 6. und 7. Zeichenklassen als ein diskonnexes Konnex aus drei nicht-identischen Teilsystemen erweist, also wiederum die Resultate in Toth (2007, 173 ff.), die notabene auch für n-adische Zeichenklassen mit $n > 3$ gelten, bestätigend. Die folgende Tafel stammt aus Toth (2007, S. 177)

1	3.1	2.1	1.1	×	1.1	1.2	1.3	1^3
2	3.1	2.1	1.2	×	2.1	1.2	1.3	$2^1 1^2$
3	3.1	2.1	1.3	×	3.1	1.2	1.3	$3^1 1^2$
4	3.1	2.2	1.2	×	2.1	2.2	1.3	$2^2 1^1$
5	**3.1**	**2.2**	**1.3**	**×**	**3.1**	**2.2**	**1.3**	$\mathbf{3^1 2^1 1^1}$
6	3.1	2.3	1.3	×	3.1	3.2	1.3	$3^2 1^1$
7	3.2	2.2	1.2	×	2.1	2.2	2.3	2^3
8	3.2	2.2	1.3	×	3.1	2.2	2.3	$3^1 2^2$
9	3.2	2.3	1.3	×	3.1	3.2	2.3	$3^2 2^1$
10	3.3	2.3	1.3	×	3.1	3.2	3.3	3^3

.

Hinter der Darstellung des semiotischen Zehnersystems als "determinanten-symmetrischem Dualitätssystem", wie es Bense (1992, S. 76) nach E. Walther abgebildet hatte

Zkl			Rth			Rpw	
3.1	2.1	1.1	1.1	1.2	1.3	9	Mittel
3.1	2.1	1.2	2.1	1.2	1.3	10	
3.1	2.1	1.3	3.1	1.2	1.3	11	
3.1	2.2	1.2	2.1	2.2	1.3	11	Objekt
3.2	2.2	1.2	2.1	2.2	2.3	12	
3.2	2.2	1.3	3.1	2.2	2.3	13	
3.1	2.3	1.3	3.1	3.2	1.3	13	Interpretant
3.2	2.3	1.3	3.1	3.2	2.3	14	
3.3	2.3	1.3	3.1	3.2	3.3	15	
3.1	2.2	1.3	3.1	2.2	1.3	12	Eigenrealität

verbirgt sich also ein asymmetrisches System von symmetrischen Dual-systemen.

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Toth, Alfred, Zwischen den Kontexturen. Klagenfurt 2007

Elementare Anforderungen an eine Logik der Ontik

1. Wie ich seit Toth (2012) gezeigt habe, ist der von Peirce und Bense begründeten Semiotik als Theorie der Zeichen eine Ontik als Theorie der Objekte gegenüberzustellen. Diese Notwendigkeit ergibt sich aus der Tatsache, daß die Dichotomie D = (Objekt, Zeichen) isomorph ist zur logischen Basisdichotomie L = (0, 1). Würde ein Objekt, wie es Peirce, Bense und ihre Nachfolger behaupten, durch Wahrnehmung automatisch zum Zeichen („Alles, was wir wahrnehmen, nehmen wir durch Zeichen wahr"), gäbe es in dieser Welt nur Zeichen, denn selbst gesetzt, es gäbe trotzdem noch Objekte, wie sollten diese ohne Wahrnehnung erkennbar sein? Die Peirce-Bense-Semiotik ist, wie übrigens die meisten Semiotiken, pansemiotisch. Diese Tatsache bedeutet also vermöge Isomorphie dasselbe, als würde man eine Logik entweder durch L = 0 oder durch L = 1 definieren, also einen vollständigen Unsinn.

2. Objekte setzen Subjekte voraus, so, wie Subjekte Objekte voraussetzen, denn Objekte können vermöge Wahrnehmung nur für Subjekte Objekte sein, und umgekehrt setzt die Wahrnehmung eines Objektes das wahrnehmende Bewußtsein eines Subjektes voraus. Daraus folgt, daß wir statt von

L = (0, 1)

auszugehen haben von

0 = f(1)

1 = f(0),

d.h.

L* = (f(1), f(0)).

Es gibt somit weder subjektive Subjekte noch objektive Objekte, sondern nur objektive Subjekte und subjektive Objekte. Die subjektiven Objekte sind die von einem Subjekt wahrgenommenen Objekte, und die objektiven Subjekte sind die von einem Objekt „affizierten" Subjekte. Das bedeutet aber, daß L* nicht nur zwei, sondern vier mögliche Relationen aufweist

L* = ((0, (0)), (0, (1)), (1, (0)), (1, (1))).

Einfacher ausgedrückt, wird die klassische Definition der 2-wertigen Logik

L = (0, 1)

durch die nicht-klassische Definition

L* = (E, (0, 1)),

darin E den Einbettungsoperator bedeutet, ersetzt. Man bedenke, daß L* immer noch 2-wertig ist, denn die erkenntnistheoretisch relativen Werte von L* unterscheiden sich von den erkenntnistheoretischen absoluten Werten von L lediglich dadurch, daß sie in funktionale Abhängigkeit voneinander gesetzt werden. Man könnte sogar soweit gehen, zu sagen, daß die vier Werte von L* zwischen den zwei Werten von L vermitteln

$0 = 0(0) \rightarrow 0(1) \rightarrow 1(0) \rightarrow 1(1) = 1.$

Die Vermittlungsstufen 0(1) und 1(0) verstoßen jedoch nicht gegen das Grundgesetzt des tertium non datur, denn mit E wird ja kein weiterer logischer Wert in L* eingeführt.

3. L* ist genau die Logik, welche man benötigt, um als Grundlage der Dichotomie

D = (Objekt, Zeichen)

bzw.

D = (Ontik, Semiotik)

zu fungieren, denn wie man leicht einsieht, ist

subjektives Objekt = Objekt → Ontik

objektives Subjekt = Zeichen → Semiotik,

d.h. die beiden Relationen

O(0) und 0(1)

sind die logischen Basisrelationen der Ontik, und die beiden Relationen

1(0) und 1(1)

sind die logischen Basisrelationen der Semiotik.

Wie Rudolf Kaehr in einer grundlegenden Arbeit gezeigt hatte, ist die von mir zuerst skizzierte „quadralektische" Logik L* kompatibel mit der polykontexturalen Diamantentheorie (vgl. Toth 2011), d.h. einem mehrwertigen logischen System, das auf der Dissemination theoretisch unendlich vieler 2-wertiger „Monokontexturen" beruht.

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Kaehr, Rudolf, Quadralectic Diamonds. Fourfoldness of Beginnings. Semiotic Studies with Toth's "Theory of the Night". ThinkartLab (Glasgow), 2011

Hierarchische Vermittlung von logischer Zweiwertigkeit

1. Im folgenden präsentieren wir ein kleines und harmlos ausschauendes, aber nichts desto weniger „gefährliches" Spiel. Wie zuletzt in Toth (2017) gezeigt, ist es möglich, eine „quadralektische" (vgl. Kaehr 2011) Logik zu konstruieren vermittels eines Einbettungsoperators, d.h. eine Logik der Basisdefinition

$L^* = (E, (0, 1))$,

ohne gegen die Grundgesetze des Denkens, in Sonderheit ohne gegen das Gesetz des tertium non datur zu verstoßen. Es ist dann

$L^* = ((0, (0)), (0, (1)), (1, (0)), (1, (1)))$

mit

$0 = 0(0) \rightarrow 0(1) \rightarrow 1(0) \rightarrow 1(1) = 1.$

2. Sei nun

$L = (A, B)$,

dann bekommen wir als Anwendung eines „Vermittlungsoperators" V

$V(L) = V(A, B) = (A, C, B)$.

Diese erststufige Anwendung von V ist also bijektiv. Die Bijektion ist jedoch bereits bei der zweitstufigen Anwendung von V aufgehoben, denn wir bekommen

$V(A, C, B) = ((A, D, C), (C, D, B))$.

Als drittstufige Anwendung von V bekommen wir

$V(A, D, C) = ((A, E, D), (D, E, C), (C, E, D), (D, E, B))$, usw.

Die Progression der V-Hierarchie von zweiwertigen Strukturen folgt also natürlich dem Schema

$2^0 = 1 \rightarrow 2^1 = 2 \rightarrow 2^2 = 4 \rightarrow 2^3 = 8 \rightarrow 2^4 = 16 \rightarrow \ldots$.

2. Gehen wir nun von der Logik aus, gilt selbstverständlich

$L = (0, 1)$ oder $L = (1, 0)$,

und das bedeutet, daß für die zusätzlich eingeführten Werte C, D, E, ... natürlich ebenfalls gelten kann

$C, D, E \in (0, 1)$.

Dieses Ergebnis ist nun in der Tat erstaunlich, denn dadurch präsentiert sich unsere obige Hierarchie wie folgt

$L = (0, 1)$ oder $(1, 0)$

$V(L) = V(0, 1)$ oder $V(1, 0) = (0, 0, 1)$ oder $(0, 1, 1)$ oder $(1, 0, 1)$ oder $(1, 1, 1)$,

d.h. bereits auf der 1. Stufe der Vermittlung ergibt sich das von uns zuletzt in Toth (2017) zugrunde gelegte „quadralektische" Schema relativer statt absoluter logischer Werte.

Verzichtet man also auf die Einführung anderer logischer Werte als 0 und 1 und wahrt die logische Zweiwertigkeit, ohne gegen das Gesetz des tertium non datur zu verstoßen, wächst die Progression der V-Hierarchie von zweiwertigen Strukturen doppelt so schnell an, d.h durch

$2^0 = 2 \rightarrow 2^1 = 4 \rightarrow 2^2 = 8 \rightarrow 2^3 = 16 \rightarrow 2^4 = 32 \rightarrow \ldots$.

Damit ist eine echte Fortsetzung des elementaren Schemas einer Logik als Grundlage für die Ontik und die Semiotik

$L^* = (E, (0, 1))$,

mit

$L^* = ((0, (0)), (0, (1)), (1, (0)), (1, (1)))$

erreicht, da ja jeder der jeweils 3 Werte einer Vermittlungstruktur eingebettet oder nicht eingebettet erscheinen kann, vgl. etwa

$L^* = (0, 0, (1)), (0, (0), 1), ((0), 0, 0); (o, (0, 0), \ldots; ((0, 0, 0))$

und da wegen iterierter Anwendung von E natürlich auch eingebettete Vermitlungsstrukturen von Formen wie z.B.

$L^* = (0, (0), ((0))), (0, ((0)), (((0))))$, usw.

denkbar sind. Man wird der Hilfe elektronischer Rechenanlagen bedürfen, um die astronomisch hohe Zahl von möglichen Einbettungsgraden vermittelter Strukturen von L* zu berechnen. Alle Einzelwerte dieser Strukturen sind und bleiben jedoch natürlich logisch zweiwertig, da die zusätzlichen Werte die beiden Werte der aristotelischen Logik sind und da E nicht gegen den logischen Drittensatz verstößt!

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Günther-Zahlen und P(ω)-Zahlen

1. Bekanntlich sind die qualitativen Zahlen der Güntherlogik und der auf ihr basierenden “Mathematik der Qualitäten“ (Kronthaler 1986) nicht als anderes als Leerformen, in die Peanozahlen eingesetzt werden und über deren Verteilung die Sterlingzahlen 2. Art (bzw. deren Summierungen, die Bell-Zahlen), Auskunft geben. Auf jeden Fall handelt es sich nicht um Zahlen, denen neben Quantität auch bereits Qualität inhäriert, sondern die letztere folgt aus einer behaupteten strukturellen Anordnung dieser soweit also immer noch rein quantitativen Zahlen. Bei Günther sind es drei Sorten: die Proto-, die Deutero- und die Trito-Zahlen, die sich auf Grund der sog. Schadach-Äquivalenzen ergeben (vgl. Schadach 1967)

1.1. Theorem der Proto-Äquivalenz

card B^A / $\underset{\sim}{p}$ = min {card A, card B}

1.2. Theorem der Deutero-Äquivalenz

$\mu_1 \sim^d \mu_1 \Leftrightarrow A/\ker \mu_1 \cong A/\ker \mu_2$

1.3. Theorem der Trito-Äquivalenz

card B^A / $\underset{\sim}{d} = \sum_{k=1}^{M} P(n,k)$.

2. Stirling-Zahlen 2. Art sind Zahlen, die angeben, auf wie viele Arten eine Menge von n Zahlen in k nicht-leere disjunkte Teilmengen zerlegt werden können. Als Beispiel stehe die Menge M= (a, b, c, d).

((a. b), (c, d))

((a, c), (b, d))

((a, d), (b, c))

((a, b, c), (d))

((a, b, d), (c))

((a, c, d), (b))

((b, c, d), (a)).

Also ist $S_{4.2} = 7$.

3. Dagegen wurde für die qualitative Arithmetik, wie sie innerhalb der von uns begründeten Ontik Anwendung findet (vgl. Toth 2015), Qualität im Sinne von Ort von Anfang an in den Zahlbegriff gebracht. Gegeben sei also eine Peano-Zahl P und ihr Ort (ω). D.h. eine qualitative ontische Zahl ist eine Zahl der Form

$P = f(\omega)$.

Dementsprechend gibt es natürlich den logischen Austausch der Spiegebild-lichkeit, wie sie in der logischen Basisdichotomie der aristotelischen Logik durch

$L = (a, b) = (b, a)$

definitorisch verankert ist, nicht mehr, denn ein Ausdruck wie

$a = b$

bedeutet ja, wie Wittgenstein einmal gesagt hatte, daß an der Stelle von a auch b und an der Stelle von b auch an auftreten könne. Und dies widerspricht ja $P = f(\omega)$.

Das bedeutet, daß wir eine Abbildung bekommen der Form

$L = (a, b) \rightarrow$

$L^* = ((a), b), (a, (b)), ((b), a), (b, (a)),$

d.h. wir gehen statt von L aus von $L^* = (L, E)$, darin E ein Einbettungsoperator ist mit

$E: a \rightarrow (a)$.

4. Sowohl im Falle der Stirling-Zahlen als auch im Falle der P(ω)-Zahlen haben wir also Mengen mit vier Elementen. Berechnen wir also die Sterling-Zahl für $L^* = ((a), b), (a, (b)), ((b), a), (b, (a))$. Man sieht allerdings sehr schnell, daß dies unmöglich ist, denn eine L*-Zahl wie ((a), b) oder ((b), a) besteht ja nicht aus zwei voneinander unabhängigen Zahlen, sondern ist eine Funktion und

somit eine Relation. Die Frage, auf wie viele Arten eine Menge von n Zahlen in k nicht-leere disjunkte Teilmengen zerlegt werden können, ist also bei ortsfunktionalen Zahlen etwa so sinnlos wie der Versuch, die Höhe eines Tisches in Sekunden oder die Länge einer Strecke in Kalorien zu berechnen. Stattdessen bekommen wir wiederum drei qualitativ differenzierbare Zahlen, die wir adjazente, subjazente und transjazente Zahlen genannt hatten:

4.1. Adjazente P(ω)-Zahlen

x_i	y_j		y_i	x_j		y_j	x_i		x_j	y_i
$Ø_i$	$Ø_j$		$Ø_i$	$Ø_j$		$Ø_j$	$Ø_i$		$Ø_j$	$Ø_i$
		$\times$			$\times$			$\times$		
$Ø_i$	$Ø_j$		$Ø_i$	$Ø_j$		$Ø_j$	$Ø_i$		$Ø_j$	$Ø_i$
x_i	y_j		y_i	x_j		y_j	x_i		x_j	y_i

4.2. Subjazente P(ω)-Zahlen

x_i	$Ø_j$		$Ø_i$	x_j		$Ø_j$	x_i		x_j	$Ø_i$
y_i	$Ø_j$		$Ø_i$	y_j		$Ø_j$	y_i		y_j	$Ø_i$
		$\times$			$\times$			$\times$		
y_i	$Ø_j$		$Ø_i$	y_j		$Ø_j$	y_i		y_j	$Ø_i$
x_i	$Ø_j$		$Ø_i$	x_j		$Ø_j$	x_i		x_j	$Ø_i$

4.3. Transjazente P(ω)-Zahlen

x_i	$Ø_j$		$Ø_i$	x_j		$Ø_j$	x_i		x_j	$Ø_i$
$Ø_i$	y_j		y_i	$Ø_j$		y_j	$Ø_i$		$Ø_j$	y_i
		$\times$			$\times$			$\times$		
$Ø_i$	y_j		y_i	$Ø_j$		y_j	$Ø_i$		$Ø_j$	y_i
x_i	$Ø_j$		$Ø_i$	x_j		$Ø_j$	x_i		x_j	$Ø_i$

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Toth, Alfred, Zur Arithmetik der Relationalzahlen I-II. In: Electronic Journal for Mathematical Semiotics, 2015

Einbettungsoperatoren

1. Die 2-wertige aristotelische Logik beruht auf einer heterarchischen Austauschrelation der Form

$L = [P, N]$

$L = [N, P]$,

denn für die der Position (P) und der Negation (N) zugeordneten Wahrheitswerte W und F gilt bekanntlich

$\neg W = F$

$\neg\neg W = W$

$\neg\neg F = F$.[12]

Daher läßt sich der Negationsoperator als 2-wertiger Reflektor

$\times W = F$

$\times F = W$

darstellen.

2. In Sonderheit verbietet also der logische Drittensatz nicht nur die Existenz eines dritten logischen Wertes, sondern auch hierarchische Austauschrelationender Form

$W^* = [W, F]$

$F^* = [F, W]$.

Läßt man nämlich Selbstenthaltung zu – dazu muß das Fundierungsaxiom der Zermelo-Fraenkelschen Mengentheorie außer Kraft gesetzt werden –, dann bekommen wir statt einer doppelten eine vierfache Opposition

[12] Die Antwort auf Wittgensteins Frage (Tractatus 5.44) , ob im Ausdruck $\neg\neg$ p p bejaht oder $\neg$p verneint wird, lautet natürlich, daß sich doppelte 2-wertige Operatoren selbst aufheben, vgl. die Körperaddition $1 + 1 = 0$.

L = [P, [N]]

L = [[P], N]

L = [N, [P]]

L = [[N], P]

und also

W* = [W, [F]]

W* = [[W], F]]

F* = [F, [W]]

F* = [[F], W].

Ganz offensichtlich sind diese Strukturen, die nur mit den zwei logischen Werten W und F operieren, dennoch nicht 2-wertig, denn ein bislang undefinierter, rechtsmehrdeutiger Einbettungsoperator

E := [x, y] → {[x, [y]], [[x], y], [y, [x]], [[y], x]}

fungiert quasi anstelle eines dritten logisches Wertes, indem er ein Tertium datur in die 2-wertige aristotelische Logik einführt. Da die Logik für jeden dyadischen Operator "Wertfunktionen", richtig: Kombinationen von Wahrheitswerten (WW, WF, FW, FF) kennt, ordnet also der Einbettungsoperator E diesen Wertkombinationen folgende Strukturen für jede der 16 dyadischen logischen Operatoren zu

E(WW) → {[W, [W]], [[W], W]}

E(WF) → {[W, [F]], [[W], F], [F, [W]], [[F], W]}

E(FW) → {[F, [W]], [[F], W], [W, [F]], [[W], F]}

E(FF) → {[F, [F]], [[F], F]}.

3. Daß die Einführung eines Einbettungsoperators zur relationalen, aber nicht materialen Erzeugung von logischer Mehrwertigkeit qua Aufhebung des

Tertium non datur-Gesetzes von größter Bedeutung für die Arithmetik ist, die ja natürlich auf der 2-wertigen Logik beruht, dürfte unmittelbar einleuchten. Nehmen wir als Beispiel

N = (1, 2, 3),

d.h. den Anfang der natürlichen Zahlen, der vermöge der Peano-Axiome in der Form

N = (|, ||, |||)

darstellbar ist, weshalb Günther (1991) von der "totalen Relationslosigkeit" der Zahl gesprochen hatte. Da Bense (1975, S. 167 ff.) die Zeichenzahlen, später auch "Primzeichen" genannt (Bense 1981, S. 17 ff.) explizit mit Hilfe der Peano-Axiome eingeführt hatte, widerspricht diese Einführung der späteren Definition Benses (1979, S. 53), die Mengeninklusionsketten der Form

$Z = (1 \subset 2 \subset 3)$

voraussetzt. (Die semiotische Drittheit schließt Zweit- und Erstheit, und die Zweitheit schließt Erstheit ein. Bense, a.a.O., sprach daher ausdrücklich vom Zeichen als einer "Relation über Relationen".) Es gilt somit selbstverständlich

$N \neq Z$,

d.h. die Zeichenzahlen sind NICHT mit Hilfe der Peano-Axiome einführbar, da für natürliche Zahlen keine Mengeninklusionen gelen. Jede natürliche Zahl $n > 1$ besitzt lediglich einen Vorgänger V(n) = (n-1) und einen Nachfolger N(n) = (n+1), schließt aber keineswegs die Menge aller ihrer Vorgänger ein, wie dies die Zeichenzahlen tun. Bense selbst (1981, S. 26) hatte zwar den Begriff der Relationszahl nur für die drittheitlich fungierende Zeichenzahlen reserviert, aber selbstverständlich stellen alle drei Zeichenzahlen Relationszahlen dar, die somit von Peanozahlen strikt zu trennen sind.

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Günther, Gotthard, Idee und Grundriß einer nicht-Aristotelischen Logik. 3. Aufl. Hamburg 1991

Einbettungsoperator und Elementschaft

1. In Toth (2014) hatten wir die Legitimität der logischen Dichotomie

$L = [0, 1]$

und aller ihr isomorphen, d.h. ontischen und semiotischen, Dichotomien bezüglich ihrer logischen 2-Wertigkeit in Frage gestellt, indem wir einen Einbettungsoperator E eingeführt hatten, der ein Objekt Ω vermöge

$E: \quad \Omega \rightarrow [\Omega]$

auf eine tiefere Einbettungsstufe abbildet, d.h. für wiederholte Anwendung gilt z.B.

$E^2(\Omega) = [[\Omega]]$

$E^2([\Omega]) = E^3(\Omega)$ $[[[\Omega]]]$, usw.

Das bedeutet in Sonderheit, daß es für jedes $K \cong L$ einen nicht-leeren Rand

$R = [\Omega, [\Omega]]$

bzw.

$R^{-1} = [[\Omega], \Omega]$

gibt, der, ohne einen zusätzlichen dritten Wert einzuführen, als tertium datur fungiert.

2. Wie man leicht zeigt, gibt es für jedes $K \cong L$ jedoch nicht nur zwei, sondern vier Einbettungsstrukturen. Sei $S^* = [S, U]$ (vgl. Toth 2012), dann bekommen wir für $E(S^*)$

$S_1^* = [S, [U]]$ $\qquad$ $S_2^* = [[U], S]$

$S_3^* = [[S], U]$ $\qquad$ $S_4^* = [U, [S]]$

mit

$S_2^* = S_1^{*-1}$

$S_4^* = S_3^{*-1}$.

Nun kann man die Zahlen 0 und 1, die als Wahrheitswerte in L erscheinen, vermöge des Satz von Wiener und Kuratowski (1917) durch

$0 := Ø$

$1 := \{Ø\}$

definieren. Wir bekommen damit sofort für L

L = [Ø, [{Ø}]]	L = [[{Ø}], Ø]
L = [[Ø], {Ø}]	L = [{Ø}, [Ø]],

und damit gibt es auch vier nicht-leere Ränder zwischen Ø und {Ø}, welche die Elementschaftsrelation Ø ∈ {Ø} formalisieren

R[Ø, [{Ø}]]	R[[{Ø}], Ø]
R[[Ø], {Ø}]	R[{Ø}, [Ø]].

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Wiener, Norbert, A simplification of the logic of relations. In: Proceedings of the Cambridge Philosophical Society 17 (1914), S. 387-390

Einbettungsoperator und Multisets

1. Wie in Toth (2015a) dargestellt wurde, kann man durch einen Einbettungsoperator E, der die Juxtaposition der Relata in der logischen Basisdichotomie L = [0, 1] aufhebt, ein Tertium als Vermittlung einführen, ohne einen dritten Wert einzuführen, denn E erzeugt eine Differenz zwischen den Werten 0 und 1. Wir bekommen dann

E(L) =

$L_1 = [0, [1]]$ $\qquad$ $L_3 = [1, [0]]$

$L_2 = [[0], 1]$ $\qquad$ $L_4 = [[1], 0]$,

d.h. ein Quadrupel von L-Strukturen, deren Werte in Subordinations- bzw. Superordinationsrelation zueinander stehen.

2. Als abstrakte Form des L-Quadrupels können wir setzen

$$\begin{array}{ll} \emptyset & \emptyset \\ \hline \emptyset & \emptyset \end{array}$$

mit einer Abbildung

f: $\quad (0, 1) \rightarrow \emptyset$

und die vier L-Strukturen des Quadrupels durch eine Art von "Tableaux" darstellen.

2.1. $L_1 = [0, [1]]$

$$\begin{array}{ll} 0 & \emptyset \\ \hline \emptyset & 1 \end{array}$$

2.2. $L_2 = [[0], 1]$

$$\begin{array}{cc} \emptyset & 1 \\ \hline 0 & \emptyset \end{array}$$

2.3. $L_3 = [1, [0]]$

$$\begin{array}{cc} 1 & \emptyset \\ \hline \emptyset & 0 \end{array}$$

2.4. $L_4 = [[1], 0]$

$$\begin{array}{cc} \emptyset & 0 \\ \hline 1 & \emptyset \end{array}$$

3. Wie man erkennt, gilt in 2.1. bis 2.4., daß keine zwei Werte übereinander stehen dürfen, d.h. es können keine zwei Werte sich gleichzeitig am selben logischen Ort befinden. Allerdings gilt dies nur in einer Cantor-Mengentheorie, nicht aber in der sog. Multiset-Theorie (vgl. Toth 2015b). Abgesehen von den vier L-Strukturen ergeben sich hier drei weitere L-Strukturen.

3.1. $L_1 = [0, 0, [1]]$

$$\begin{array}{cc} 0 & 0 \\ \hline \emptyset & 1 \end{array}$$

3.2. $L_2 = [0, [1, 1]]$

$$\begin{array}{cc} 0 & \emptyset \\ \hline 1 & 1 \end{array}$$

3.3. $L_3 = [0, 0, [1, 1]]$

$$\begin{array}{cc} 0 & 0 \\ \hline 1 & 1 \end{array}$$

Literatur

Toth, Alfred, Leerstellen bei nicht-leeren Rändern. In: Electronic Journal for Mathematical Semiotics, 2015a

Toth, Alfred, Multiset-Relationen. In: Electronic Journal for Mathematical Semiotics, 2015b

Iteration des ortsfunktionalen Einbettungsoperators

1. Der in Toth (2014) in die Ontik eingeführte Einbettungsoperator bildet die Menge der Peanozahlen P = (0, 1), die man als Modell der 2-wertigen aristotelischen Logik L = [0, 1] interpretieren kann, auf ein Quadrupel von 1-fach eingebetteten Strukturen ab

$E \to L = [0, 1] =$

$$\begin{pmatrix} L_1 = [0, [1]] & L_1^{-1} = [[1], 0] \\ L_2 = [[0], 1] & L_2^{-1} = [1, [0]] \end{pmatrix}.$$

Im Falle der adjazenten Zählweise, auf die wir uns hier beschränken wollen, ergeben sich damit $2^3 = 8$ Zahlenfelder.

$$\begin{array}{ccccccccccc} x_i & y_j & & y_i & x_j & & y_j & x_i & & x_j & y_i \\ \emptyset_i & \emptyset_j & & \emptyset_i & \emptyset_j & & \emptyset_j & \emptyset_i & & \emptyset_j & \emptyset_i \\ & & \times & & & \times & & & \times & & \\ \emptyset_i & \emptyset_j & & \emptyset_i & \emptyset_j & & \emptyset_j & \emptyset_i & & \emptyset_j & \emptyset_i \\ x_i & y_j & & y_i & x_j & & y_j & x_i & & x_j & y_i \end{array}$$

2. Iteriert man E, so erhält man vermöge der Abbildung

$$E^2 \to \begin{pmatrix} L_1 = [0, [1]] & L_1^{-1} = [[1], 0] \\ L_2 = [[0], 1] & L_2^{-1} = [1, [0]] \end{pmatrix} =$$

bereits eine große Menge weiterer ortsfunktionaler Einbettungsstrukturen, die man wie folgt subkategorisieren kann.

2.1. 0- vs. 2-stufige Einbettung

[0, [[1]]]	[[[1], 0]
[[[0], 1]	[1, [[0]]]

2.2. 1- vs. 2-stufige Einbettung

[[0], [[1]]]   [[[1]], [0]]

[[[1]], [0]]   [[0], [[1]]]

2.3. 0-, 1- und 2-stufige Einbettung

[0, [0], [[1]]]   [[[1]], [0], 0]   [0, [1], [[1]]]   [[[1]], [1], 0]

[1, [1], [[0]]]   [[[0], [1], 1]   [1, [0], [[0]]]   [[[0], [0], 1] , usw.

Im Falle der adjazenten Zählweise, auf die wir uns hier wiederum beschränken wollen, ergeben sich nun $3^3 = 27$ Zahlenfelder, von denen wir nur die allgemeine Form angeben wollen.

0_i	1_j	2_k	$Ø_i$	$Ø_j$	$Ø_k$	$Ø_i$	$Ø_j$	$Ø_k$
$Ø_j$	$Ø_k$	$Ø_i$	0_j	1_k	2_i	$Ø_j$	$Ø_k$	$Ø_i$
$Ø_k$	$Ø_i$	$Ø_j$	$Ø_k$	$Ø_i$	$Ø_j$	0_k	1_i	2_j
	$\times$			$\times$			$\times$	
0_j	1_k	2_i	$Ø_j$	$Ø_k$	$Ø_i$	$Ø_j$	$Ø_k$	$Ø_i$
$Ø_k$	$Ø_i$	$Ø_j$	0_k	1_i	2_j	$Ø_k$	$Ø_i$	$Ø_j$
$Ø_i$	$Ø_j$	$Ø_k$	$Ø_i$	$Ø_j$	$Ø_k$	0_i	1_j	2_k
	$\times$			$\times$			$\times$	
0_i	1_j	2_k	$Ø_i$	$Ø_j$	$Ø_k$	$Ø_i$	$Ø_j$	$Ø_k$
$Ø_j$	$Ø_k$	$Ø_i$	0_j	1_k	2_i	$Ø_j$	$Ø_k$	$Ø_i$
$Ø_k$	$Ø_i$	$Ø_j$	$Ø_k$	$Ø_i$	$Ø_j$	0_k	1_i	2_j

3. 3-stufige Einbettung tritt somit ein, sobald der Einbettungsoperator zu E^3 iteriert wird. Allgemein kann man natürlich n Einbettungsstufen durch n-fache Iteration, d.h. durch E^n, erzeugen. Dabei werden also objektive Objekte und subjektive Objekte, die den Werten 0 und 1 bzw. 1 und 0 korrespondieren (vgl.

Toth 2015a, b), immer weiter einander angenähert, indem das Objekt immer mehr Subjektanteile und das Subjekt immer mehr Objektanteile erhält. Man kann dies schematisch wie folgt andeuten.

Für E^0

Für E^1

Für E^2

, usw.

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Die Logik von Hermann Hermann

1. Bekanntlich kann man die klassische, d.h. 2-wertige aristotelische Logik in ihrer einfachsten Form als Relation

$L = [0, 1]$

darstellen (vgl. Toth 2015a). Darin stehen die Werte 0 und 1 für logische Position und Negation und damit für erkenntnistheoretisches Objekt und Subjekt. Nun hatte bereits Günther die Besonderheit von L in unüberbietbarer Weise charakterisiert: "Beide Werte einer solchen Logik aber sind metaphysisch äquivalent. Das heißt, man kann sie beliebig miteinander vertauschen. Sie verhalten sich zueinander in einer totalen logischen Disjunktion, wie rechts und links. Es gibt keinen theoretischen Grund, welche Seite rechts und welche Seite links von der Zugspitze ist. Die Benennung beruht auf einer willkürlichen Entscheidung, und wenn man seinen Standpunkt wechselt, sind die rechte und die linke Seite miteinander vertauscht (Günther 2000, S. 230 f.). Damit gilt also

$(L = L^{-1}) = [0, 1] = [1, 0]$,

und dies ist deshalb der Fall, weil das Grundgesetz des Ausgeschlossenen Dritten eine Vermittlung der beiden Werte verbietet. Genau genommen, bedeutet dies aber, daß nicht nur eine substantielle Vermittlung der Formen

$L^* = [0, 2, 1]$

$L^* = [1, 2, 0]$,

sondern auch eine differentielle der Formen

$0 = f(1)$

$1 = f(0)$

verboten ist. Daraus wiederum folgt, daß 0 und 1 absolute Kategorien sind, d.h. es handelt sich nicht nur um ein Objekt, sondern um ein objektives Objekt und nicht nur um ein Subjekt, sondern um ein subjektives Subjekt. In Sonderheit sind also die beiden vermittelten Kategorien des subjektiven Objektes $O = f(1)$

und des objektiven Subjektes $1 = f(0)$ ausgeschlossen. Systemtheoretisch bedeutet dies, daß der Rand zwischen 0 und 1 leer ist, d.h. daß gilt

$R[0, 1] = R[1, 0] = Ø$.

Kronthaler hatte diesen Sachverhalt wie folgt erkannt: "Die aristotelische Logik besitzt nur deshalb zwei Werte, weil es sich bei ihr um einen Abbildungsprozeß handelt. Man kann etwa HABEN, was 1-wertig ist, aber nicht ABBILDEN. Der zweite Wert spielt aber nur eine Hilfsrolle, er designiert nichts, sondern tritt nur als Hintergrund auf; er wiederholt nur. 1-wertiges Sein ist AUTO-referentiell. Es verweist auf nichts außerhalb seiner eigenen Kontexturalität. Einfach deshalb, weil es nichts außerhalb gibt!" (1986, S. 8).

2. Führt man einen dritten Wert, d.h. eine substantielle Vermittlung, in L ein, so löst man überhaupt kein Problem, denn das Tertium non datur wird dann, je nach der Anzahl der gewählten vermittelnden Werte, zu einem Quartum, Quintum ... non datur verschoben. Setzt man hingegen differentielle Vermittlung an, dann ergeben sich vier mögliche Strukturen über L

$L_1 = [0, [1]]$ $\quad$ $L_1^{-1} = [[1], 0]$

$L_2 = [[0], 1]$ $\quad$ $L_2^{-1} = [1, [0]]$.

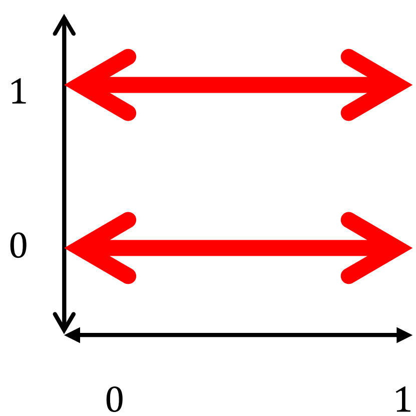
Es ist leicht einzusehen, daß die beiden Werte in diesem Quadrupel nun nicht nur koordiniert wie in L, sondern auch sub- und superordiniert auftreten können, d.h. man kommt nicht mehr mit einer Peanolinie aus, sondern benötigt zum Zählen solcher logischer Wertzahlen 2-dimensionale Zahlenfelder. Wie in Toth (2015b-d) gezeigt, kann zwischen horizontaler, vertikaler und diagonaler bzw. adjazenter, subjazenter und transjazenter Zählweise unterschieden werden.

2.1. Adjazente Zählweise

2.1.1. Zahlenfelder

x_i	y_j		y_i	x_j		y_j	x_i		x_j	y_i
$Ø_i$	$Ø_j$		$Ø_i$	$Ø_j$		$Ø_j$	$Ø_i$		$Ø_j$	$Ø_i$
		$\times$			$\times$			$\times$		
$Ø_i$	$Ø_j$		$Ø_i$	$Ø_j$		$Ø_j$	$Ø_i$		$Ø_j$	$Ø_i$
x_i	y_j		y_i	x_j		y_j	x_i		x_j	y_i

2.1.2. Zahlenschema

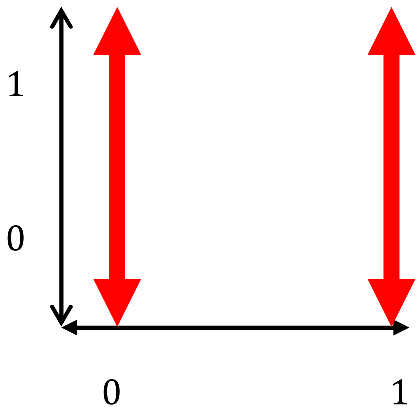


2.2. Subjazente Zählweise

2.2.1. Zahlenfelder

x_i	$Ø_j$		$Ø_i$	x_j		$Ø_j$	x_i		x_j	$Ø_i$
y_i	$Ø_j$		$Ø_i$	y_j		$Ø_j$	y_i		y_j	$Ø_i$
		$\times$			$\times$			$\times$		
y_i	$Ø_j$		$Ø_i$	y_j		$Ø_j$	y_i		y_j	$Ø_i$
x_i	$Ø_j$		$Ø_i$	x_j		$Ø_j$	x_i		x_j	$Ø_i$

2.2.2. Zahlenschema

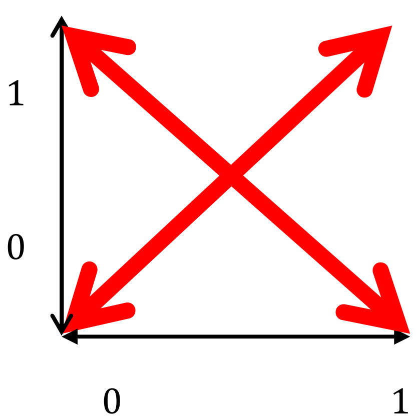


2.3. Transjazente Zählweise

2.3.1. Zahlenfelder

$$\begin{array}{ccccccccccc}
x_i & Ø_j & & Ø_i & x_j & & Ø_j & x_i & & x_j & Ø_i \\
Ø_i & y_j & & y_i & Ø_j & & y_j & Ø_i & & Ø_j & y_i \\
 & & \times & & & \times & & & \times & & \\
Ø_i & y_j & & y_i & Ø_j & & y_j & Ø_i & & Ø_j & y_i \\
x_i & Ø_j & & Ø_i & x_j & & Ø_j & x_i & & x_j & Ø_i
\end{array}$$

2.3.2. Zahlenschema



3. Eine Logik, welche auf dem L-Quadrupel und nicht auf L aufgebaut ist, ist also eine Logik, in welcher die Basiskategorien nicht mehr das objektive Objekt und

das subjektive Subjekt, sondern das subjektive Objekt und das objektive Subjekt sind. Wie man ohne lange Erörterung einsehen dürfte, gelten also für eine solche Logik, in der die Objektposition Subjektanteile und die Subjektposition Objektanteile besitzt, die Theoreme der klassischen Logik nicht mehr. Nehmen wird als repräsentatives Beispiel die bekannten Sätze

(1) $\neg p \rightarrow (p \rightarrow q)$

(2) $q \rightarrow (p \rightarrow q)$,

d.h. ex falso sequitur quodlibet (1) und verum sequitur e quolibet (2). Diese Sätze sind geradezu Paradebeispiele einer auf absoluten Kategorien aufgebauten Nonsenslogik. Sie besagen nämlich, daß aus dem Nichts alles, d.h. also in Sonderheit auch das Sein, folgt, und daß umgekehrt natürlich das Sein aus allem, in Sonderheit also auch aus dem Nichts, folgt. Natürlich – die beiden unvermittelten Werte sind ja reflexionsidentisch, da weder $p = f(q)$ noch $q = f(p)$ gelten darf. Ebenfalls unmittelbar einleuchten dürfte, daß diese beiden Sätze in einer auf vermittelten Kategorien basierten Logik nicht mehr länger gültig sein können. Wenn ich kein Geld in meinem Portemonnaie habe, kann ich es auch nicht ausgeben. Bei Achternbusch heißt es: Du hast keine Chance – aber nutze sie! Umgekehrt kann ich das Geld in meinem Portemonnaie nicht zum Verschwinden bringen, ohne es auszugeben. Die aristotelische Logik, indem sie nicht von einem wahrgenommenen, d.h. subjektiven Objekt als erkenntnistheoretischer Basis der logischen Position und einem wahrnehmenden, d.h. objektiven Subjekt als erkenntnistheoretischer Basis der logischen Negation ausgeht, beschreibt die ontische Wirklichkeit daher keineswegs in einer abstrakten, sondern in einer unsinnigen und sogar falschen Form. Wenn man weiter bedenkt, daß die aristotelische Logik die Basis aller Wissenschaften, in Sonderheit also auch der Mathematik, bildet, kann man einen Hochschein davon bekommen, wie katastrophal die Auswirkungen dieser Nicht-Abbildung der Ontik sind.

Die hier skizzierte und ansatzweise illustrierte nicht-klassische Logik, die auf vermittelten Kategorien, d.h. auf subjektiven Objekten qua wahrgenommenen Objekten auf auf objektiven Subjekten qua Zeichen, basiert und damit als qualitative Logik die Grundlage des Dualschemas von Ontik und Semiotik bildet, sei nun als "Logik von Hermann Hermann" illustriert. Gemeint ist der Protagonist der von R.W. Faßbinder 1978 verfilmten Erzählung des Nobelpreisträgers Vladimir Nabokov, "Despair". Die Hauptrolle des Hermann Hermann spielte der britische Schauspieler Sir Dirk Bogarde, daher wurde der Film auf Englisch gedreht. Bereits im Namen der Hauptfigur zeigt sich die qualitative Ungleichheit

Hermann Hermann ≠ Hermann Hermann,

die weit über die Nicht-Identität eines gleichen Namens, der als Vornamen und zugleich als Nachnamen verwendet wird, hinausgeht (vgl. Lambert Lambert in Claude Berris Film "Tchao Pantin" von 1983, in dem Coluche die Hauptrolle spielte). Hermann Hermann wird nämlich zum nicht-identischen Doppelgänger von Felix Weber, so daß eine auf Nicht-Identität basierende chiastische Relation entsteht, ähnlich derjenigen, die zwischen vier identischen Personen in E.T.A. Hoffmanns "Prinzessin Brambilla" konstruiert wird (vgl. Toth 2007). Der folgende Dialogausschnitt mit den dazu passenden Bildern wurde aus dem Originalfilm R.W. Faßbinders herauskopiert. Die Eingangsfrage stammt von der Polizei, die den von seinem Schwager verratenen Hermann Hermann gefunden hat und nun in Schwerstbewaffnung vor der Tür einer Elendsabsteige, der letzten Zuflucht des völlig Hilflosen, auf ihn wartet. Die übrigen Dialogteile stammen alle von Hermann Hermann.

"Hermann Hermann? " –

"Yes ... No."

"How childish."

"Good people, we are making a film here. In a minute, I will be coming out."

"I will be coming out. But you must keep the ... policemen back. So that I can get away. ... I am a film actor.

I'm coming out. Don't look at the camera.

I'm coming out."

Vom Standpunkt der Logik vermittelter Kategorien gilt

Sir Dirk Bogarde = subjektives Objekt

Hermann Hermann = objektives Subjekt,

denn, wie Max Bense sich in einer seiner letzten Vorlesungen ausgedrückt hatte, macht sich der Schauspieler selbst zum Zeichen. Wenn also Hermann Hermann im Film sagt, er sei ein Schauspieler, so ist diese Aussage falsch, denn im Film ist er ja die Doppelperson Hermann Hermann = Felix Weber. Es liegt also eine ganz besondere Spielart des Epimenides-Paradoxes vor. Dieses auf unvermittelten Kategorien basierende Paradox lautet bekanntlich in seiner simpelsten Form

"Ich lüge",

und diese Aussage ist wahr gdw. wenn sie falsch ist und falsch gdw. sie wahr ist (da lügen = nicht die Wahrheit sagen bedeutet). Im Falle von Hermann Hermann gilt aber: Die Aussage "Ich bin Filmschauspieler" ist in der semiotischen Welt des Films falsch, aber in der ontischen Welt wahr, denn Sir Dirk Bogarde war ja tatsächlich Filmschauspieler. Diese Differenz zwischen einer Logik, welche die semiotische Welt der Zeichen, d.h. der objektiven Subjekte, und einer Logik, welche die ontische Welt der Objekte, d.h. der subjektiven Objekte, unterscheiden kann, existiert wegen der Nicht-Vermitteltheit der Kategorien in der aristotelischen Logik überhaupt nicht. Die Transformation des Kreter-Paradoxes aus der hermetischen aristotelischen Logik auf eine in Ontik und Semiotik geteilte Wirklichkeit ist dort überhaupt nicht einmal ansatzweise darstellbar.

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Systemik, Ontik, Semiotik, Logik und Erkenntnistheorie

1. Daß die Semiotik die „tiefste fundierende Wissenschaft“ sei, geht schon auf Charles S. Peirce zurück und wurde zuletzt ausführlich von Bense (1986) behandelt.

2. Wie in meinen bisherigen Arbeiten, wird dieses „Axiom“ hier bestritten, und es wird vorgeschlagen, statt der Dichotomie von Objekt und Zeichen (vgl. Bense 1967, S. 9)

$D = (O, Z)$

von der viel fundamentaleren Dichotomie von Außen und Innen

$E = (A, I)$

auszugehen. Da sowohl D als auch E isomorph sind zur – ebenfalls als fundamental aufgefaßten – logischen Dichotomie von Position und Negation

$F = (P, N)$

bzw.

$F = (0, 1)$,

folgt somit

$D \cong E \cong F$.

Da es die Logik mit Aussagen zu tun hat, die den Zeichenbegriff voraussetzen, und da feststeht, daß E die tiefste aller drei zu einander isomorphen Relationen ist, haben wir ferner

$D \lesssim E \lesssim F$.

3. Nun haben wir aber bereits für die Dichotomien, für welche bekanntlich die Grundgesetze des Denkens gültig sind, immer zwei Möglichkeiten, sie als Systeme zu definieren

$D = O^* = (O, Z)$

$D = Z^* = (Z, O)$

$E = A^* = (A, I)$

$E = I^* = (I, A)$

$F = 0^* = (0, 1)$

$F = 1^* = (1, 0)$,

denn "beide Werte einer solchen Logik aber sind metaphysisch äquivalent. Das heißt, man kann sie beliebig miteinander vertauschen. Sie verhalten sich zueinander in einer totalen logischen Disjunktion, wie rechts und links. Es gibt keinen theoretischen Grund, welche Seite rechts und welche Seite links von der Zugspitze ist. Die Benennung beruht auf einer willkürlichen Entscheidung, und wenn man seinen Standpunkt wechselt, sind die rechte und die linke Seite miteinander vertauscht (Günther 2000, S. 230 f.).

4. Die „Spiegelbildlichkeit" der Werte von D, E und F, zu der sich übrigens noch diejenige der erkenntnistheoretischen Dichotomie von Objekt und Subjekt

$G = O^* = (O, S)$

$G = S^* = (S, O)$

gesellt, ist jedoch zu nichts nütze, da das Spiegelbild eines Etwas nichts Anderes reflektieren kann als das, was der andere Wert bereits ist oder hat (vgl. Kronthaler 1986). Sollen die beiden Werte der vier Dichotomien mehr sein als bloße Reflexionen des Einen vom Andern oder des Andern vom Einen, muß also ihre POSITION relevant werden, d.h. es muß gelten

$D = O^* = (O, Z) \quad \ncong \quad D = Z^* = (Z, O)$

$E = A^* = (A, I) \quad \ncong \quad E = I^* = (I, A)$

$F = 0^* = (0, 1) \quad \ncong \quad F = 1^* = (1, 0)$,

$G = O^* = (O, S) \quad \ncong \quad G = S^* = (S, O)$.

Ohne jedoch das Tertiumgesetz zu verletzen, gibt es, wie bereits in Toth (2015) dargelegt, wurde, die Möglichkeit, statt ein materielles ein differentielles „Tertium“ einzuführen. Dafür benötigen wir einen Einbettungsoperator E (vgl. Toth 2014).

$E \rightarrow F = (0, 1) \neq F^{-1} = (1, 0) =$

$$\begin{pmatrix} L_1 = (0, (1)) & L_1^{-1} = ((1), 0) \\ L_2 = ((0), 1) & L_2^{-1} = (1, (0)) \end{pmatrix}.$$

Für jedes L_i gilt somit zusätzlich zu F

$0 = f(1)$

$1 = f(0)$,

und somit ist

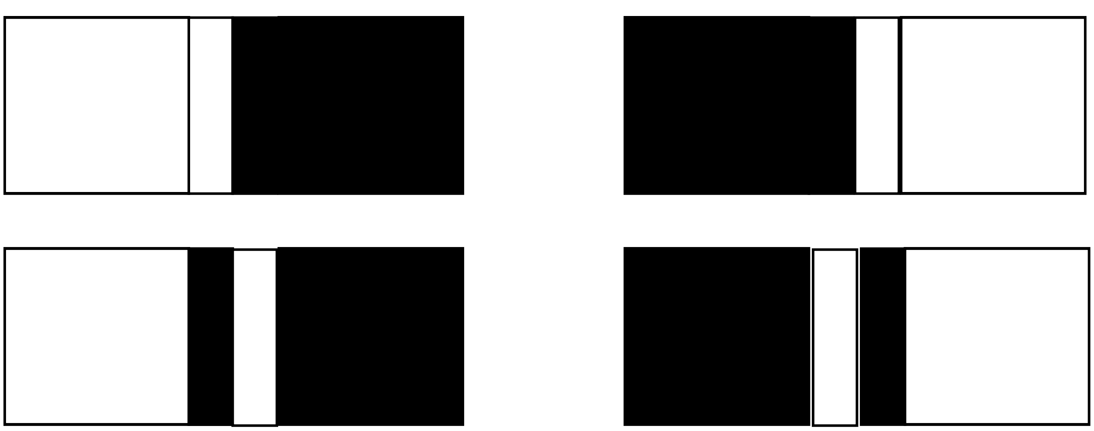
$(x \in 0) \subset 1$

$(y \in 1) \subset 0$,

d.h. 0 hat 1-Anteile, und 1 hat 0-Anteile. Und das, was hier anhand von F dargestellt wurde, gilt vermöge

$D \cong E \cong F \cong G$

natürlich auch für D, E und G. Man kann diese durch E erwirkte Abbildung der Paare auf Quadrupel schematisch wie folgt darstellen.



Die Werte in einer solchen Semiotik, Systemik, Logik und Erkenntnistheorie sind also vermöge E vermittelt. In Sonderheit erhalten wir also als vermittelnde Instanz einen „Rand" R, für den gilt

$R[0, 1] \neq R[1, 0] \neq \emptyset$,

während für $L = [0, 1]$ natürlich gilt

$R[0, 1] = R[1, 0] = \emptyset$.

Wegen

$D \lesssim E \lesssim F$

(worin allerdings die hierarchische Stellung von G unklar ist), können wir nun das folgende neue hierarchische System aufstellen.

Logik / Erkenntnistheorie (F, G)
↑
Semiotik / Ontik (D)
↑
Systemik (E)

Die tiefste fundierende Wissenschaft ist also die Systemik, die sich mit der Differenz von Außen und Innen befaßt und in der der Einbettungsoperator bewirkt, daß es einen Rand gibt zwischen Außen und Innen.

Eine mathematische Besonderheit dieses durch den Einbettungsoperator induzierten Randes in D, E, F und G ist übrigens, daß er iterierbar ist, und zwar zweiseitig, vgl. etwa für F

	(0, (0))	(0, (1))	((0), 1)	((1), 0)
0_λ	(0, ((0, (0)))	(0, (0, (1)))	(0, ((0), 1))	(0, ((1), 0))
0_ρ	((0, (0)), 0)	((0, (1)), 0)	(((0), 1), 0)	(((1), 0), 0)
1_λ	(1, ((0, (0)))	(1, (0, (1)))	(1, ((0), 1))	(1, ((1), 0))
1_ρ	((0, (0)), 1)	((0, (1)), 1)	(((0), 1), 1)	(((1), 0), 1),

worin gilt

$E \rightarrow E \rightarrow ... \rightarrow E = E^n$.

Wie man sich leicht vorstellen kann, entstehen durch Abbildung von E^n auf D, E, F und G sehr rasch hochkomplexe systemische, semiotische/ontische, logische und erkenntnistheorietische Systeme, welche die Komplexität von F = (0, 1) weit übersteigen, ohne dabei an den Grundgesetzen des Denkens zu rütteln, wie dies etwa bei der polykontexturalen Logik und Ontologie von Günther, Kaehr und Kronthaler der Fall ist.

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Systemtheorie des Bildnisses von Dorian Gray

1. Das Bildnis des Dorian Gray wurde bereits, damals noch rein semiotisch, in Toth (2009) behandelt. Im folgenden soll exemplarisch gezeigt werden, wie viel präziser und fundamentaler dieses Thema mit der seither entwickelten ontischen Systemtheorie behandelt werden kann.

2. Wie in Toth (2018a, b) gezeigt wurde, stellt die systemische Dichotomie

$S = (A, I)$

die Basisdichotomie der logischen, semiotischen, erkenntnistheoretischen, allgemein aller ihr isomorphen Dichotomien dar.

Logik / Erkenntnistheorie

↑

Semiotik / Ontik

↑

Systemik

Betrachtet man nun aber S vom Standpunkt der Ontik, so ist die Dichotomie unvollständig, denn um überhaupt Außen und Innen zu unterscheiden, wird ein Rand als Vermittlung benötigt, etwa eine Wand, die das Außen vom Innen oder das Innen vom Außen trennt.

Da für die logische Dichotomie

$L = (0, 1)$,

bekanntlich gilt

$L = (0, 1) = L^{-1} = (1, 0)$,

da es ja egal ist, welcher der beiden Werte als Position bzw. als Negation designiert wird, gilt die Gleichheit der zu einander konversen Relationen vermöge Isomorphie für alle mit L isomorphen Dichotomien. Wenn wir jedoch S als Basisdichotomie nehmen, ist nicht von S = (A, I) auszugehen, sondern von

$S^* = (A, R, I)$

mit

$R(A, I) \neq R(I, A) \neq \emptyset$,

während für S = (A, I) natürlich gilt

$R(A, I) = R(I, A) = \emptyset$.

Ohne jedoch das Tertiumgesetz zu verletzen, gibt es, wie bereits in Toth (2015) dargelegt wurde, die Möglichkeit, statt einem materiellen ein differentielles „Tertium“ einzuführen. Dafür benötigen wir einen Einbettungsoperator E (vgl. Toth 2014).

$E \rightarrow S = (A, I) \neq S^{-1} = (I, A) =$

$$\begin{pmatrix} S_1 = (A, (I)) & S_1^{-1} = ((I), A) \\ S_2 = ((A), I) & S_2^{-1} = (I, (A)) \end{pmatrix},$$

wodurch A und I nun nicht mehr, wie in S, spiegelbildlich, sondern durch die durch E induzierte Einbettung positionsgebunden sind. Ontisch gesehen ist dieser Sachverhalt wiederum unmittelbar klar: Das Innere eines Hauses ist natürlich dem Äußeren ungleich, und eine Wand trennt Außen und Innen, Innen und Außen (und selbst die Wand läßt eine eindeutige Unterscheidung von Außenseite und Innenseite zu). Aus diesem Grunde ist die Konversion der Randrelation, in diesem Falle eine Umstülpung, unmöglich.

Setzen wir nun statt S = (A, I)

S = (-A, I)

oder

S = (A, -I),

so bekommen wir

$$\left(\begin{array}{ll} S_1 = (-A, (I)) & S_1^{-1} = ((I), -A) \\ S_2 = ((-A), I) & S_2^{-1} = (I, (-A)) \end{array}\right)$$

$$\left(\begin{array}{ll} S_1 = (A, (-I)) & S_1^{-1} = ((-I), A) \\ S_2 = ((A), -I) & S_2^{-1} = (-I, (A)) \end{array}\right),$$

also ein Paar von Quadrupeln. Drückt man dieses in Form von relationalen Einbettungszahlen (vgl. Toth 2012) aus, so erhält man

$$\left(\begin{array}{ll} S_1 = (-1, (1)) & S_1^{-1} = ((1), -1) \\ S_2 = ((-1), 1) & S_2^{-1} = (1, (-1)) \end{array}\right)$$

$$\left(\begin{array}{ll} S_1 = (1, (-1)) & S_1^{-1} = ((-1), 1) \\ S_2 = ((1), -1) & S_2^{-1} = (-1, (1)) \end{array}\right),$$

d.h. in drei Zählweisen zweidimensionale Zahlen, die sowohl positive als auch negative Werte auf beiden Einbettungsstufen und für beide möglichen Positionen besitzen:

Adjazente Zählweise

$$\begin{array}{cccccccccccc} \pm1 & \pm1 & & \pm1 & \pm1 & & \pm1 & \pm1 & & \pm1 & \pm1 \\ \emptyset & \emptyset & & \emptyset & \emptyset & & \emptyset & \emptyset & & \emptyset & \emptyset \\ & & \times & & & \times & & & \times & & \\ \emptyset & \emptyset & & \emptyset & \emptyset & & \emptyset & \emptyset & & \emptyset & \emptyset \\ \pm1 & \pm1 & & \pm1 & \pm1 & & \pm1 & \pm1 & & \pm1 & \pm1 \end{array}$$

Subjazente Zählweise

±1 Ø   Ø ±1   Ø ±1   ±1 Ø

±1 Ø   Ø ±1   Ø ±1   ±1 Ø

× × ×

±1 Ø   Ø ±1   Ø ±1   ±1 $Ø_i$

±1 Ø   Ø ±1   Ø ±1   ±1 $Ø_i$

Transjazente Zählweise

±1 Ø   Ø ±1   Ø ±1   ±1 Ø

Ø ±1   ±1 Ø   ±1 Ø   Ø ±1

× × ×

Ø ±1   ±1 $Ø_j$ ±1 Ø   Ø ±1

±1 Ø   Ø ±1   Ø ±1   ±1 Ø,

Da im Bildnis des Dorian Gray Bild, d.h. Zeichen, und Person, d.h. Objekt, vertauscht werden, sind somit nicht nur alle reflexiven, sondern auch alle chiastischen Relationen aller drei ontischen Zahlenfelder vertauscht.

Adjazente Zählweise

±1 ±1   ±1 ±1   ±1 ±1   ±1 ±1

Ø Ø   Ø Ø   Ø Ø   Ø Ø

↕ ↗↙ ↕ ↗↙ ↕ ↗↙ ↕

Ø Ø   Ø Ø   Ø Ø   Ø Ø

±1 ±1   ±1 ±1   ±1 ±1   ±1 ±1

Subjazente Zählweise

± 1	$\varnothing$	$\varnothing$	± 1	$\varnothing$	± 1	± 1	$\varnothing$
± 1	$\varnothing$	$\varnothing$	± 1	$\varnothing$	± 1	± 1	$\varnothing$

↕ ↗↙ ↕ ↗↙ ↕ ↗↙ ↕

± 1	$\varnothing$	$\varnothing$	± 1	$\varnothing$	± 1	± 1	$\varnothing_i$
± 1	$\varnothing$	$\varnothing$	± 1	$\varnothing$	± 1	± 1	$\varnothing_i$

Transjazente Zählweise

± 1	$\varnothing$	$\varnothing$	± 1	$\varnothing$	± 1	± 1	$\varnothing$
$\varnothing$	± 1	± 1	$\varnothing$	± 1	$\varnothing$	$\varnothing$	± 1

↕ ↗↙ ↕ ↗↙ ↕ ↗↙ ↕

$\varnothing$	± 1	± 1	$\varnothing_j$	± 1	$\varnothing$	$\varnothing$	± 1
± 1	$\varnothing$	$\varnothing$	± 1	$\varnothing$	± 1	± 1	$\varnothing$.

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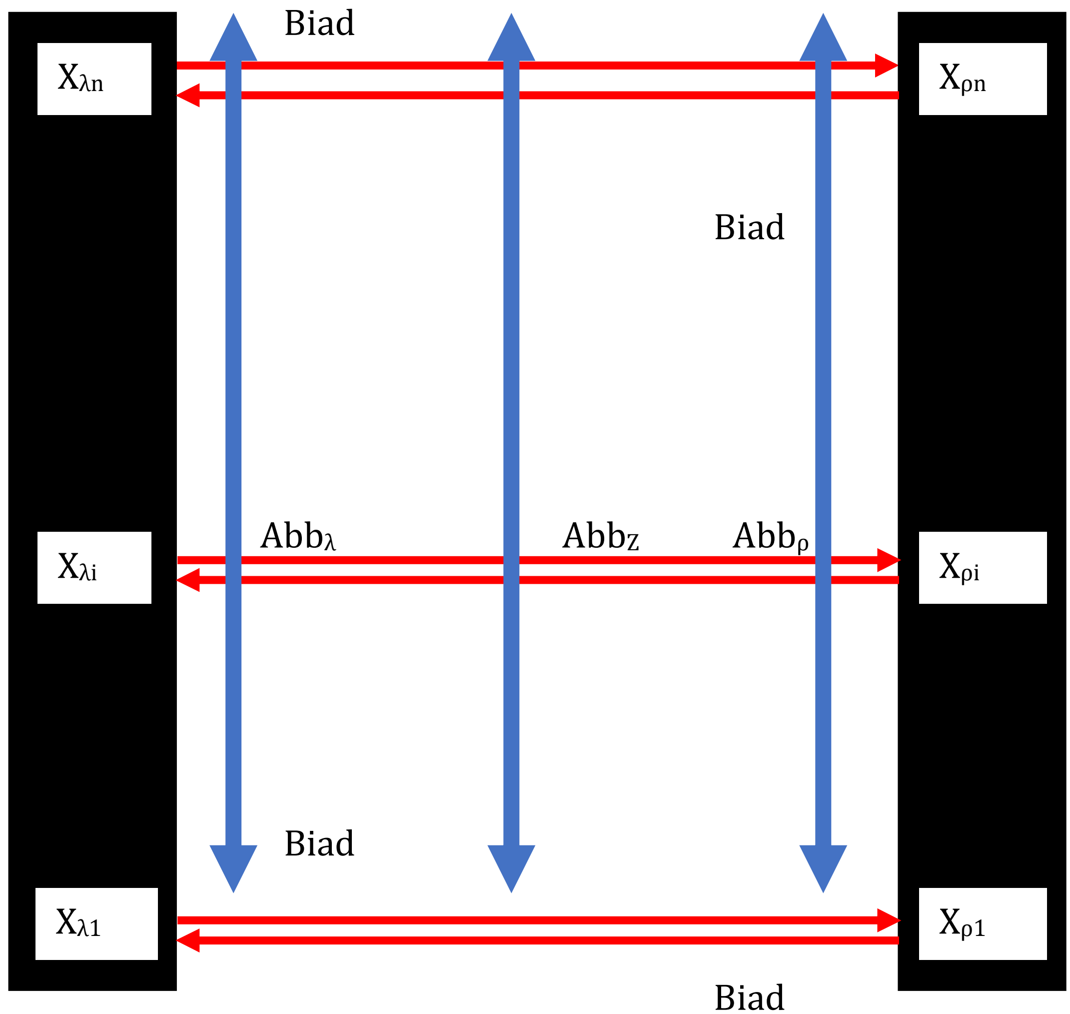
Colinearität und Biadessivität

1. In Toth (2018a) wurde Colinearität von ontischen Abbildungen durch

$O = (X_\lambda, (Abb_Z)_n, Z_\rho)$,

wobei n ≥ 1 ist, definiert und das folgende ontotopologische Modell dazu eingeführt.



2. Im folgenden wird gezeigt, daß dieses Modell so allgemein ist, daß es durch einfache raumsemiotische (vgl. Bense/Walther 1973, S. 80) Adaptationen als Modell für Biadessivität (vgl. Toth 2014a) dienen kann. In einem weiteren Schritt wird gezeigt, daß das gleiche Modell kompatibel ist mit der Vermittlungsfunktion von Rändern (vgl. Toth 2018b), denn die Dichotomie von Außen und Innen ist, da sie die Existenz einer „Differenz“ bzw. eines ontischen Etwas

voraussetzt, welche überhaupt erst Außen und Innen sich voneinander unterscheiden läßt, nicht-isomorph zur logischen Basisdichotomie

$L = (0, 1)$,

für die bekanntlich gilt

$L = (0, 1) = L^{-1} = (1, 0)$,

da es ja egal ist, welcher der beiden Werte als Position bzw. als Negation designiert wird. D. h. es ist nicht von

$S = (A, I)$,

sondern von

$S^* = (A, R, I)$

mit

$R(0, 1) \neq R(1, 0) \neq Ø$

auszugehen, während für $L = (0, 1)$ natürlich gilt

$R(0, 1) = R(1, 0) = Ø$.

Ohne jedoch das Tertiumgesetz zu verletzen, gibt es, wie bereits in Toth (2015) dargelegt wurde, die Möglichkeit, statt einem materiellen ein differentielles „Tertium" einzuführen. Dafür benötigen wir einen Einbettungsoperator E (vgl. Toth 2014).

$E \rightarrow F = (0, 1) \neq F^{-1} = (1, 0) =$

$$\begin{pmatrix} L_1 = (0, (1)) & L_1^{-1} = ((1), 0) \\ L_2 = ((0), 1) & L_2^{-1} = (1, (0)) \end{pmatrix}.$$

Für jedes L_i gilt somit

$0 = f(1)$

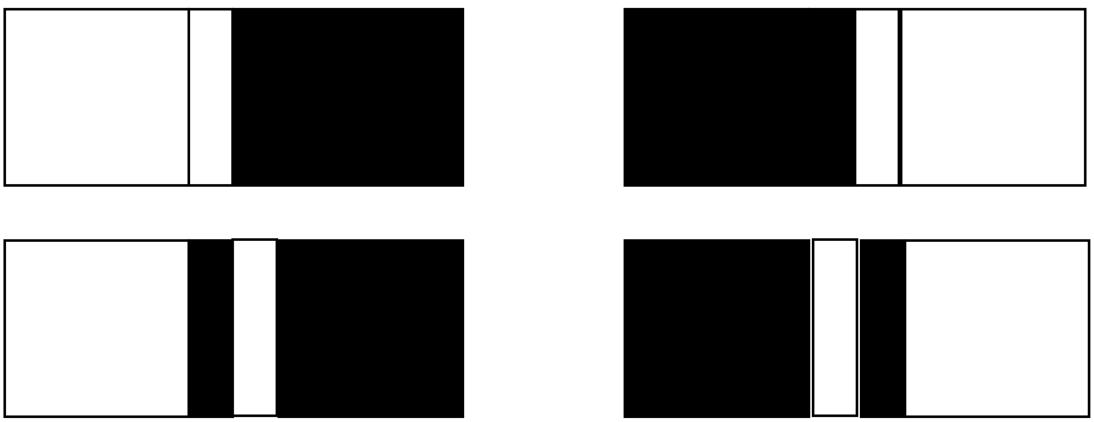
$1 = f(0)$,

und somit ist

$(x \in 0) \subset 1$

$(y \in 1) \subset 0,$

d.h. 0 hat 1-Anteile, und 1 hat 0-Anteile. Man kann diese durch E erwirkte Abbildung der Paare auf Quadrupel schematisch wie folgt darstellen



.

Diese Ränder, welche vermöge Isomorphie auch für die übrigen klassischen Dichotomien wie Position und Negation, Objekt und Zeichen, Objekt und Subjekt usw. gelten, die somit als durch E vermittelte Relationen eingeführt werden, lassen sich nun als biadessive Relationen wie folgt definieren

Biad = (X, R, Y)

mit

$R(X, Y) \neq R(Y, X) \neq \emptyset,$

wodurch X und Y nun nicht mehr, wie in L, spiegelbildlich, sondern durch die durch E induzierte Einbettung positionsgebunden sind. Ontisch gesehen ist dieser Sachverhalt unmittelbar klar: Das Innere eines Hauses ist natürlich dem Äußeren ungleich. Entsprechend ist die Konversion der Randrelation, in diesem Falle eine Umstülpung, möglich.

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Dieses Buch schließt – unüblicherweise – mit einer Aufgabe:

Vermöge des Satzes von Wiener und Kuratowski (1917) läßt sich jedes n-tupel als Paar darstellen. Man beweise nun, daß sich jedes Paar als Quadrupel, d.h. als quadralektische Relation, darstellen läßt.